DYNAMIC HOLOGRAPHIC QCD*

Nick Evans

STAG Research Centre and Physics and Astronomy
University of Southampton, Highfield SO17 1BJ, UK

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I introduce holographic techniques for modelling strongly coupled gauge theories and AdS/QCD. Dynamic AdS/QCD is a variant in which the formation of the chiral condensate is dynamically determined. As an example, I use the model, based on perturbative computations of the running of the anomalous dimension of the quark mass γ, to study $\mathrm{SU}(N_\mathrm{c})$ gauge theory with N_f quark flavours.

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Holography [1] is a new technique for modelling strongly coupled gauge dynamics. It is particularly useful for studying renormalization group flow. Here I will review holographic AdS/QCD models [2]. I will present a simple model of the dependence of $\mathrm{SU}(N_\mathrm{c})$ gauge theory on the number of quark flavours, N_f, that displays many of the expected features such as a conformal window, walking and QCD-like dynamics [3].

1. Holography

In holographic models [1], the renormalization group scale of the theory is treated as a space-time dimension. The conformal symmetry of the classical theory is realized as a symmetry of the AdS_5 metric

$$ds^2 = r^2 dx^2_{3+1} + \frac{dr^2}{r^2}\,. \tag{1}$$

We can think of the space as a box with r corresponding to energy scale. At any fixed r, we see a $3+1d$ theory living in the x_{3+1} directions — large r is the UV of the theory, whilst small r is the IR. The dilatation symmetries in the classical gauge theory act on the space and the fields as $x \to e^{\alpha}x, A^{\mu} \to e^{-\alpha}A^{\mu}$ but are realized on the AdS space as $x \to e^{\alpha}x, r \to e^{-\alpha}r$.

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Fields in the AdS space represent gauge invariant operators and sources in the gauge theory such as the quark condensate $\langle \bar{q}q \rangle$ or mass m. A scalar field, ϕ, in the AdS space dual to such an operator/source has an action

$$S = \int d^4x \, dr \, \sqrt{-g} \left[(\partial_\mu \phi)^2 - M^2 \phi^2 \right] \tag{2}$$

which leads to solutions $\phi \sim \frac{a}{r^\Delta} + \frac{b}{r^{(4-\Delta)}}$, $\Delta(\Delta - 4) = M^2$. The integration constants, a, b, have the correct dimensions to play the roles of the source and its associated operator. The relation between the dimension of an operator and the mass of the dual scalar is key to much of our discussion to come.

It is worth stressing that although this discussion appears rigorous, we made a big leap of faith to write down a weakly coupled theory of the AdS space. The only justification for this is that there is strong evidence that this is correct for the strongly coupled $\mathcal{N} = 4$ super-Yang–Mills. We can hope the ideas will apply to a wider set of strongly coupled gauge theories.

Traditional AdS/QCD: The first simple phenomenological model of QCD using holography assumed the space is AdS_5 with a hard cut off at some r_0 to represent the mass gap scale [2]. The large r UV description of QCD remains an AdS dual inspite of the fact that QCD becomes a weakly coupled theory of quarks and gluons — the AdS dual should be strongly coupled. One gets away with ignoring this because the AdS description comes from $\mathcal{N} = 4$ SYM which is conformal (like QCD's UV) and preserves the perturbative dimension of the operators we will include.

We include a mass squared -3 $N_\mathrm{f} \times N_\mathrm{f}$ scalar field, X, that describes the quark mass and condensate (here the integration constants equivalent to a, b above are input and fitted to QCD) — X's phase is the pion; a vector gauge field to describe the operator $\bar{q}\gamma^\mu q$ (and hence the ρ mesons); an axial gauge field to describe the operator $\bar{q}\gamma^5\gamma^\mu q$ (the a mesons). The action is

$$S = \int_{r_0}^{\infty} d^4x \, dr \, \sqrt{-g} \, \mathrm{Tr} \left[|DX|^2 + 3|X|^2 - \frac{1}{2g_5^2} \left(F_\mathrm{V}^2 + F_\mathrm{A}^2 \right) \right] . \tag{3}$$

For example, we can find the ρ meson mass by solving the vector equation of motion assuming a solution of the form $V \sim V(r) e^{-iq \cdot x}$, $q^2 = -M_\rho^2$. The UV boundary condition which corresponds to the operator is the normalizable solution $V \sim 1/r^2$. We can then numerically shoot into the IR boundary at r_0. In this simple model, one must pick a boundary condition at the wall, *e.g.* Neumann. Only for particular choices of M_ρ^2 will this boundary condition be achieved, which picks out the meson spectrum. Substituting the r dependent wave functions back into the action and integrating over r generates a four dimensional action with predictions for the couplings of the theory.

The model has four parameters — the hard wall r_0, the quark mass, the quark condensate and g_5 (this latter was fixed in the original paper by matching to the two point function of an external vector field). An example of the fit from [2] to a simple set of parameters is shown in following table.

	Value [MeV]	Fit [MeV]		Value [MeV]	Fit [MeV]
m_π	139.6 ± 0.0004	141	f_π	92.4 ± 0.35	84.0
m_ρ	775.8 ± 0.5	832	F_ρ	345 ± 8	353
m_a	1230 ± 40	1220	F_a	433 ± 13	440

The fit is surprisingly good. One would like to make a real effective field theory with the ability to estimate the errors due to missed terms. We would need to include all operators important in the vacuum and match the running of all operators' dimensions to the perturbative QCD results in the UV. However, the number of operators and couplings in the AdS Lagrangian grows very fast and it rapidly becomes un-predictive. AdS/QCD is therefore a model but the goodness of fit suggests that it can be a useful model in cases where the lattice is incapable of computing. The method is likely to be most effective in near conformal strongly coupled theories. The N_f dependence of QCD is then an interesting problem — the lattice has only just begun to study the problem and for some N_f there is believed to be an IR conformal regime.

2. SU(N_c) gauge theory with varying N_f

The two loop running of the gauge coupling in QCD (with general N_c and N_f) is given by

$$\mu \frac{d\alpha}{d\mu} = -b\alpha^2 - c\alpha^3 \,, \tag{4}$$

$$b = \frac{1}{6\pi}(11N_\mathrm{c} - 2N_F)\,, \qquad c = \frac{1}{24\pi^2}\left(34N_\mathrm{c}^2 - 10N_\mathrm{c}N_\mathrm{f} - 3\frac{N_\mathrm{c}^2-1}{N_\mathrm{c}}N_F\right) . \tag{5}$$

The one loop result for the anomalous dimension of the quark mass is

$$\gamma = \frac{3(N_\mathrm{c}^2-1)}{4N_\mathrm{c}\pi}\alpha \,. \tag{6}$$

Asymptotic freedom is present provided $N_\mathrm{f} > 11N_\mathrm{c}/2$. There is naively an IR fixed point with value $\alpha_* = -b/c$. The fixed point begins at perturbative values of the coupling for $N_\mathrm{f} \simeq 11N_\mathrm{c}/2$ then rises to infinity at

$N_{\rm f} \sim 2.6N_{\rm c}$. At some critical value of $N_{\rm f}$, the behaviour presumably changes to QCD-like dynamics with a break from IR conformal to chiral symmetry breaking behaviour [4]. For theories lying just below the critical value of $N_{\rm f}$, γ is expected to run (at a scale $\Lambda_{\rm running}$) to an IR fixed point which in the deep IR is just sufficient to trigger chiral symmetry breaking (at a scale $\Lambda_{\rm IR}$). The IR condensate will have dimension $3-\gamma$. The condensate measured in the UV is dimension 3 and will therefore take the form $\langle \bar{q}q\rangle_{\rm UV} \simeq \Lambda_{\rm IR}^{3-\gamma}\Lambda_{\rm running}^{\gamma}$. Theories with this enhancement of the UV condensate are called walking gauge theories [5]. These are the behaviours with $N_{\rm f}$ we would like to study using holography. To do so, we need a holographic model that dynamically predicts chiral symmetry breaking.

3. Dynamic AdS/QCD

Dynamic AdS/QCD was introduced in detail in [3] based on top-down models of chiral symmtry breaking that lie close in theory space to $\mathcal{N}=4$ SYM [6]. The model maps onto the action of a probe D7 brane in an AdS geometry expanded to quadratic order. The gauge theory dynamics is included through the running of the anomalous dimension of the quark mass which is encoded through an AdS scalar's mass term that depends on the radial AdS coordinate ρ. The five dimensional action of our effective holographic theory is

$$S = \int d^4x\, d\rho\, {\rm Tr}\, \rho^3 \left[\frac{1}{\rho^2+|X|^2}|DX|^2 + \frac{\Delta m^2}{\rho^2}|X|^2 + \frac{1}{2g_5^2}\left(F_{\rm V}^2+F_{\rm A}^2\right)\right] . \quad (7)$$

X, $F_{\rm V}$ and $F_{\rm A}$ play the same roles as the equivalent fields in the simplest AdS/QCD model.

The five dimensional metric used to contract indices is

$$ds^2 = \frac{d\rho^2}{(\rho^2+|X|^2)} + \left(\rho^2+|X|^2\right)dx^2. \quad (8)$$

$|X|$ enters into the effective radial coordinate in the space, *i.e.* there is an effective $r^2=\rho^2+|X|^2$.

The key features of the model beyond the simple AdS/QCD model are:

- X is now a dynamical field whose profile is determined by its equations of motion. The quark mass is chosen in the UV and we impose $X'(0)=0$ — the condensate is then a prediction of the dynamics.
- The v.e.v. of X enters into the AdS metric and the presence of a mass or condensate generates an IR wall in r automatically.

- The specific gauge theory dynamics is introduced through $\Delta m^2(r)$ using the relation $\Delta(\Delta - 4) = M^2$ and $\Delta = 3 - \gamma(r)$ for the quark condensate.

We input the dynamics through our assumed form for $\gamma(r)$. We will use the one loop result in (6) with the two loop running of α in (4) (we set $\mu = r$). It is important to stress that these perturbative results are not expected to be applicable in the non-perturbative regime but they are a decent guess as to the dynamics involved. To demonstrate the strength of the model I will now report on some of its predictions and successes.

$\gamma = 1$ criticality condition: A key question is for what value of $N_{\rm f}$ does chiral symmetry breaking set in for massless quarks. When does the field X experience an unstable potential and obtain a v.e.v.? In AdS instability for a scalar field occurs when $M^2 = -4$ (the Breitenlohner–Freedman bound [8]). Given $\Delta(\Delta - 4) = M^2$ this corresponds to $\Delta = 2$ or $\gamma = 1$. This seems a robust prediction of the AdS description. It also matches the condition obtained from solving gap equations [7]. Using our perturbative ansatz for the running of γ, a fixed point value of 1 occurs for $N_{\rm f} \simeq 4N_{\rm c}$.

Hyperscaling relations in the conformal window: In the region $4N_{\rm c} \leq N_{\rm f} \leq 11N_{\rm c}/2$, the massless theory has an IR conformal fixed point. To extract predictions it is then useful to put in a quark mass as a scale and look at the dependence of quantities on the mass. If in the conformal regime we consider the case with fixed but non-zero $\gamma < 1$, then the solution of the equation of motion for X is

$$|X| = \frac{m}{\rho^{\gamma}} + \frac{\bar{q}q}{\rho^{2-\gamma}}\,. \tag{9}$$

If we impose the on mass shell boundary condition $X'(\rho = X) = 0$, then we find

$$\bar{q}q \sim m^{\frac{3-\gamma}{1+\gamma}}\,. \tag{10}$$

The power is that expected from dimensional analysis and the model correctly reproduces the expected hyperscaling relations.

Walking dynamics: For $N_{\rm f}$ just below $4N_{\rm c}$, we expect walking behaviour. It is straightforward to compute the UV quark condensate and compare it to an IR quantity such as f_π^3. In Fig. 1 (left) we plot this ratio against $N_{\rm f}$ (which we treat as a continuous variable) for SU(3) gauge theory. We see that this ratio indeed diverges and the model reproduces walking behaviour. Since, at the phase transition, $\langle\bar{q}q\rangle_{\rm UV}$ grows but the height of the effective potential between the true minimum and $\langle\bar{q}q\rangle_{\rm UV} = 0$ is determined by the IR scale, we expect the effective potential to be very flat and the σ mode in the model to be light relative to the rest of the spectrum. In Fig. 1 (right) we observe this phenomena in our model also. Further analysis of the spectrum can be found in [3].

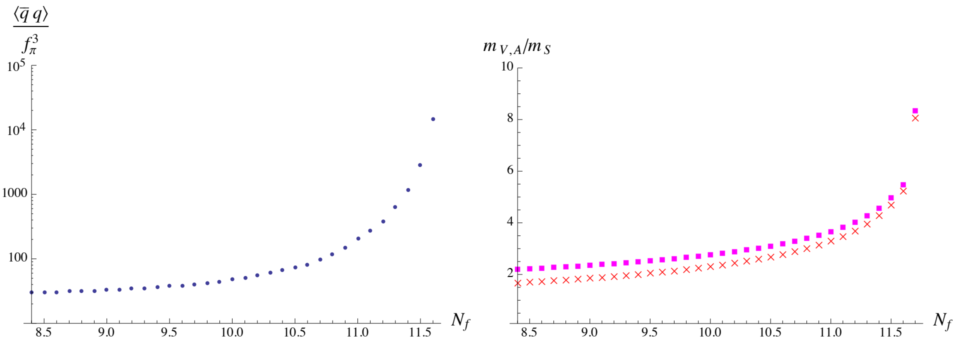

Fig. 1. Plot of (left) $\langle \bar{q}q\rangle/f_\pi^3$ and (right) the vector and axial meson masses over the σ mass against $N_{\rm f}$ for SU(3) gauge theory.

Dynamic AdS/QCD is therefore an easy to compute toy description of the N_F dependence of gauge theory. We hope that it will be a useful guide for lattice practitioners studying this problem and for Beyond the Standard Model physics. An alternative holographic model of this dynamics is explored in [9].

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