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THE $\Sigma_{\pi N}$ TERM, CHIRAL MULTIPLET MIXING AND HIDDEN STRANGENESS IN THE NUCLEON*

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We calculate the $\Sigma_{\pi N}$ term in the chiral mixing approach to baryons, *i.e.*, with $\mathrm{SU_L}(3)\times\mathrm{SU_R}(3)$ chiral multiplets $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$, $[\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ and $[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$, admixed in the baryons, using known constraints on the current quark masses m_u^0, m_d^0. We show that the $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ multiplet's contribution is enhanced by a factor of $\frac{57}{9}\simeq 6.33$, due to $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$ algebra, that leads to $\Sigma_{\pi N}\geq\left(1+\frac{48}{9}\sin^2\theta\right)\frac{3}{2}\left(m_u^0+m_d^0\right)=60$ MeV, in general accord with "experimental" values of $\Sigma_{\pi N}$. The chiral mixing angle θ is given by $\sin^2\theta=\frac{3}{8}\left(g_\mathrm{A}^{(0)}+g_\mathrm{A}^{(3)}\right)$, where $g_\mathrm{A}^{(0)}=0.33\pm0.08$, or 0.28 ± 0.16 is the flavor-singlet axial coupling, and $g_\mathrm{A}^{(3)}=1.267$, is the third component of the octet one. These results show that there is no need for $q^4\bar{q}$ components, and in particular, no need for an $s\bar{s}$ component in the nucleon.

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1. Introduction

The nucleon $\Sigma_{\pi N}$ term is a "theoretical measure" of its current quarks' mass contribution to the total nucleon mass. The difference of the value extracted from the measured πN scattering partial wave analyses from 25 MeV has been interpreted as an increase of Zweig-rule-breaking in the nucleon, or equivalently to an increased $s\bar{s}$ content $y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{u}u|N\rangle}$ of the nucleon, Refs. [1–3]. As all "measurements" of $\Sigma_{\pi N}$ have yielded values ranging from 55 MeV to 75 MeV [4], that are substantially larger than the expected 25 MeV, it has consequently appeared that the $s\bar{s}$ content of the nucleon must be (very) large.

A number of experiments have measured the $s\bar{s}$ contributions to nucleon observables other than the $\Sigma_{\pi N}$ term [5]. No experiment has found a result larger than a few % of the u (and/or $\bar{u}$) and d (and/or $\bar{d}$) contributions[1], thus making the $s\bar{s}$ content of the nucleon effectively negligible $y \simeq 0$. Thus, the enigma has deepened: how is it possible to have such a large $\Sigma_{\pi N}$ term without any $s\bar{s}$ content in other observables? In the meantime, the nucleon $\Sigma_{\pi N}$ term has been shown as an important ingredient in searches for (supersymmetric) cold dark matter, Ref. [7] and in the QCD phase diagram, thereby only increasing the stakes.

In this report, we show explicitly an alternative mechanism of hadronic $\Sigma_{\pi N}$ term enhancement with strangeness content $y = 0$ and pin-point the source of the enhancement to the $(6,3) \equiv [(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$, or $(1,\frac{1}{2}) \equiv [(\mathbf{1},\frac{\mathbf{1}}{\mathbf{2}})\oplus(\frac{\mathbf{1}}{\mathbf{2}},\mathbf{1})]$ chiral component (in the $\mathrm{SU_L}(3)\times\mathrm{SU_R}(3)$ or $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$ notations, respectively) of the nucleon. This component contributes about three quarters of the enhanced value of $\Sigma_{\pi N} \geq 55$ MeV, which would otherwise be ≥ 14 MeV, while keeping a vanishing $s\bar{s}$ component in the nucleon. The same $(1,\frac{1}{2})$ chiral component is crucial for the proper description of the nucleon's isovector axial coupling $g_{\mathrm{A}}^{(3)} = 1.267$.

We show in some detail how the $\Sigma_{\pi N}$ term enhancement emerges from the $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$ chiral algebra. To that end, we use a hadronic two-flavor $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$ chiral mixing model, in which the $s\bar{s}$ content of the nucleon vanishes, $y = 0$, *per definitionem*. Baryons in the spontaneously broken symmetry phase may be effectively described by a few chiral components: it was shown in Refs. [8–10], that several nucleon's properties can be successfully described by mixing of three chiral multiplet components. Of two historical chiral mixing scenarios [8–10], only the Harari one [9, 10], described by

$$|N\rangle \;=\; \sin\theta|(6,3)\rangle + \cos\theta\big(\cos\varphi|(3,\bar{3})\rangle + \sin\varphi|(\bar{3},3)\rangle\big)\,, \qquad (1)$$

[1] This makes these effects compatible with the (much more) mundane isospin-violating corrections, from which they are indistinguishable [6].

has survived the inclusion of the baryons' anomalous magnetic moments in the three-flavor case [11]. Here we use the original $\mathrm{SU_L}(3)\times\mathrm{SU_R}(3)$ notation to distinguish between the two kinds of $(\frac{1}{2},0)$ multiplets in $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$, though we shall use only the two-flavor multiplets.

2. Calculation

To calculate the nucleon $\Sigma_{\pi N} = \langle N|\Sigma|N\rangle$ term, we use the Σ operator defined as the double commutator

$$\Sigma \;=\; \tfrac{1}{3}\delta^{ab}\left[Q_5^a,\left[Q_5^b,H_{\chi\mathrm{SB}}\right]\right] \tag{2}$$

of the axial charges Q_5^a and the chiral symmetry breaking Hamiltonian $H_{\chi\mathrm{SB}}$[2]. It was introduced by Dashen [13] as a way of separating out the explicit chiral $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$ symmetry breaking part $H_{\chi\mathrm{SB}}$ from the total Hamiltonian. Ensuring that the (spontaneously broken) chiral symmetry is properly implemented is particularly important in a calculation at the hadron level. We have developed in Refs. [11, 15–21] a (linear realization) chiral Lagrangian that reproduces the results of the phenomenological chiral mixing method.

We follow Ref. [12], and use an explicit χSB "bare" nucleon mass and the corresponding χSB Hamiltonian density

$$\mathcal{H}^{\mathrm{N}}_{\chi\mathrm{SB}} \;=\; \sum_{i=1}^{3}\bar{N}_i M^0_{N_i}N_i + \bar{\Delta}_{(1,\frac{1}{2})}M^0_{\Delta(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})}\,,$$

where i stands for the three chiral multiplets $(1,\frac{1}{2})$, $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$. *A priori*, we do not know the values of the "current" nucleon masses, except for a lower limit: they cannot be smaller than three isospin-averaged current quark masses: $M^0_{N_i}\geq 3\bar{m}^0_q = \frac{3}{2}\left(m^0_u+m^0_d\right)$. For simplicity's sake, we shall assume, as a first approximation, that all three chiral components have the same "current" nucleon mass $M^0_N = M^0_{N(6,3)} = M^0_{N(1,\frac{1}{2})} = M^0_{\Delta(1,\frac{1}{2})} = M^0_{(3,\bar{3})} = M^0_{(\frac{1}{2},0)} = M^0_{(\bar{3},3)} = M^0_{(0,\frac{1}{2})} = \frac{3}{2}\left(m^0_u+m^0_d\right)$.

The chiral $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$ generators Q_5^a and their commutators with the nucleon N and Δ fields were worked out in Refs. [16–19]:

$$\begin{aligned}\left[Q_5^a,N_{(1,\frac{1}{2})}\right] \;&=\; \gamma_5\left(\frac{5}{3}\frac{\tau^a}{2}N_{(1,\frac{1}{2})}+\frac{2}{\sqrt{3}}T^a\Delta_{(1,\frac{1}{2})}\right)\,,\\ \left[Q_5^a,\Delta_{(1,\frac{1}{2})}\right] \;&=\; \gamma_5\left(\frac{2}{\sqrt{3}}T^{\dagger a}N_{(1,\frac{1}{2})}+\frac{1}{3}t^a_{(\frac{3}{2})}\Delta_{(1,\frac{1}{2})}\right)\,,\end{aligned}$$

[2] For normalization and notational conventions, see Ref. [12].

$$\begin{aligned}\left[Q_5^a, N_{(\frac{1}{2},0)}\right] &= \gamma_5 \frac{\tau^a}{2} N_{(\frac{1}{2},0)}\,,\\ \left[Q_5^a, N_{(0,\frac{1}{2})}\right] &= -\gamma_5 \frac{\tau^a}{2} N_{(0,\frac{1}{2})}\,,\end{aligned} \tag{3}$$

where $a = 1, 2, 3$, $t^i_{(\frac{3}{2})}$ are the isospin-$\frac{3}{2}$ generators of the SU(2) group and T^i are the so-called iso-spurion (2×4) matrices, that are related to the SU(2) Clebsch–Gordan coefficients $\langle \frac{3}{2} I_3(\Delta) | 1 I_3(i) \frac{1}{2} I_3(N)\rangle$, with the following properties (see Appendix B of Ref. [18])

$$\begin{aligned}T^{i\dagger}T^k &= \frac{3}{4}\delta^{ik} - \frac{1}{6}\left\{t^i_{(\frac{3}{2})}, t^j_{(\frac{3}{2})}\right\} + \frac{i}{3}\,\epsilon^{ijk} t^k_{(\frac{3}{2})}\,,\\ T^i T^{k\dagger} &= P^{ik}_{\frac{3}{2}}\,.\end{aligned} \tag{4}$$

The chiral $\mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$ double commutators for the $\left[(1,\frac{1}{2}) \oplus (\frac{1}{2},1)\right]$ chiral multiplet are

$$\begin{aligned}\left[Q_5^b, \left[Q_5^a, \bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})}\right]\right] &= \frac{41}{9}\delta^{ab} \bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})}\\ &+ \bar{\Delta}_{(1,\frac{1}{2})}\left(2\delta^{ab} - \frac{4}{9}\left\{t^a_{(\frac{3}{2})}, t^b_{(\frac{3}{2})}\right\}\right)\Delta_{(1,\frac{1}{2})} + \ldots\,,\end{aligned} \tag{5}$$

where $\ldots$ stand for the off-diagonal terms, such as $\bar{N}_{(1,\frac{1}{2})}(\ldots)\Delta_{(1,\frac{1}{2})}$, and their Hermitian conjugates.

We contract Eq. (5) with $\frac{1}{3}\delta^{ab}$ (where summation over repeated indices is understood) to find

$$\frac{1}{3}\delta^{ab}\left[Q_5^b, \left[Q_5^a, \bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})}\right]\right] = \frac{41}{9}\bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})} + \frac{8}{9}\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})} + \ldots\,, \tag{6}$$

where we have used the identity $t^a_{(\frac{3}{2})} t^a_{(\frac{3}{2})} = \frac{15}{4}\mathbf{1}_{4\times 4}$, and similarly for the Δ-field contribution

$$\frac{1}{3}\delta^{ab}\left[Q_5^b, \left[Q_5^a, \bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})}\right]\right] = \frac{16}{9}\bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})} + \frac{13}{9}\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})} + \ldots \tag{7}$$

This finally leads to

$$\begin{aligned}\Sigma_{\pi N} &= \sin^2\theta\left(\frac{41}{9} M^0_{N(1,\frac{1}{2})} + \frac{16}{9} M^0_{\Delta(1,\frac{1}{2})}\right)\\ &\quad + \cos^2\theta\left(\cos^2\varphi M^0_{N(\frac{1}{2},0)} + \sin^2\varphi M^0_{N(\frac{1}{2},0)}\right)\,,\end{aligned} \tag{8}$$

which is our basic result here.

3. Result and discussion

Inserting our simplifying assumption that all the "current nucleon" masses are equal, one finds the final result

$$\Sigma_{\pi N} = \left(1 + \frac{16}{3}\sin^2\theta\right) M_N^0 . \tag{9}$$

Note that the enhancement term $\frac{16}{3}\sin^2\theta$ is due to the factor $\frac{41+16}{9} = \frac{19}{3} \approx 6.33$ appearing in Eq. (8) of the $[(\mathbf{1}, \frac{\mathbf{1}}{\mathbf{2}}) \oplus (\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{1})]$ chiral multiplet which, in turn, is due to the iso-spurion matrices T^i. Thus, the enhancement factor $\frac{19}{3}$ in Eq. (8) and consequently also the $\frac{16}{3}\sin^2\theta$ in Eq. (9), are of $\mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$ algebraic origin. This leaves ample room for improvement of the $\Sigma_{\pi N}$ predictions, irrespective of the specific value of the chiral mixing angle θ, within the chiral $\mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$ algebra approach.

The relevant chiral mixing angle θ has been extracted in Refs. [15–18], as $\frac{8}{3}\sin^2\theta = g_{\rm A}^{(0)} + g_{\rm A}^{(3)}$, a function of the isovector $g_{\rm A}^{(3)}$, and the flavor-singlet $g_{\rm A}^{(0)}$ axial coupling, where $g_{\rm A}^{(0)} = 0.28 \pm 0.16$, according to Ref. [22], or $g_{\rm A}^{(0)} = 0.33 \pm 0.03 \pm 0.05$, according to Ref. [23]. Here we have taken the values of current quark masses from PDG2012 [24]: $m_u^0 = 2.3 \times 1.35$ MeV and $m_d^0 = 4.8 \times 1.35$ MeV, yielding $\frac{1}{2}\left(m_u^0 + m_d^0\right) \approx 4.73$ MeV, substantially lower than before (*cf.* 7.6 MeV in Ref. [25]), and inserted them into the current nucleon mass to find $M_N^0 = \frac{3}{2}\left(m_u^0 + m_d^0\right) \approx 14.2$ MeV and $\Sigma_{\pi N} = 59.5 \pm 2.3$ MeV, with $g_{\rm A}^{(0)} = 0.33 \pm 0.03 \pm 0.05$ [23], or $\Sigma_{\pi N} = 58.0 \pm 4.5$ MeV, with $g_{\rm A}^{(0)} = 0.28 \pm 0.16$ [22], in fair agreement with the "observed" $\Sigma_{\pi N}$ value range (55–75) MeV, see Ref. [4].

The above result of Eq. (9) ought to be viewed as a lower bound on the "true" $\Sigma_{\pi N}$ value, as we have assumed that all current nucleon masses $M_{N_i}^0$ equal three times the isospin-averaged current quark mass $\bar{m}_q^0$, which is appropriate only when all chiral components of the nucleon correspond to three-quark fields. That condition is not necessary, however, because some $q^4\bar{q}$ baryon fields belong to the same chiral multiplets [21], and such fields have a larger current mass $M_{N'}^0 = 5\bar{m}_q^0$, that consequently leads to a higher value of $\Sigma_{\pi N}$, but merely a sufficient one, as all of these chiral multiplets exist as bi-local three-quark fields [20].

In summary, we have shown that the "observed" values of $\Sigma_{\pi N} \geq 55$ MeV are readily obtained in the chiral-mixing approach without any strangeness content in the nucleon, as a natural consequence of the substantial chiral $(6,3) = [(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})] \to (1, \frac{1}{2})$ multiplet component. The precise value of $\Sigma_{\pi N}$ is a linear function, Eq. (9), of the sum of the flavor-singlet $g_{\rm A}^{(0)}$, and the isovector $g_{\rm A}^{(3)}$ axial coupling of the nucleon.

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