

INFLUENCE OF COMMUNITY STRUCTURE ON COOPERATIVE DYNAMICS IN COUPLED SOCIO-ECOLOGICAL SYSTEMS*

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(Received February 10, 2014)

We explore the influence of community structure on the effectiveness of social sanction in promoting cooperative behavior in populations sharing common pool resources. We found that the formation of community within a population is not necessarily associated with a higher level of cooperation. In fact, our results show that defectors tend to survive better in populations with weak community structure. Nonetheless, as we further strengthened the community structure within a social network, we uncovered the occurrence of a transition towards a regime of greater cooperation. In this respect, our results provide deeper insights into the manner in which governance structures can have important influence on the management of coupled socio-ecological systems.

DOI:10.5506/APhysPolBSupp.7.257

PACS numbers: 87.23.Kg, 89.75.Fb

* Presented at the Summer Solstice 2013 International Conference on Discrete Models of Complex Systems, Warszawa, Poland, June 27–29, 2013.

1. Introduction

Most of Earth's ecosystems have been seriously degraded by human activities such as large-scale harvesting of timber and freshwater use. Since actions involving resource management require effective cooperation between different user groups who have open access to the common pool resource, sustainable management of natural resources has always been a challenge. Recently, Tavoni *et al.* [1] have introduced an analytical model to study how cooperative behavior can be promoted in coupled socio-ecological systems through social ostracism against defectors who overuse the common pool resources. Interestingly, the effectiveness of social sanction in promoting cooperation is found to be dependent on the average number of social interactions within the population [2]. In this paper, we further extend the study of cooperative behavior in the harvesting of common pool resource to populations with community structures.

In several studies, community structure is observed to enhance cooperation within social network [3, 4]. Nonetheless, in networks with low level of structural heterogeneity, community structure is found to have negative effects on cooperation. On the other hand, the positive effects of community structure on cooperation appears only when the structural heterogeneity is high enough [5] as observed in studies carried out in evolutionary prisoner's dilemma game. In this paper, we explore the effect of community structure on cooperation among members of a population who share a common pool resource by coupling cooperative dynamics of the social network to dynamics of the resource. In particular, we investigate the influence of different degree of community structure, as measured by the network modularity, on the effectiveness of social sanction in the promotion of cooperative behavior.

2. A model of coupled social and ecological dynamics

In the model introduced in Ref. [1], agents who share a common pool resource adopt one of the two extracting strategies: cooperation or defection. Agents that cooperate harvest the resource at a socially agreed-upon acceptable level by putting in an effort of e_c . On the other hand, defectors choose to extract more resource by putting in a greater level of effort $e_d = \mu e_c$, where $\mu > 1$. The total number of agents is denoted by N and a fraction f_c of them adopt the cooperative strategy. Hence, the mean effort exerted by the agents is given by

$$E = N[f_c e_c + [1 - f_c] e_d]. \quad (1)$$

Let R be the resource level, the production F can then be represented as

$$F = \gamma E^\alpha R^\beta \quad (2)$$

with $E \geq 0$, $F > 0$ and $\alpha + \beta < 1$ to guarantee the existence of an optimal e_c which maximize the population's production gain. For each cooperator, a production gain of

$$\pi_c = \frac{e_c}{E}F - we_c \quad (3)$$

is obtained. The production gain for an agent who defect is

$$\pi_d = \frac{e_d}{E}F - we_d. \quad (4)$$

Note that w is the opportunity cost of labor.

In addition, social sanction against agents who overuse the resource is included in the model such that each defector pays an extra cost, which is captured by the following ostracism function

$$O(n_c) = h \exp(t \exp(g n_c)). \quad (5)$$

Here, h gives the maximum sanction, t is the sanctioning effectiveness threshold, g denotes the growth rate of the function. Note that the ostracism is modeled by the Gompertz growth function with $t < 0$ and $g < 0$ such that the maximum sanction is bounded by h . The ostracism process operates only when the cooperator community is sufficiently large. It increases rapidly when the size of the cooperator community is beyond a certain threshold, and it then saturates as the proportion of cooperator further increases. In our simulation, by choosing $t = -150$, social sanction operates only when $n_c > 0.3$. In Eq. (5), n_c denotes the fraction of cooperator in an agent's neighborhood. An agent's neighborhood includes only neighbors that connect directly to the agent. The sanctions imposed on the norm violator is dependent on the composition of its neighborhood and the pay-off for defectors with n_c fraction of cooperative neighbors is given by

$$U_d(n_c) = \pi_d - O(n_c) \frac{\pi_d - \pi_c}{\pi_d}. \quad (6)$$

Note that agents who cooperate do not pay the extra cost. Hence, the pay-off for a cooperator is given by

$$U_c = \pi_c. \quad (7)$$

Through Eqs. (1)–(7), we model how the individual pay-off depends on the extraction pattern and the resource level. Differences in the pay-off of cooperating and defecting agents are assumed to exert evolutionary pressure on the population composition to the advantage of the agents earning the highest pay-offs. With this, we study the influence of ecological feedback on the cooperation of the social system. On the other hand, condition of the ecological system is dependent on the composition of the population.

Dynamics of the common pool resource, including its refurbishment, natural depreciation and degradation due to extractive effort is modeled by the following differential equation

$$\dot{R} = c - d \left[\frac{R}{R_{\max}} \right]^\kappa - qER. \quad (8)$$

Here, the resource is growing with a positive rate c up to the maximum storage capacity $R_{\max}$. The parameter d governs the discharge of the growth rate, while appropriation of the resource is given by qER where q is a technological factor. We have set $q = 1$ and $\kappa = 2$. The other parameters used are: $R_{\max} = 200$, $c = d = 50$, $w = 15$, $\gamma = 10$, $\alpha = 0.6$, $\beta = 0.2$, $h = 0.34$, $g = -10$ and $t = -150$. For this set of parameters, the effort that maximize the population's pay-off is found to be $0.483/N$ [1] and we will use this value as e_c in our simulations.

The evolution of the composition of the population is governed by the replicator dynamics. During each iteration, each individual is allowed to compare its pay-offs with an agent randomly matched from its connected neighbors and then choose whether to switch its strategies. If the pay-off of the neighbor is lower, the agent keeps its strategy. If the pay-off of the neighbor is higher, the probability that an individual switches its strategy is proportional to the difference between the pay-offs. Note that we have modeled the population interaction structure by means of a network. Here, we consider both random and scale-free networks with different community structure. Communities are represented by densely connected sub-graphs in a social network. The strength of division of a network into communities is measured by the modularity [6]

$$Q = \frac{1}{2m} \sum_{i,j} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j). \quad (9)$$

In Eq. (9), A_{ij} are the elements of the adjacency matrix of an undirected network with m number of links. k_i is the degree of node i . $\delta(c_i, c_j)$ is equal to 1 if i and j belong to the same community and is zero otherwise. Networks with high modularity have dense connections between the nodes within communities but sparse connections between nodes in different communities. Hence, for populations with high modularity, the agents tend to form groups. They connect densely with their group members but rarely interact with agents in the other groups.

3. Results

We have computed the replicator dynamics for the Erdos–Renyi networks with size $N = 200$ and average degree $k = 99$ for various values of μ . The

initial fraction of population that cooperate is set to 0.8. The fractions of cooperators at equilibrium are shown in Fig. 1. For populations with social connection represented by random network without community structure, cooperation predominates when $\mu \leq 2.5$. A transition from cooperation to defection takes place at $\mu = 2.5$. When weak community structure is introduced, social sanction is found to be less effective in promoting cooperation. As shown in Fig. 1, a transition from cooperation to defection takes place at $\mu = 2.3$ for population with $Q = 0.1$. When the modularity increases to $Q = 0.2$, this value drops to $\mu = 2.2$. Nevertheless, an enhancement of cooperation is found for $\mu \geq 2.4$ when the modularity is further increased to $Q = 0.35$. In addition, cooperation is found to be further enhanced for $Q > 0.35$. Note that the result shown in Fig. 1 is for populations made up of two communities. Similar simulations are then performed for populations consisting of four communities with $Q = 0.2, 0.4, 0.6$ and 0.72 . The result

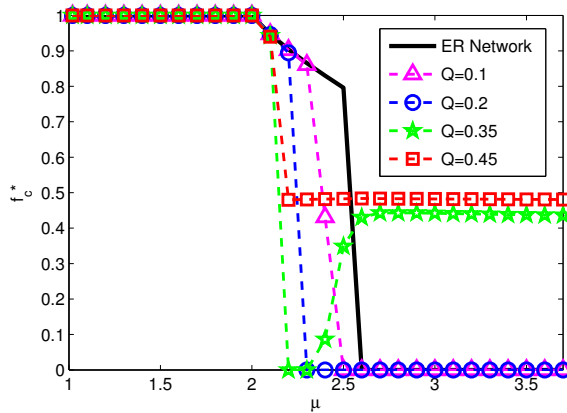


Fig. 1. (Color on-line) A plot of the fraction of cooperators at equilibrium (f_c^*) for various values of μ . The initial fraction of population that cooperate (f_{c_0}) is set to 0.8. Interaction within the population is represented by a random network (solid line) with size $N = 200$ and average degree $k = 99$. Then, different levels of modularity are introduced into the network structure. For this, two pairs of nodes are chosen randomly from the two communities, if all nodes from different communities are connected and nodes in the same community are not connected, connections are rewired such that nodes in the same community connect to each other. The process is repeated until the network possess the specific degree of modularity. Specifically, networks with $Q = 0.1$ (triangles), $Q = 0.2$ (circles), $Q = 0.35$ (stars), and $Q = 0.45$ (squares) are considered. Here, $Q = 0.1$ and 0.2 correspond to weak community structure. Note that the networks consist of two communities of the same size and each data point gives the result averaged over 1000 simulations.

is shown in Fig. 2. As shown, social sanction is less effective in the promotion of cooperation in population with weak community structure. However, its effectiveness in promoting cooperation is found to enhance when the strength of the community structure within the social network becomes strong enough.

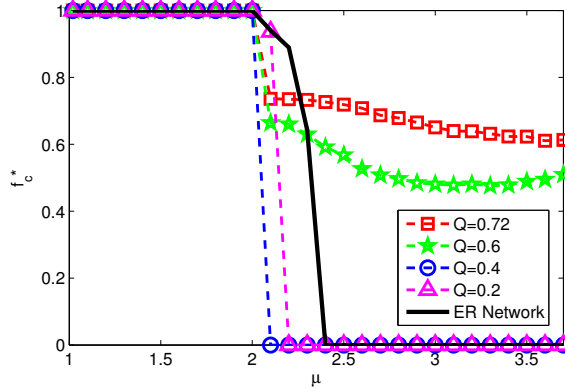


Fig. 2. (Color on-line) A plot of f_c^* for various values of μ . Here, $f_{c_0} = 0.8$. Interaction within the population is modeled by a random network (solid line) with size $N = 200$ and average degree $k = 49$. Rewiring is then carried out to introduce different level of modularity into the network structure. Specifically, networks with $Q = 0.2$ (triangles), $Q = 0.4$ (circles), $Q = 0.6$ (stars), and $Q = 0.72$ (squares) are considered. Here, $Q = 0.2$ and 0.4 correspond to weak community structure. Note that the networks consist of four communities of the same size and each data point gives the result averaged over 1000 simulations.

Next, we generalize our results to more realistic social networks with heterogeneous structure [7, 8] and communities of different sizes. For this, social networks of size $N = 500$ and average degree $k = 50$ are generated through the benchmark approach [9]. Here, we consider populations which consist of six communities with $Q \approx 0.22, 0.48$ and 0.51 . The result is shown in Fig. 3. Cooperation survives better in social network with heterogeneous structure. However, the influence of community structure on the effectiveness of social sanction in promoting cooperative behavior in populations sharing a common pool resource persists when heterogeneous structure is present. In particular, cooperation is enhanced only for population with strong community structure.

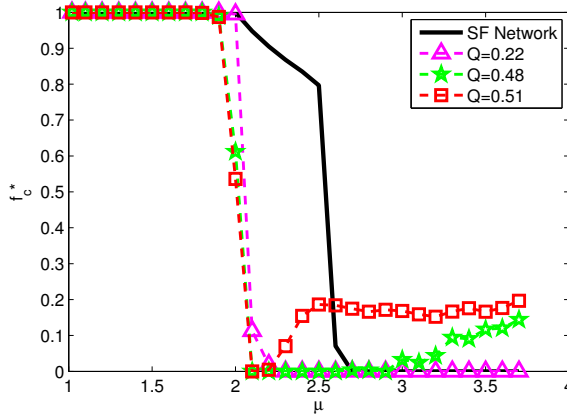


Fig. 3. (Color on-line) A plot of f_c^* for various values of μ . Here, $f_{c_0} = 0.8$. Interaction within the population is represented by a scale-free network (solid line) with size $N = 500$ and average degree $k = 50$. Rewiring is then carried out to introduce different level of modularity into the network structure. Specifically, networks with $Q = 0.22$ (triangles), $Q = 0.48$ (stars) and $Q = 0.51$ (squares) are considered. Here, $Q = 0.22$ corresponds to weak community structure. Note that the networks contain six communities of different sizes and each data point gives the result averaged over 1000 simulations.

4. Summary and conclusion

We have explored the influence of community structure on the effectiveness of social ostracism in the promotion of cooperative behavior among harvesters who shared a common pool resource. In particular, we have investigated the promotion of cooperation in social networks with various level of modularity. Our results show that defectors can survive better in population with weak community structure. In fact, the presence of weak community structure has facilitated the tendency of defectors to cluster together to minimize the effect of social sanction from the community of cooperators. As a result, social sanction is less effective in promoting cooperation in population with weak community structure. Nonetheless, cooperation can be enhanced when the strength of the community structure within the social network becomes large enough. In this case, agents are found to be densely connected within the same community while sparsely connected with agents from a different community. This leads to clusters of isolated social networks of high network degree. Since the effectiveness of social sanction in promoting cooperation increases with an increase in the number of social ties [2], this explains the enhanced cooperation observed in networks with strong community structure as illustrated in Figs. 1, 2 and 3.

N.N. Chung would like to acknowledge the financial support of Temasek Laboratories@NUS for conference travel via Project POD0613356 (funded by the Defense Science and Technology Agency of Singapore).

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