Vol. 7 (2014) *Acta Physica Polonica B Proceedings Supplement* No 3

VORTICES AND CHIRAL SYMMETRY BREAKING*

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(Received May 28, 2014)

We analyze the creation of near-zero modes from would-be zero modes of various topological charge contributions from classical center vortices in SU(2) lattice gauge theory. We show that colorful spherical vortex and instanton congurations have very similar Dirac eigenmodes and give rise to a finite density of near-zero modes, leading to chiral symmetry breaking via the Banks–Casher formula. We discuss the influence of magnetic vortex fluxes on quarks and how center vortices may break chiral symmetry.

DOI:10.5506/APhysPolBSupp.7.457
PACS numbers: 11.15.Ha, 12.38.Gc

1. Introduction

A well established theory of spontaneous chiral symmetry breaking (χSB) relies on instantons, which are localized in space-time and carry a topological charge of modulus 1. According to the Atiyah–Singer index theorem, a zero mode of the Dirac operator arises, which is concentrated at the instanton core. In the instanton liquid model overlapping would-be zero modes split into low-lying nonzero modes which create the chiral condensate.

Center vortices are promising candidates for explaining confinement. The vortex model of confinement is theoretically appealing and was confirmed by a multitude of numerical calculations, both in lattice Yang–Mills theory and within a corresponding infrared effective model, see *e.g.* [1, 2]. Lattice simulations indicate that vortices are responsible for topological

* Presented by M. Faber at "Excited QCD 2014", Bjelašnica Mountain, Sarajevo, Bosnia and Herzegovina, February 2–8, 2014.

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charge and χSB as well [3–5], and thus unify all nonperturbative phenomena in a common framework. A similar picture to the instanton liquid model exists insofar as lumps of topological charge arise at the intersection and writhing points of vortices. The colorful, spherical SU(2) vortex was introduced in a previous article of our group [6] and may act as a prototype for this picture, as it contributes to the topological charge by its color structure, attracting a zero mode like an instanton. We show how the interplay of various topological structures from center vortices (and instantons) leads to near-zero modes, which by the Banks–Casher relation are responsible for a finite chiral condensate.

2. Free Dirac eigenmodes

The chiral density of free overlap eigenmodes obtained numerically using the MILC code are shown in Fig. 1. The modes are found with the Ritz functional algorithm with random start and for degenerate eigenvalues the eigenmodes span a randomly oriented basis in the degenerate subspace. Therefore, the numerical modes presented in Fig. 1 are linear combinations of plane waves with momenta $\pm p_\mu$ and show plane wave oscillations of $2p_\mu$ in the chiral density. For $12^3 \times 24$ lattices, the first eight degenerate modes consist of plane waves with $p_4 = \pm\pi/24$, hence there is one sine (cosine) oscillation in time direction, the next eight have $p_4 = \pm 3\pi/24$, *i.e.*, three oscillations in the time direction. The oscillations of χ_{R} and χ_{L} are separated by half an oscillation length, *i.e.*, the maxima of ρ_+ correspond to minima of ρ_- and *vice versa*, accordingly, the scalar density is constant as expected for free eigenmodes.

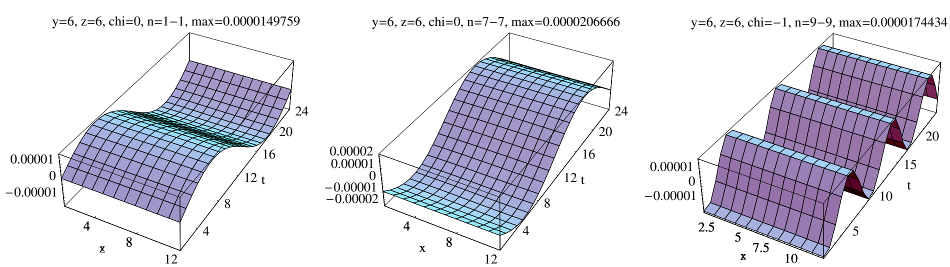
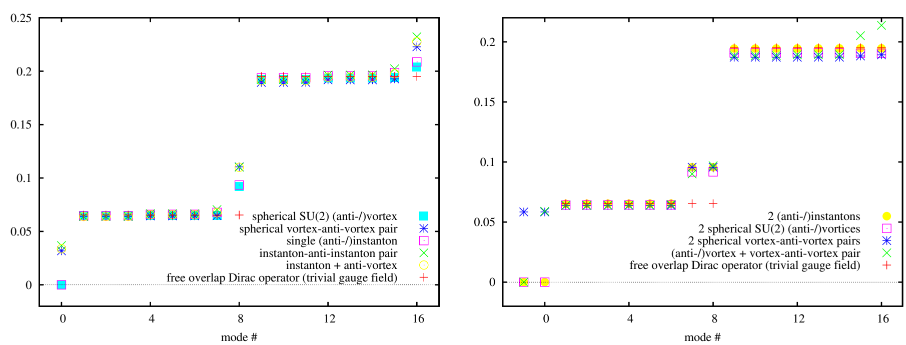


Fig. 1. Chiral density of the low-lying eigenmodes of the free overlap Dirac operator: $\rho_5\#1$ (left), $\rho_5\#7$ (center) $\rho_5\#9$ (right). The modes clearly show the plane wave behavior with oscillations of $2p_\mu$ (see the text).

3. The colorful spherical vortex

The spherical vortex was introduced in [6] and analyzed in more detail in [7–9]. It is constructed with t-links in a single time slice at fixed $t = t_i$, given by $U_t(x^\nu) = \exp\left(i\alpha(|\vec{r} - \vec{r}_0|)\vec{r}/r \cdot \vec{\sigma}\right)$, where $\vec{r}$ is the spatial part of x_ν. The profile function $\alpha(r)$ changes from π to 0 in radial direction for the negative spherical vortex, or from π to 2π for the positive (anti-)vortex. This gives a hedgehog-like configuration, since the color vector points in (or against) the radial direction at the vortex radius R. The hedgehog-like structure is crucial for our analysis. The t-links of the spherical vortex define a map $S^3 \to \mathrm{SU}(2)$, characterized by a winding number $N = -1$ for positive (anti-) and $N = +1$ for negative spherical vortices. Obviously, such windings influence the Atiyah–Singer index theorem giving a topological charge $Q = -1$ for positive and $Q = +1$ for negative spherical vortices (anti-vortices) and attract Dirac zero modes similar to instantons. In [9] we showed that the spherical vortex is, in fact, a vacuum-to-vacuum transition in the time direction which can even be regularized to give the correct topological charge also from gluonic definitions. Figure 2 (a) shows that a spherical vortex has nearly exactly the same eigenvalues as an instanton. We further plot the spectra of instanton–anti-instanton, spherical vortex–anti-vortex and instanton–anti-vortex pairs. We again see nearly exactly the same eigenvalues for instanton or spherical vortex pairs, instead of two would-be zero modes there is a pair of near-zero modes for each pair.



(a) (b)

Fig. 2. The lowest overlap eigenvalues for instanton and spherical vortex configurations compared to the eigenvalues of the free (overlap) Dirac operator.

The chiral density plots in Fig. 3 for the instanton–anti-instanton pair and Fig. 4 for the spherical vortex–anti-vortex pair show, besides the similar densities, that the near-zero modes are a result of two chiral parts corresponding to the two constituents of the pairs. The nonzero modes can be identified with the free overlap modes, as they show plane-wave behavior.

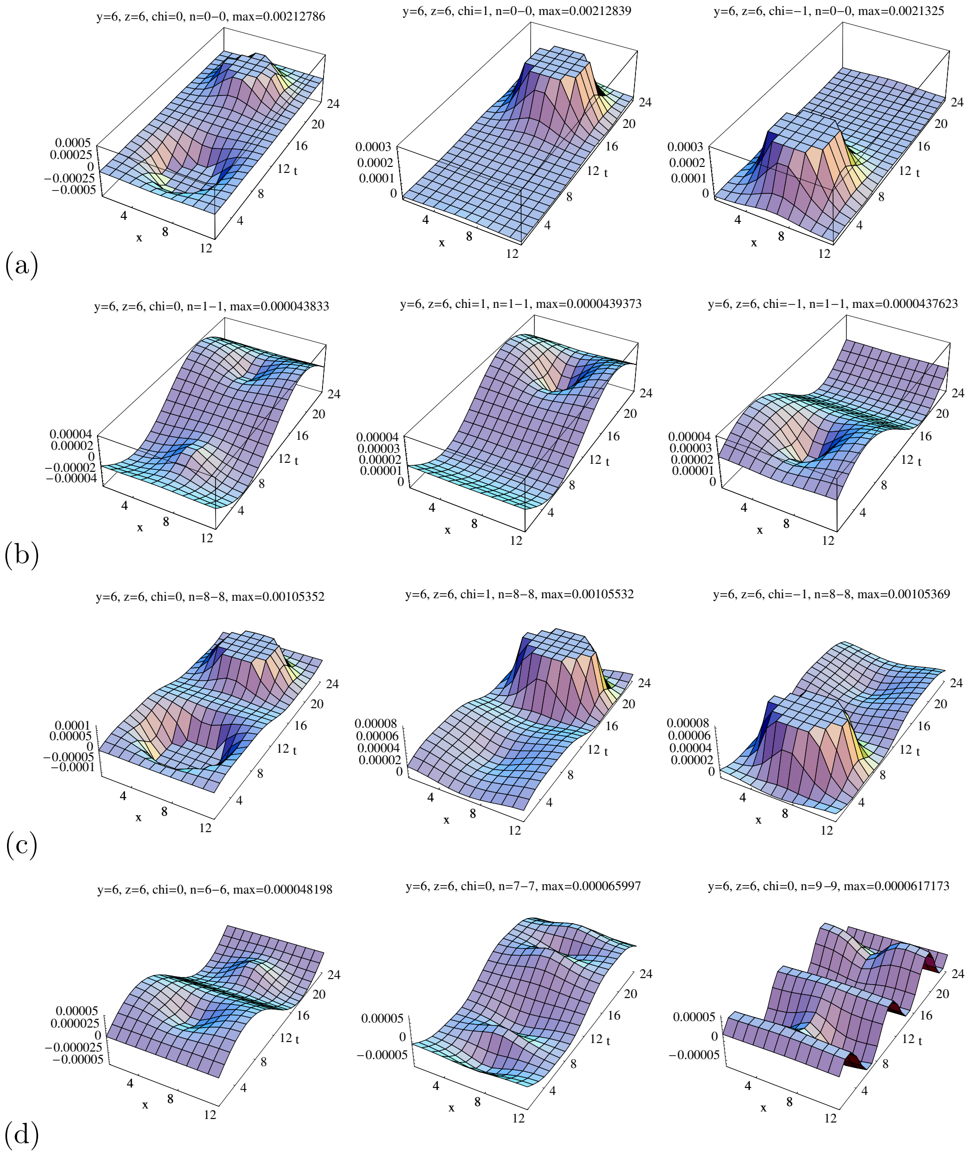


Fig. 3. Chiral densities (ρ_5 left, ρ_+ center and ρ_- right column) of the (a) lowest (near-zero), (b) second-lowest (nonzero) and (c) eighth (nonzero) eigenmode of the overlap Dirac operator for an instanton–anti-instanton pair. (d) ρ_5 of the sixth (left), seventh (center) and ninth (right) eigenmode.

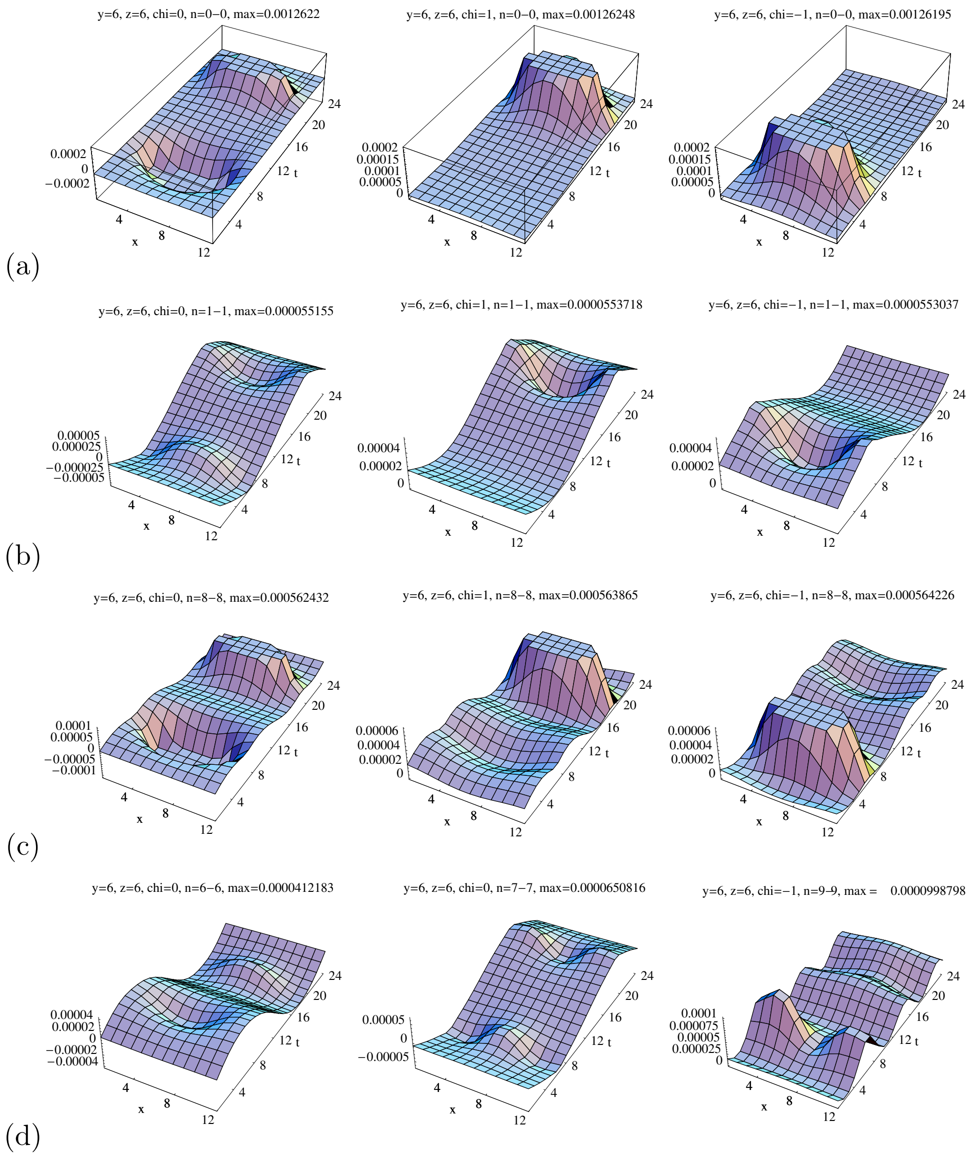


Fig. 4. Same as Fig. 3 but for a spherical vortex–anti-vortex pair. Chiral densities (ρ_5 left, ρ_+ center and ρ_- right column) of the (a) lowest (near-zero), (b) second-lowest (nonzero) and (c) eighth (nonzero) eigenmode. (d) ρ_5 of the sixth (left), seventh (center) and ninth (right) eigenmode.

In Fig. 2 (b), we plot the eigenvalues of two (anti-)instantons and two spherical (anti-)vortices giving topological charge $Q = 2$ ($Q = -2$) and therefore two zero modes, two vortex–anti-vortex pairs with two pairs of near-zero modes and a configuration with two vortices and an anti-vortex (*i.e.*, a single vortex plus one vortex–anti-vortex pair) giving one zero mode ($Q = 1$) and one pair of near-zero modes. The results show that we may draw the same conclusions for spherical vortices as for instantons concerning the creation of near-zero modes.

4. Conclusions

Instantons and spherical vortices show similar response to fermions and the instanton liquid model can be extended to colorful spherical center vortices. In Monte Carlo configurations we do not, of course, find perfectly spherical vortices, as one does not find perfect instantons. The general picture of topological charge from center vortices can provide a general picture of χSB: just like instantons, any source of topological charge can attract (would-be) zero modes and produce a finite density of near-zero modes leading to chiral symmetry breaking via the Banks–Casher relation. For more details, see [10].

The work of R.H. was funded by the Erwin Schrödinger Fellowship of the Austrian Science Fund under Contract No. J3425-N27.

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