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PROBABILITY DISTRIBUTION OF CONSERVED CHARGES IN THE PRESENCE OF THE CHIRAL PHASE TRANSITION*

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We discuss the influence of the chiral phase transition on the structure of the probability distributions of conserved charges within the quark–meson model based on the functional renormalization group approach. By considering the ratio of the probability distribution of the net-baryon number to the Skellam function, we quantify characteristic features of the distribution that are related to the $O(4)$ criticality at the chiral crossover. We explore the corresponding ratios for data obtained at RHIC by STAR Collaboration and discuss their possible interpretation.

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1. Introduction

One of the goals of experiments with ultrarelativistic heavy ion collisions is to explore the structure of the QCD phase diagram. Fluctuations of conserved charges were shown to be promising observables to study remnants of the critical phenomena in relativistic heavy ion collisions due to

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deconfinement of quarks and the chiral symmetry restoration [1, 2]. A particular role was attributed to the higher order cumulants of the net-baryon number and electric charge fluctuations [2–4].

At small values of the quark chemical potential, the Lattice QCD (LQCD) calculations show, that there is a chiral crossover, which appears in the critical region of the second order phase transition belonging to the universality class of the 3-dimensional $O(4)$ spin systems [5]. Thus, a promising approach for probing the phase boundary in heavy ion collisions is to explore the fluctuations of the chiral phase transition, assuming the $O(4)$ criticality.

Owing to the proximity of the chemical freeze-out to the chiral crossover, at small values of the baryonic chemical potential, one may expect that the critical fluctuations are reflected in data on conserved charges [6]. A baseline for the non-critical properties of the cumulants of charge fluctuations is provided by the hadron resonance gas (HRG), which reproduces the particle yields at chemical freeze-out in heavy ion collisions [7], as well as the LQCD equation of state in the hadronic phase [8].

Fluctuations of conserved charges are directly linked to the corresponding probability distribution $P(N)$. Thus, the critical properties of cumulants of conserved charges must be also reflected in the probability distribution. The baseline for the non-critical behavior of the probability distribution is constituted by the Skellam function which is derived from the HRG at the chemical freeze-out.

Recently, the effect of the chiral phase transition on the net-baryon number probability distribution was examined within the framework of the mean-field and scaling theory of phase transitions [9], as well as within the effective chiral models [10, 11]. It was found that the critical behavior of the cumulants is a direct consequence of the change of the corresponding probability distribution.

In this contribution, we summarize the main results which indicate the essential properties of the net-baryon number probability distribution due to remnants of the $O(4)$ criticality. In particular, we show that at vanishing chemical potential, the residual $O(4)$ critical fluctuation at physical pion mass leads to a specific narrowing of the probability distribution, relative to the Skellam function. This corresponds to a negative structure of the sixth order cumulant at the chiral crossover [10]. At finite chemical potential, the ratio of $P(N)$ to the Skellam function, exhibits a characteristic asymmetry around the mean charge value M. For $N < M$, the probability ratio is enhanced near the $O(4)$ pseudocritical point, while for $N > M$ it is suppressed. This asymmetry of the distribution ratio is enhanced with increasing chemical potential along the chemical freeze-out line.

The influence of the $O(4)$ criticality on the probability distribution of the net-baryon number is calculated at finite temperature and quark–chemical potential, and at a physical value of the pion mass in the quark–meson model within the functional renormalization group scheme [9–11].

2. Modeling the net-baryon number probability distribution

In the grand canonical ensemble specified by the temperature T, sub-volume V and chemical potential μ, the probability distribution for the conserved charge N is given by

$$P(N;T,V,\mu) = \frac{Z(T,V,N)e^{\mu N/T}}{\mathcal{Z}(T,V,\mu)}\,, \tag{1}$$

where the canonical partition function $Z(T,V,N)$ is obtained by a projection of the grand partition function $\mathcal{Z}(T,V,\mu)$, as [7]

$$Z(T,V,N) = \frac{1}{2\pi}\int\limits_0^{2\pi} d\left(\frac{\mu_{\mathrm{I}}}{T}\right) e^{-iN\frac{\mu_{\mathrm{I}}}{T}}\,\mathcal{Z}(T,V,\mu=i\mu_{\mathrm{I}})\,. \tag{2}$$

The above equations link the grand canonical partition function in a finite volume V, at imaginary chemical potential, to thermodynamics at fixed net charge N, and its probability distribution.

We extract the characteristic features of the probability distribution near the chiral crossover, within the $O(4)$ universality class, by applying the Functional Renormalization Group (FRG) approach to the quark–meson (QM) model [12]. At vanishing and moderate values of μ, the quark–meson model in the chiral limit exhibits the second order phase transition, belonging to the $O(4)$ universality class [13]. For a physical pion mass, the chiral symmetry is explicitly broken and the transition is of the crossover type. Nevertheless, remnants of the $O(4)$ criticality remain in various observables [6]. Thus, also the probability distribution of the net-quark number is expected to exhibit characteristic features reflecting the critical behavior of the underlying $O(4)$ transition.

We obtain the thermodynamics of the quark–meson model by computing the thermodynamic potential within the FRG approach, as discussed in Ref. [13]. Following [12], we consider the scale dependent effective action in the local potential approximation. Using the so-called optimized cutoff functions, one obtains the evolution equation for the scale dependent thermodynamic potential density $\Omega_k(\rho)$, with the reduced field variable $\rho = (\sigma^2 + \vec{\pi}^2)/2$, as [13]

$$\begin{aligned}\frac{12\pi^2}{k^4}\partial_k\Omega_k(\rho) &= \frac{3}{E_\pi}\left\{1+2n_{\rm B}(E_\pi)\right\}\\ &+\frac{1}{E_\sigma}\left\{1+2n_{\rm B}(E_\sigma)\right\}-\frac{2\nu_q}{E_q}\left\{1-n_{\rm F}\left(E_q^+\right)-n_{\rm F}(E_q^-)\right\}\,,\end{aligned} \tag{3}$$

where $n_{\rm B}$ and $n_{\rm F}$ are the Bose and the Fermi distribution functions, respectively and $\nu_q = 2N_{\rm c}N_{\rm f} = 12$ is the quark degeneracy. The single particle energies of pion, sigma meson and quark/antiquark are given by

$$E_\pi = \sqrt{k^2+\bar{\Omega}'_k}\,,\quad E_\sigma = \sqrt{k^2+\bar{\Omega}'_k+2\rho\bar{\Omega}''_k}\,,\quad E_q^\pm = \sqrt{k^2+2g^2\rho}\pm\mu\,, \tag{4}$$

where $\bar{\Omega}'_k$ and $\bar{\Omega}''_k$ denote the first and the second derivatives of $\bar{\Omega}_k = \Omega_k + h\sqrt{2\rho_k}$, with respect to ρ. The flow equation (3) is solved by using the Taylor expansion method. Expanding the potential up to the third order in ρ around the scale dependent potential minimum ρ_k,

$$\Omega_k(\rho) = \sum_{n=0}^{3}\frac{a_n(k)}{n!}(\rho-\rho_k)^n\,, \tag{5}$$

and using Eq. (3), one finds the flow equations for the coefficients

$$\begin{aligned} d_k a_{0,k} &= \frac{c}{\sqrt{2\rho_k}}\,d_k\rho_k + \delta_k\Omega_k\,,\\ d_k\rho_k &= -\frac{1}{\left(c/(2\rho_k)^{3/2}+a_{2,k}\right)}\delta_k\Omega'_k\,,\\ d_k a_{2,k} &= a_{3,k}\,d_k\rho_k + \delta_k\Omega''_k\,,\\ d_k a_{3,k} &= \delta_k\Omega'''_k\,, \end{aligned} \tag{6}$$

where $d_k = d/dk$. The flow equations are solved numerically starting at the ultraviolet cutoff scale $\Lambda = 1.0$ GeV [13]. We eliminate a_1 by means of the scale independent relation, $h = a_1(k)\sqrt{2\rho_k}$.

There are four initial conditions for the flow equations which are fixed at the scale $k=\Lambda$. Within this scheme, the initial value of a_0 is just a constant shift of thermodynamic potential density Ω. We note, however, that such a cutoff at a finite momentum leads to unphysical behavior of thermodynamic quantities at high temperatures. This problem can be amended by accounting for the μ- and T-dependent contribution of the momenta beyond the cutoff scale [3]. Following Ref. [3], we include the high-momentum contribution approximately by using the flow equation for non-interacting massless quarks and gluons,

$$\partial_k \Omega_k^\Lambda(T,\mu) = \frac{k^3}{12\pi^2}\left\{2\left(N_c^2-1\right)\left[1+2n_{\rm B}(k)\right] -\nu_q\left[1-n_{\rm F}\left(k^+\right)-n_{\rm F}\left(k^-\right)\right]\right\} . \tag{7}$$

By integrating the flow equation (7) from $k=\infty$ to $k=\Lambda$, we obtain $\Omega^\Lambda(T,\mu)$ which is then used as an initial condition $a_0(\Lambda)$ for the solution of the flow equations (6).

We set $a_3(\Lambda)=0$ and fix $\rho_{k=\Lambda}$ and $a_2(\Lambda)$ by requiring, that in vacuum, the pion $m_\pi=135$ MeV and the sigma $m_\sigma=640$ MeV masses are reproduced. The strength of the Yukawa coupling, $g=3.2$, is fixed by the constituent quark mass, $M_q(T=\mu=0)=g\sigma_{k=0}(T=\mu=0)=300$ MeV with $\sigma_{k=0}(T=\mu=0)=f_\pi=93$ MeV. The full thermodynamic potential density of the quark–meson model $\Omega(T,\mu)$, which includes thermal and quantum fluctuations of the meson and quark fields, is then obtained from $\Omega(T,\mu)=\lim_{k\to 0}\Omega_k$, where Ω_k is the solution of the flow equation (3).

With the grand canonical thermodynamic potential density obtained in the FRG approach, $\Omega=-(T/V)\log\mathcal{Z}$, we calculate the canonical partition function (2) and the corresponding probability distribution from Eq. (1). The $P^{\rm QM}(N)$ obtained in this way, contains all information about criticality at the $O(4)$ chiral crossover.

3. The O(4) criticality in the probability distribution

To unravel the influence of the chiral transition on the probability distribution $P^{\rm QM}(N)$, one needs to establish the reference distribution, which does not include the effect of critical fluctuations. At temperatures below the pseudocritical or chiral crossover temperature $T_{\rm pc}$, the thermodynamic potential is well described by the quasi-ideal quark gas with a dynamically generated mass. Consequently, at fixed T and V, the natural reference for $P(N)$ is the probability distribution of an ideal gas of quarks and antiquarks, *i.e.* the Skellam distribution [14]. The Skellam distribution is then determined entirely by the mean number of quarks $b=\langle N_q\rangle$ and antiquarks $\bar{b}=\langle N_{\bar{q}}\rangle$

$$P^{\rm S}(N)=\left(\frac{b}{\bar{b}}\right)^{N/2} I_N\left(2\sqrt{b\bar{b}}\right)e^{-\left(b+\bar{b}\right)}\,, \tag{8}$$

where $I_N(x)$ is the modified Bessel function of the first kind.

In the following, we compare the probability distribution $P^{\rm QM}(N)$ with the non-critical Skellam function $P^{\rm S}(N)$, constructed with the same mean and variance. At the $O(4)$ chiral transition, both the mean and variance are finite, thus the above comparison is the sensitive method to identify criticality which appears in the tail of the net-quark distribution.

In general, the probability distribution $P(N)$ depends on the volume parameter. However, as shown in [9], the volume dependence of the rescaled distribution $\sqrt{V}P(N/\sqrt{V})$ is strongly reduced. This approximate scaling is valid for both, the Skellam and the $P(N)$ distribution calculated within the QM model. Thus, in the ratios of $P^{\rm QM}(N)$ and Skellam, the leading volume dependence is canceled.

Figure 1 (left) shows the ratio of $P^{\rm QM}(N)$, computed in the quark–meson model at $\mu = 0$ within the FRG approach [10], and the Skellam distribution $P^{\rm S}(N)$, calculated with the same mean M and variance σ, as a function of $\delta N/N_6$. We have normalized $\delta N = N - M$ by the minimal value of the net-quark number $N > N_6$ which is needed in the Skellam distribution $P^{\rm S}(N)$ to saturate the sixth order cumulant [11].

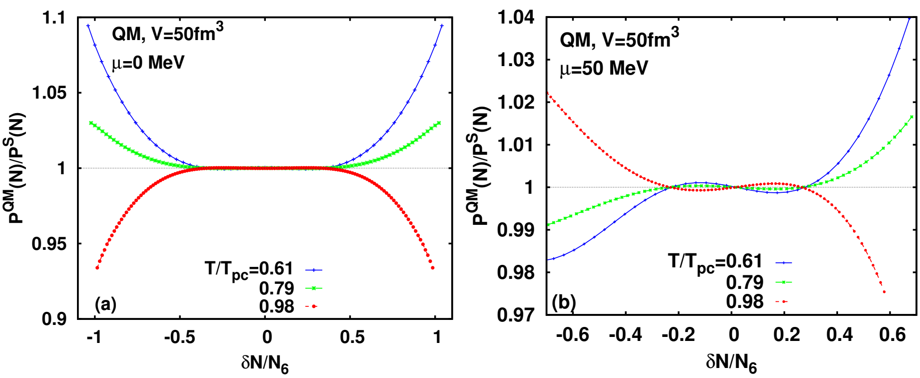

Fig. 1. The ratio of the probability distribution obtained in the quark–meson model $P^{\rm QM}(N)$ and the Skellam distribution $P^{\rm S}(N)$ with the same mean and variance as $P^{\rm QM}(N)$. The left panel shows the ratio at $\mu = 0$ for different temperatures $T/T_{\rm pc}$, expressed in units of the pseudocritical temperature $T_{\rm pc}$, while in the right panel, shows the same ratio at $\mu = 50$ MeV. The δN and N_6 are introduced in the text.

The $P^{\rm QM}/P^{\rm S}$ ratio in figure 1 (left) is shown for different temperatures normalized by $T_{\rm pc}$. This ratio exhibits a characteristic dependence on temperature, as $T_{\rm pc}$ is approached from below. For $T \simeq T_{\rm pc}$, the $P^{\rm QM}(N)/P^{\rm S}(N)$ is less than unity, indicating the narrowing of the probability distribution for larger $|\delta N|$, owing to the $O(4)$ criticality. Indeed, the decrease of the probability ratio for $\delta N/N_6 \simeq 1$ near $T_{\rm pc}$ is responsible for the negative values of the sixth order cumulant, which are characteristic of the chiral crossover in the $O(4)$ universality class [10]. Consequently, shrinking probability distribution, relative to the Skellam function, can indeed be considered as a necessary condition to observe the $O(4)$ criticality [10].

At finite chemical potential, the probability distribution $P(N)$ of the net-baryon number, is no longer symmetric around the mean. Thus, it is not *a priori* clear, how the distribution is modified by the $O(4)$ criticality.

The asymmetry of $P(N)$ at $\mu \neq 0$, appears due to the fugacity factor $e^{\mu N/T}$ in Eq. (1), which suppresses the contribution from $N < 0$ and enhances that from $N > 0$. Consequently, at finite chemical potential, the tail of the probability distribution $P(N)$ is enhanced, and criticality is expected to appear at smaller $|\delta N|$, and thus also in lower order cumulants.

Figure 1 (right) shows the $P^{\mathrm{QM}}(N)/P^{\mathrm{S}}(N)$ ratio obtained in the QM model at $\mu = 50$ MeV. Below the pseudocritical temperature T_{pc}, the distribution is asymmetric, with an enhanced tail relative to the Skellam function for positive and suppressed tail for negative δN. However, as T_{pc} is approached, there is a qualitative change of the properties of the distribution, resulting in the narrowing for positive δN. Moreover, a comparison of Fig. 1 (left) and (right) shows, that at finite μ, the narrowing of $P(N)$ starts at smaller $\delta N/N_6$. The stronger narrowing of the distribution is consistent with the fact, that at finite μ, already the third cumulant, exhibits the $O(4)$ critical behavior. On the other hand, for negative δN the ratio of the distributions exhibits the opposite behavior, reflecting the asymmetry of the probability distribution at non-zero net-baryon density. The narrowing of the probability ratio seen in Fig. 1 (right) is the characteristic feature of $P(N)$ due to the $O(4)$ criticality. Evidently, the deviation of $P^{\mathrm{QM}}(N)/P^{\mathrm{S}}(N)$ from unity near $T_{\mathrm{pc}}(\mu)$ is increasing with increasing $|\delta N/N_6|$ and μ.

4. The O(4) criticality and heavy ion collisions

In heavy ion collisions, particle yields, charge densities and their variance are described consistently by the HRG model on the same chemical freeze-out line in the (T, μ_{B})-plane [6, 15]. For a given collision energy and centrality, there is a unique point on the freeze-out line. If the freeze-out takes place sufficiently close to the chiral crossover, the critical fluctuations are expected to leave a characteristic imprint in the cumulants and the corresponding probability distributions.

In Fig. 2 (left), we illustrate the expected structure in the QM model of the probability distribution at chemical freeze-out, in the (T, μ) plane. The freeze-out line was defined by the condition of a fixed variance of the net-baryon number per unit volume. The μ-dependence of the probability ratio, with the narrowing of the distribution for positive and broadening for negative δN with increasing μ, is characteristic for the critical region. As shown in Fig. 1, the distribution in a non-critical system exhibits the opposite trend, with broadening for positive and narrowing for negative δN.

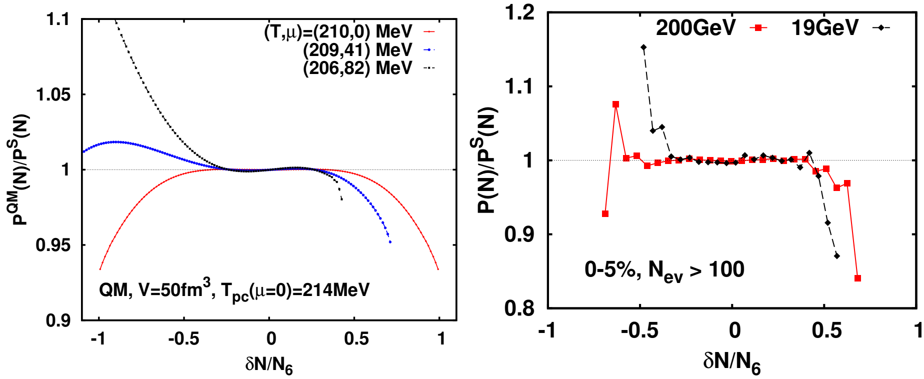

Fig. 2. Left figure: The probability ratio $P^{\mathrm{QM}}(N)/P^{\mathrm{S}}(N)$ for different (T,μ) points. The points lie on an approximate freeze-out line, specified in the text. Right figure: Ratios of the efficiency uncorrected probability distributions of the net-proton number $P(N)$ by STAR Collaboration [15] to the Skellam function $P^{\mathrm{S}}(N)$ with the same mean and variance as $P(N)$. The data are for the most central Au–Au collisions, with the number of events $N_{\mathrm{ev}} > 100$.

In general, the measurement of higher order cumulants, which are particularly sensitive to criticality, needs high statistics data owing to the increasing importance of the tail of the distribution. Furthermore, the experimental conditions, such as *e.g.* the acceptance corrections, must be under control to make a meaningful comparison of the measured cumulants and their probability distribution with theoretical predictions [16–18].

Recently, STAR Collaboration has presented results on the probability distribution of the net-proton number and the corresponding cumulants up to the fourth order, measured in heavy ion collisions for different energies and centralities [15]. Also preliminary results on the sixth order cumulant have been presented by STAR [18].

While the cumulant ratios measured by STAR [15] were efficiency corrected and tested against possible modifications due to volume fluctuations and accepted kinematical windows, the data on the probability distributions of the net-proton number are *uncorrected*. Therefore, the significance of a direct comparison of model predictions with the measured probability distribution is, *a priori*, not robust.

Nevertheless, we have verified, that data on $P(N)$ obtained by STAR [15] are dominated by physics. The contribution of volume fluctuations is small, and the ratios of cumulants computed directly from the uncorrected $P(N)$ [15], exhibit similar systematics as the efficiency corrected ratios. This means, that data may yield at least a qualitative indication, whether the measured distributions contain some remnants of the chiral criticality.

Figure 2 (right) shows the probability ratio $P(N)/P^{\rm S}(N)$ obtained from the uncorrected data in the highest centrality bin, for $\sqrt{s_{NN}} = 200$ and 19.6 GeV [15]. The probability ratio is constructed using the same method as in Fig. 1. In order to avoid large uncertainties, we have restricted to data with more than 100 events. Consequently, the probability distributions are limited to $|\delta N/N_6| < 0.5$. This implies, that the present statistics does not allow a reliable estimate of the sixth order cumulant. Nonetheless, the ratios in Fig. 2 (right) clearly exhibit structures, which are qualitatively similar to that shown in Figs. 1 and 2 (left). Thus, such structure in the net-proton number probability distribution, could be the first indication of the underlying $O(4)$ criticality in heavy ion data. In particular, the narrowing of the probability distribution relative to the Skellam function for the positive δN, and an earlier drop of the ratio below unity for $\sqrt{s} = 19.6$ GeV, are characteristic signatures expected from the $O(4)$ criticality.

There are several potential contributions to the cumulants and the probability distribution from sources other than critical fluctuations [19, 21, 22], as well as experimental issues *e.g.* regarding efficiency corrections [20]. Thus, the final conclusion on the criticality of $P(N)$ can be drawn only, when the role of these effects is established.

In [15], the different cumulant ratios were analyzed with efficiency and centrality bin width corrections. By constructing such cumulant ratios from the uncorrected $P(N)$ data discussed above, we have found, that deviations from the Skellam distribution are slightly smaller than that seen in the corrected ratios. However, the systematics and the energy dependence is almost the same. Therefore, we regard the results shown in Fig. 2 (right) as the lower limit for the possible deviations from the Skellam function.

The present method provides a transparent framework to search for criticality in the probability distribution of conserved charges. If the narrowing of $P(N)$ relative to the Skellam, as seen in Fig. 2 (right), is still observed after all experimental corrections are included, then this will provide an evidence for remnants of the chiral crossover transition in the experimental data.

5. Summary

We have discussed the properties of the net-baryon number probability distribution $P(N)$ near the chiral crossover at vanishing and at finite baryon chemical potential. The critical properties of $P^{\rm QM}(N)$ in the quark–meson model were obtained within the functional renormalization group approach.

We have shown that the ratio of $P^{\rm QM}(N)$ to Skellam function $P^{\rm S}(N)$, constructed with the same mean M and variance as $P^{\rm QM}(N)$, clearly exhibits the influence of the $O(4)$ criticality. At vanishing chemical potential, there is a characteristic reduction of this ratio below unity near the pseudocritical phase boundary. At finite chemical potential, the ratio — $P^{\mathrm{QM}}(N)/P^{\mathrm{S}}(N)$ exhibits the characteristic asymmetry in $\delta N = N - M$. For $\delta N < 0$, the probability ratio is enhanced near the $O(4)$ pseudocritical point, while for $\delta N > 0$ it is suppressed. Such asymmetry of the distribution is enhanced with increasing μ along the freeze-out line.

The potential relevance of these results in heavy ion experiments was also discussed. We have constructed the corresponding ratios of the net-proton number distributions obtained by the STAR Collaboration and Skellam function and discussed their interpretation. We found, that presently available and still efficiency uncorrected data on the probability distribution, are qualitatively consistent with the expectation, that there are remnants of the $O(4)$ criticality in the tail of distributions. However, to make the final conclusion one would need the efficiency corrected and high statistic data.

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