

PROBING DECONFINEMENT WITH POLYAKOV LOOP SUSCEPTIBILITIES*

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We present new lattice results on Polyakov loop susceptibilities in the $SU(3)$ pure gauge system. These observables reflect the spontaneous breaking of $Z(3)$ center symmetry and can serve as excellent probes for deconfinement. An effective model is formulated for the Polyakov loop, with its parameters constrained by existing quenched lattice data, including fluctuations.

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1. Introduction

Deconfinement can be described by the spontaneous breaking of $Z(3)$ center symmetry [1–5]. This symmetry, however, is explicitly broken by dynamical quarks. The problem here is that string breaking will occur at long distances for the static quark potential, even when the temperature is below criticality. If one aims at studying the confining part of the heavy quark potential, and probe the related symmetry breaking phase transition, it is useful to focus first on QCD in the limit of exact $Z(3)$ symmetry — $SU(3)$ pure gauge theory.

The relevant observables to study deconfinement are the Polyakov loop and its susceptibilities. The Polyakov loop measures the free energy of a static quark immersed in a hot gluonic medium [5, 6], and can be used to define an order parameter for the deconfinement transition. At low temperatures its thermal expectation value vanishes, signaling color confinement; at high temperatures it is nonzero, resulting in a finite energy of a static quark

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and consequently the deconfinement of color. The Polyakov loop susceptibility, on the other hand, represents fluctuations of the order parameter. It features a peak at the transition temperature, and a width that signals the temperature window in which phase change occurs.

The basic thermodynamic functions of the SU(3) pure gauge theory, such as pressure and entropy, are well established within the lattice approach [7, 8]. However, it is less clear for the temperature dependence of the renormalized Polyakov loop and its susceptibilities. Careful study of these quantities can enhance our understanding of the QCD phase structure.

2. Polyakov loop and its susceptibilities on the lattice

On an $N_\sigma^3 \times N_\tau$ lattice, the Polyakov loop is defined as the trace of the product over temporal gauge links

$$L_{\vec{x}}^{\text{bare}} = \frac{1}{3} \text{Tr} \prod_{\tau=1}^{N_\tau} U_{(\vec{x}, \tau), 4}, \quad (1)$$

$$L^{\text{bare}} = \frac{1}{N_\sigma^3} \sum_{\vec{x}} L_{\vec{x}}^{\text{bare}}. \quad (2)$$

The bare Polyakov loop requires renormalization to give a physical, N_τ -independent result. We perform the following multiplicative renormalization [9]

$$L^{\text{ren}} = (Z(g^2))^{N_\tau} L^{\text{bare}}, \quad (3)$$

and introduce the ensemble average of its modulus, $\langle |L^{\text{ren}}| \rangle$. This quantity is well defined in the continuum and thermodynamic limits, and is an order parameter for the spontaneous breaking of Z(3) center symmetry.

We now define the corresponding susceptibilities. In the SU(3) gauge theory, the Polyakov loop operator is complex. One can explore its fluctuations along the longitudinal and transverse directions, as well as that of its absolute value

$$T^3 \chi_L = \frac{N_\sigma^3}{N_\tau^3} \left[\langle (L_L^{\text{ren}})^2 \rangle - \langle L_L^{\text{ren}} \rangle^2 \right], \quad (4)$$

$$T^3 \chi_T = \frac{N_\sigma^3}{N_\tau^3} \left[\langle (L_T^{\text{ren}})^2 \rangle - \langle L_T^{\text{ren}} \rangle^2 \right], \quad (5)$$

$$T^3 \chi_A = \frac{N_\sigma^3}{N_\tau^3} \left(\langle |L^{\text{ren}}|^2 \rangle - \langle |L^{\text{ren}}| \rangle^2 \right), \quad (6)$$

where $L_L = \text{Re}(\tilde{L})$ and $L_T = \text{Im}(\tilde{L})$. Here, we have introduced the Z(3)-transformed Polyakov loop, $\tilde{L} = L e^{2\pi ni/3}$, with $n = 0, \pm 1$. The phase of the transformation is chosen such that the transformed Polyakov loop is located in the main sector, defined by $-\pi/3 < \arg(\tilde{L}) < \pi/3$.

We note the factor of N_τ^{-3} in defining the various susceptibilities. This is required to formulate the quantities in the continuum. It is completely analogous to the standard procedure of multiplying factor of N_τ^4 in extracting the continuous free energy density f/T^4 on the lattice [10]. Our lattice results for these susceptibilities are collected in Fig. 1.

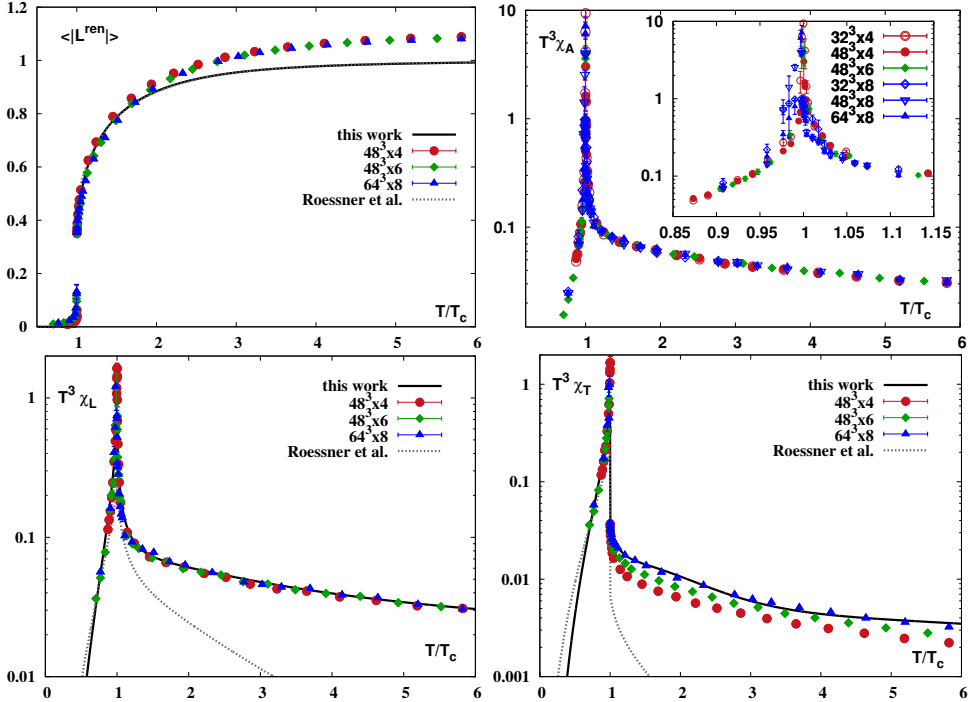


Fig. 1. The temperature dependence of the modulus of renormalized Polyakov loop and its susceptibilities in the SU(3) pure gauge theory. The lines are model results discussed in Sec. 3.

Many features of the susceptibilities can be interpreted by Z(3) center symmetry and properties of Polyakov loop distribution function [11]. One quantity of interest to study deconfinement is the relative strength of transverse to longitudinal susceptibility, *i.e.* $R_T = \chi_T/\chi_L$.

Deep in the confining region, we find that $\chi_L \approx \chi_T$. This is naturally explained by the restriction of symmetry. As the vacuum is Z(3)-symmetric below the critical temperature T_c , the expectation value of any symmetry breaking operator must vanish. In particular,

$$V \left(\langle \tilde{L}^2 \rangle - \langle \tilde{L} \rangle^2 \right) = \chi_L - \chi_T = 0. \quad (7)$$

It follows that $\chi_L = \chi_T$ in the confining phase.

This simple observation defies any perturbative treatment of the correlation functions. From the perturbative point of view, longitudinal and transverse correlators are given by two and three gluon exchange respectively [12]. The lattice results, on the contrary, suggest that they are determined by the same non-perturbative scale. There are some evidences that such a long distance scale is set by the effective string tension [13]. Further study to sort out their relations is underway.

At high temperatures, $Z(3)$ symmetry is spontaneously broken and we observe $\chi_L \gg \chi_T$. The fact that they are distinct can be interpreted as a signal for deconfinement. Their relative sizes, however, are not restricted by symmetry. What can be inferred from the data is that correlation function of the transverse (imaginary) part of the Polyakov loop is more heavily screened than that of the longitudinal (real) direction in the deconfined medium. This aspect of screening property has been confirmed by other lattice group [14]. We note, however, that a residual N_τ dependence remains in our results. Therefore, we cannot yet draw any firm conclusions about the continuum extrapolation of R_T .

Further work is needed for the physical interpretations of these observables, and to analyze in detail the systematic uncertainties involved in extracting these quantities.

3. Effective models for the Polyakov loop

Effective potentials, with quarks and Polyakov loop as degrees of freedom, have been constructed to describe QCD thermodynamics [15–20]. Many of these models have their parameters tuned to match the lattice results on thermodynamic pressure and Polyakov loop. Fluctuation effects, on the other hand, have yet to be included. The new susceptibility data here can help to better constrain these seemingly arbitrary model parameters [21].

We focus first on the most commonly used Polyakov loop models: the polynomial [15] and the logarithmic potential [16], the latter imposes the restriction of $SU(3)$ Haar measure. While both models by construction can describe the lattice data on equation of state and Polyakov loop, they predict very different results for the susceptibilities. In particular, the polynomial model asserts $R_T > 1$ for $T > T_c$, while the logarithmic model gives $R_T < 1$. In this regard, the lattice data presented here clearly prefers the latter. This is also in line with theoretical expectation: latter model is preferred as it restricts the Polyakov loop to the target region [17].

However, as seen from Fig. 1, the logarithmic model cannot reproduce the susceptibilities quantitatively. For this, one needs to take the lattice data on fluctuations into account. We consider the following $Z(3)$ -symmetric model

$$\begin{aligned} \frac{U(L, \bar{L})}{T^4} = & -\frac{1}{2}a(T)\bar{L}L + b(T)\ln M_H(L, \bar{L}) \\ & + \frac{1}{2}c(T)(L^3 + \bar{L}^3) + d(T)(\bar{L}L)^2, \end{aligned} \quad (8)$$

where M_H is the SU(3) Haar measure, expressed by the Polyakov loop and its complex conjugate as

$$M_H = 1 - 6\bar{L}L + 4(L^3 + \bar{L}^3) - 3(\bar{L}L)^2. \quad (9)$$

The restriction of Polyakov loop to the target region is naturally enforced by this term.

The model parameters can be uniquely determined from the lattice data on equation of state, expectation value of Polyakov loop, together with the susceptibilities. Details of this model are discussed in Ref. [21].

Figure 2 shows that there is a satisfactory description of lattice results on pressure and interaction measure, up to high temperatures. Our model parameters are tuned to describe the most recent [7], rather than previous [8], lattice data on the thermodynamic pressure. This explains the small differences between model results.

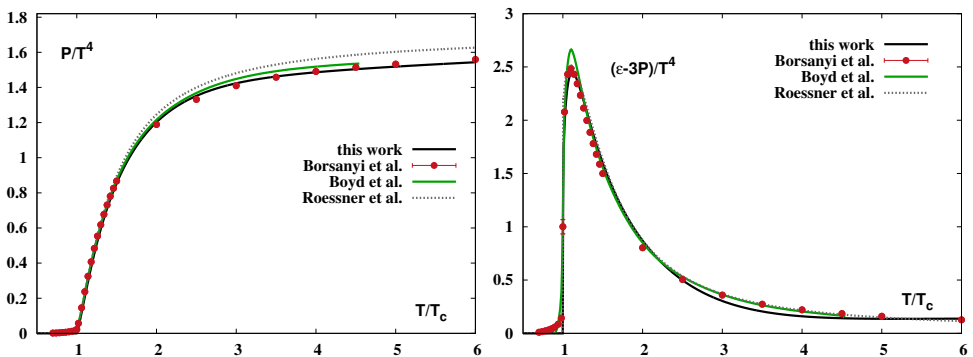


Fig. 2. (Color online) Left-hand figure: Lattice QCD data for thermodynamic pressure obtained in the SU(3) gauge theory. The points are from Ref. [7], whereas the gray/green line is the parametrization of lattice data from Ref. [8]. The black and the dashed lines are obtained in the Polyakov loop models introduced in Sec. 3. Right-hand figure: As in the left-hand figure but for the interaction measure $(\epsilon - 3P)/T^4$, where ϵ is the energy density.

The essential distinction between the two models lies in the prediction for Polyakov loop fluctuations. Figure 1 shows that both the longitudinal and the transverse susceptibilities are well reproduced by the new model, while the logarithmic model underestimates both of them.

The model developed here is open to a more realistic effective description of QCD thermodynamics with quarks.

4. Conclusions

The new lattice results on Polyakov susceptibilities allow us to probe deeper into the deconfinement transition. The observables show a narrow width extending to about $1.2 T_c$. They are sensitive to the spontaneous breaking of $Z(3)$ center symmetry and can reflect screening properties of the deconfined medium.

Preliminary results for the ratios of susceptibilities are obtained in (2+1)-flavor lattice QCD simulations [21]. In particular, the value of R_T in the high temperature phase deviates substantially from the pure gauge limit. For this, there is as yet no good theoretical understanding. An exploratory first step would be to include a small explicit breaking term in the pure glue potential and study how the picture changes from the pure gauge theory.

On the lattice, more work is needed for the robust extractions of these gluonic correlation functions, as well as for the detailed understanding of their systematic uncertainties.

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REFERENCES

- [1] J. Greensite, *Prog. Part. Nucl. Phys.* **51**, 1 (2003).
- [2] E.S. Swanson, *AIP Conf. Proc.* **717**, 636 (2004).
- [3] G. 't Hooft, *Nucl. Phys.* **B138**, 1 (1978).
- [4] B. Svetitsky, L.G. Yaffe, *Nucl. Phys.* **B210**, 423 (1982).
- [5] L.D. McLerran, B. Svetitsky, *Phys. Lett.* **B98**, 195 (1981); *Phys. Rev.* **D24**, 450 (1981).
- [6] A.M. Polyakov, *Phys. Lett.* **B72**, 477 (1978).
- [7] Sz. Borsanyi *et al.*, *J. High Energy Phys.* **1207**, 056 (2012).
- [8] G. Boyd *et al.*, *Nucl. Phys.* **B469**, 419 (1996).

- [9] O. Kaczmarek, F. Karsch, P. Petreczky, F. Zantow, *Phys. Lett.* **B543**, 41 (2002).
- [10] F. Karsch, *Lect. Notes Phys.* **583**, 209 (2002).
- [11] P.M. Lo *et al.*, *Phys. Rev.* **D88**, 014506 (2013).
- [12] S. Nadkarni, *Phys. Rev.* **D33**, 3738 (1986).
- [13] O. Kaczmarek, F. Karsch, E. Laermann, M. Lutgemeier, *Phys. Rev.* **D62**, 034021 (2000).
- [14] Y. Maezawa *et al.* [WHOT-QCD Collaboration], *Phys. Rev.* **D81**, 091501 (2010).
- [15] C. Ratti, M.A. Thaler, W. Weise, *Phys. Rev.* **D73**, 014019 (2006).
- [16] S. Roessner, C. Ratti, W. Weise, *Phys. Rev.* **D75**, 034007 (2007).
- [17] K. Fukushima, *Phys. Lett.* **B591**, 277 (2004).
- [18] C. Sasaki, B. Friman, K. Redlich, *Phys. Rev.* **D75**, 074013 (2007).
- [19] K. Fukushima, C. Sasaki, *Prog. Part. Nucl. Phys.* **72**, 99 (2013).
- [20] A. Dumitru *et al.*, *Phys. Rev.* **D70**, 034511 (2004).
- [21] P.M. Lo *et al.*, *Phys. Rev.* **D88**, 074502 (2013).