

LOW- x EVOLUTION EQUATION FOR PROTON GREEN FUNCTION*

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In the next-to-leading order, we discuss the low- x evolution equation for the baryon Wilson loop operator, which is a natural model for the Green function describing proton scattering in the Regge limit.

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1. Introduction

The description of the proton scattering in the framework of k_T -factorization can be addressed within the high energy operator expansion developed in [1]. In that paper, this method was applied to derive the full leading order (LO) hierarchy of the low- x evolution equations for Wilson lines with arbitrary indices and to the most important case of the color dipole. In the next-to-leading order (NLO), the evolution equation for the color dipole was derived in [2–4] and the connected evolution of the 3 Wilson lines was found in [5]. Finally, the full NLO hierarchy of the low- x evolution equations was written in [6] and the JIMWLK Hamiltonian equivalent to it in [7].

In this framework, the amplitude in the Regge limit can be written as a convolution of the impact factors and the matrix elements of the Wilson line operators. The impact factors consist of the wavefunctions of the incoming and outgoing particles, which describe their splitting into the quarks and gluons propagating through the shockwave formed by the other particle. The Wilson lines describe this propagation. For the virtual photon or meson scattering, the relevant Wilson line operator is a color dipole. In the

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proton case assuming SU(3) symmetry, it is the baryon or 3-quark Wilson loop (3QWL) $\varepsilon^{i'j'h'}\varepsilon_{ijh}U_{1i'}^iU_{2j'}^jU_{3h'}^h$. Its leading order (LO) linear evolution equation was studied in the C-odd case within the JIMWLK formalism and proved equivalent to the C-odd BKP equation [8, 9] in [10] and its nonlinear evolution equation was derived within Wilson line approach [1] in [11]. The connected contribution to the kernel of the equation was calculated in [5]. In the momentum representation, the evolution of this operator was first studied in [12], and the nonlinear equation was worked out in [13]. In the C-odd case, the linear NLO evolution equation for the odderon Green function was obtained in [14]. Here in (7) and (16), the NLO evolution equation for the 3QWL operator is presented. These results are based on the study in progress [15].

We use the following notation. We introduce the light cone vectors n_1 and n_2

$$n_1 = (1, 0, 0, 1), \quad n_2 = \frac{1}{2}(1, 0, 0, -1), \quad n_1^+ = n_2^- = n_1 n_2 = 1, \quad (1)$$

and for any vector p , we have

$$p k = p^\mu k_\mu = p^+ k^- + p^- k^+ - \vec{p} \vec{k} = p_+ k_- + p_- k_+ - \vec{p} \vec{k}. \quad (2)$$

We define the 3QWL operator as

$$B_{123} = \varepsilon^{i'j'h'}\varepsilon_{ijh}U_{1i'}^iU_{2j'}^jU_{3h'}^h = U_1 \cdot U_2 \cdot U_3, \quad (3)$$

where the Wilson line

$$U_i = U(\vec{r}_i, \eta) = P e^{ig \int_{-\infty}^{+\infty} b_\eta^-(r^+, \vec{r}) dr^+}, \quad (4)$$

and b_η^- is the external shock wave field built from only slow gluons

$$b_\eta^- = \int \frac{d^4 p}{(2\pi)^4} e^{-ipz} b^-(p) \theta(e^\eta - p^+) . \quad (5)$$

The parameter η separates the slow gluons entering the Wilson lines from the fast ones in the impact factors. The field

$$b^\mu(r) = b^-(r^+, \vec{r}) n_2^\mu = \delta(r^+) b(\vec{r}) n_2^\mu. \quad (6)$$

We denote $\vec{r}_1, \vec{r}_2, \vec{r}_3$ as the coordinates of the quarks within the 3QWL, and \vec{r}_0, \vec{r}_4 as the coordinates of the gluons. Hereafter we set $N_c = 3$ explicitly.

2. NLO evolution equation for the 3QWL

Taking the LO equation from [11] (2.35) and using the results of [4, 6] and [5], we can write the NLO evolution equation for 3QWL operator [15] as

$$\begin{aligned}
\frac{\partial B_{123}}{\partial \eta} = & \frac{\alpha_s(\mu^2)}{8\pi^2} \int d\vec{r}_0 \left[(B_{100}B_{320} + B_{200}B_{310} - B_{300}B_{210} - 6B_{123}) \right. \\
& \times \left\{ \frac{\vec{r}_{12}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2} - \frac{3\alpha_s}{4\pi} \beta \left[\ln \left(\frac{\vec{r}_{01}^2}{\vec{r}_{02}^2} \right) \left(\frac{1}{\vec{r}_{02}^2} - \frac{1}{\vec{r}_{01}^2} \right) - \frac{\vec{r}_{12}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2} \ln \left(\frac{\vec{r}_{12}^2}{\vec{\mu}^2} \right) \right] \right\} \\
& - \frac{\alpha_s}{\pi} \ln \frac{\vec{r}_{20}^2}{\vec{r}_{21}^2} \ln \frac{\vec{r}_{10}^2}{\vec{r}_{21}^2} \left\{ \frac{1}{2} \left[\frac{\vec{r}_{13}^2}{\vec{r}_{10}^2 \vec{r}_{30}^2} - \frac{\vec{r}_{32}^2}{\vec{r}_{30}^2 \vec{r}_{20}^2} \right] (B_{100}B_{320} - B_{200}B_{310}) - \frac{\vec{r}_{12}^2}{\vec{r}_{10}^2 \vec{r}_{20}^2} \right. \\
& \times \left. \left(9B_{123} - (B_{100}B_{320} + B_{200}B_{130}) + \frac{1}{2}B_{300}B_{120} \right) \right\} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \\
& - \frac{\alpha_s^2 n_f}{16\pi^4} \int d\vec{r}_0 d\vec{r}_4 \left[\left\{ \left(\frac{1}{3} (U_1 U_0^\dagger U_4 + U_4 U_0^\dagger U_1) \cdot U_2 \cdot U_3 - \frac{1}{9} B_{123} \text{tr} (U_0^\dagger U_4) \right. \right. \right. \\
& + (U_1 U_0^\dagger U_2) \cdot U_3 \cdot U_4 + \frac{1}{6} B_{123} - \frac{1}{4} (B_{013}B_{002} + B_{001}B_{023} - B_{012}B_{003}) \\
& \left. \left. \left. + (1 \leftrightarrow 2) + (0 \leftrightarrow 4) \right\} L_{12}^q + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right] \\
& - \frac{\alpha_s^2}{8\pi^4} \int d\vec{r}_0 d\vec{r}_4 \left[\left\{ \tilde{L}_{12} (U_0 U_4^\dagger U_2) \cdot (U_1 U_0^\dagger U_4) \cdot U_3 \right. \right. \\
& + L_{12} \left[(U_0 U_4^\dagger U_2) \cdot (U_1 U_0^\dagger U_4) \cdot U_3 + \text{tr} (U_0 U_4^\dagger) (U_1 U_0^\dagger U_2) \cdot U_3 \cdot U_4 \right. \\
& - \frac{3}{4} [B_{144}B_{234} + B_{244}B_{134} - B_{344}B_{124}] + \frac{1}{2} B_{123} \left. \right] + (M_{13} - M_{12} - M_{23} + M_2) \\
& \times \left[(U_0 U_4^\dagger U_3) \cdot (U_2 U_0^\dagger U_1) \cdot U_4 + (U_1 U_0^\dagger U_2) \cdot (U_3 U_4^\dagger U_0) \cdot U_4 \right] \\
& \left. \left. \left. + (\text{all 5 permutations } 1 \leftrightarrow 2 \leftrightarrow 3) \right\} + (0 \leftrightarrow 4) \right]. \quad (7)
\end{aligned}$$

Here,

$$\begin{aligned}
L_{12} = & \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{24}^2}{\vec{r}_{14}^2 \vec{r}_{02}^2} \right) \left[\frac{\vec{r}_{12}^2}{8\vec{r}_{04}^2} \left(\frac{1}{\vec{r}_{02}^2 \vec{r}_{14}^2} - \frac{1}{\vec{r}_{01}^2 \vec{r}_{24}^2} \right) + \frac{1}{\vec{r}_{01}^2 \vec{r}_{24}^2 - \vec{r}_{02}^2 \vec{r}_{14}^2} \right. \\
& \times \left. \left(-\frac{\vec{r}_{12}^4}{8} \left(\frac{1}{\vec{r}_{01}^2 \vec{r}_{24}^2} + \frac{1}{\vec{r}_{02}^2 \vec{r}_{14}^2} \right) + \frac{\vec{r}_{12}^2}{\vec{r}_{04}^2} - \frac{\vec{r}_{02}^2 \vec{r}_{14}^2 + \vec{r}_{01}^2 \vec{r}_{24}^2}{4\vec{r}_{04}^4} \right) \right] + \frac{1}{2\vec{r}_{04}^4}, \quad (8)
\end{aligned}$$

$$\tilde{L}_{12} = \frac{\vec{r}_{12}^2}{8\vec{r}_{04}^2} \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{24}^2}{\vec{r}_{14}^2 \vec{r}_{02}^2} \right) \left[\frac{\vec{r}_{12}^2 \vec{r}_{04}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2 \vec{r}_{14}^2 \vec{r}_{24}^2} - \frac{1}{\vec{r}_{01}^2 \vec{r}_{24}^2} - \frac{1}{\vec{r}_{02}^2 \vec{r}_{14}^2} \right], \quad (9)$$

$$L_{12}^q = \frac{1}{\vec{r}_{04}^4} \left[\frac{\vec{r}_{02}^2 \vec{r}_{14}^2 + \vec{r}_{01}^2 \vec{r}_{24}^2 - \vec{r}_{04}^2 \vec{r}_{12}^2}{2(\vec{r}_{02}^2 \vec{r}_{14}^2 - \vec{r}_{01}^2 \vec{r}_{24}^2)} \ln \left(\frac{\vec{r}_{02}^2 \vec{r}_{14}^2}{\vec{r}_{01}^2 \vec{r}_{24}^2} \right) - 1 \right], \quad (10)$$

$$M_{12} = \frac{\vec{r}_{12}^2}{16 \vec{r}_{04}^2} \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{02}^2}{\vec{r}_{14}^2 \vec{r}_{24}^2} \right) \left[\frac{\vec{r}_{12}^2 \vec{r}_{04}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2 \vec{r}_{14}^2 \vec{r}_{24}^2} - \frac{1}{\vec{r}_{01}^2 \vec{r}_{24}^2} - \frac{1}{\vec{r}_{02}^2 \vec{r}_{14}^2} \right], \quad (11)$$

$$M_2 = \frac{1}{4 \vec{r}_{04}^2 \vec{r}_{34}^2} \ln \left(\frac{\vec{r}_{02}^2}{\vec{r}_{24}^2} \right) \left[\frac{\vec{r}_{12}^2 \vec{r}_{23}^2 \vec{r}_{04}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2 \vec{r}_{24}^2} - \frac{\vec{r}_{14}^2 \vec{r}_{23}^2}{\vec{r}_{01}^2 \vec{r}_{24}^2} - \frac{\vec{r}_{03}^2 \vec{r}_{12}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2} + \frac{\vec{r}_{13}^2}{\vec{r}_{01}^2} \right]. \quad (12)$$

The $\overline{\text{MS}}$ renormalization scale μ^2 is related to scale $\tilde{\mu}^2$ through

$$\beta \ln \frac{1}{\tilde{\mu}^2} = \left(\frac{11}{3} - \frac{2}{3} \frac{n_f}{3} \right) \ln \left(\frac{\mu^2}{4e^{2\psi(1)}} \right) + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9} \frac{n_f}{3}, \quad (13)$$

$$\beta = \left(\frac{11}{3} - \frac{2}{3} \frac{n_f}{3} \right). \quad (14)$$

The evolution equation for the composite 3QWL operator B_{123}^{conf}

$$B_{123}^{\text{conf}} = B_{123} + \frac{\alpha_s 3}{8\pi^2} \int d\vec{r}_4 \left[\frac{\vec{r}_{12}^2}{\vec{r}_{41}^2 \vec{r}_{42}^2} \ln \left(\frac{a \vec{r}_{12}^2}{\vec{r}_{41}^2 \vec{r}_{42}^2} \right) \left(-B_{123} + \frac{1}{6} (B_{144} B_{324} + B_{244} B_{314} - B_{344} B_{214}) \right) + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right] \quad (15)$$

reads [15]

$$\begin{aligned} \frac{\partial B_{123}^{\text{conf}}}{\partial \eta} &= \frac{\alpha_s (\mu^2)}{8\pi^2} \int d\vec{r}_0 \left[((B_{100} B_{320} + B_{200} B_{310} - B_{300} B_{210}) - 6B_{123})^{\text{conf}} \right. \\ &\times \left(\frac{\vec{r}_{12}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2} - \frac{3\alpha_s}{4\pi} \beta \left[\ln \left(\frac{\vec{r}_{01}^2}{\vec{r}_{02}^2} \right) \left(\frac{1}{\vec{r}_{02}^2} - \frac{1}{\vec{r}_{01}^2} \right) - \frac{\vec{r}_{12}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2} \ln \left(\frac{\vec{r}_{12}^2}{\tilde{\mu}^2} \right) \right] \right) + (1 \leftrightarrow 3) \\ &+ (2 \leftrightarrow 3) \left. \right] - \frac{\alpha_s^2}{32\pi^3} \int d\vec{r}_0 \left(B_{003} B_{012} \left[\frac{\vec{r}_{32}^2}{\vec{r}_{03}^2 \vec{r}_{02}^2} \ln^2 \left(\frac{\vec{r}_{32}^2 \vec{r}_{10}^2}{\vec{r}_{13}^2 \vec{r}_{20}^2} \right) \right. \right. \\ &- \left. \left. \frac{\vec{r}_{12}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2} \ln^2 \left(\frac{\vec{r}_{12}^2 \vec{r}_{30}^2}{\vec{r}_{13}^2 \vec{r}_{20}^2} \right) \right] + (\text{all 5 permutations } 1 \leftrightarrow 2 \leftrightarrow 3) \right) \\ &- \frac{\alpha_s^2 n_f}{16\pi^4} \int d\vec{r}_0 d\vec{r}_4 \left[\left\{ \left(\frac{1}{3} (U_1 U_0^\dagger U_4 + U_4 U_0^\dagger U_1) \cdot U_2 \cdot U_3 - \frac{1}{9} B_{123} \text{tr} (U_0^\dagger U_4) \right. \right. \right. \\ &+ (U_1 U_0^\dagger U_2) \cdot U_3 \cdot U_4 + \frac{1}{6} B_{123} - \frac{1}{4} (B_{013} B_{002} + B_{001} B_{023} - B_{012} B_{003}) \\ &\left. \left. \left. \right\} \right] \end{aligned}$$

$$\begin{aligned}
& + (1 \leftrightarrow 2) \Big) + (0 \leftrightarrow 4) \Big\} L_{12}^q + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \Big] \\
& - \frac{\alpha_s^2}{8\pi^4} \int d\vec{r}_0 d\vec{r}_4 \left(\left\{ \tilde{L}_{12}^C \left(U_0 U_4^\dagger U_2 \right) \cdot \left(U_1 U_0^\dagger U_4 \right) \cdot U_3 \right. \right. \\
& + L_{12}^C \left[\left(U_0 U_4^\dagger U_2 \right) \cdot \left(U_1 U_0^\dagger U_4 \right) \cdot U_3 + \text{tr} \left(U_0 U_4^\dagger \right) \left(U_1 U_0^\dagger U_2 \right) \cdot U_3 \cdot U_4 \right. \\
& \left. \left. - \frac{3}{4} [B_{144}B_{234} + B_{244}B_{134} - B_{344}B_{124}] + \frac{1}{2} B_{123} \right] \right. \\
& + M_{12}^C \left[\left(U_0 U_4^\dagger U_3 \right) \cdot \left(U_2 U_0^\dagger U_1 \right) \cdot U_4 + \left(U_1 U_0^\dagger U_2 \right) \cdot \left(U_3 U_4^\dagger U_0 \right) \cdot U_4 \right] \\
& \left. + (\text{all 5 permutations } 1 \leftrightarrow 2 \leftrightarrow 3) \Big\} + (0 \leftrightarrow 4) \right). \quad (16)
\end{aligned}$$

Here, the composite operator $((B_{100}B_{320} + B_{200}B_{310} - B_{300}B_{210}) - 6B_{123})^{\text{conf}}$ is defined according to model

$$O^{\text{conf}} = O + \frac{1}{2} \frac{\partial O}{\partial \eta} \Bigg|_{\frac{\vec{r}_{mn}^2}{\vec{r}_{im}^2 \vec{r}_{in}^2} \rightarrow \frac{\vec{r}_{mn}^2}{\vec{r}_{im}^2 \vec{r}_{in}^2} \ln \left(\frac{\vec{r}_{mn}^2}{\vec{r}_{im}^2 \vec{r}_{in}^2} \right)} , \quad (17)$$

$$L_{12}^C = L_{12} + \frac{\vec{r}_{12}^2}{4\vec{r}_{01}^2 \vec{r}_{04}^2 \vec{r}_{24}^2} \ln \left(\frac{\vec{r}_{02}^2 \vec{r}_{14}^2}{\vec{r}_{04}^2 \vec{r}_{12}^2} \right) + \frac{\vec{r}_{12}^2}{4\vec{r}_{02}^2 \vec{r}_{04}^2 \vec{r}_{14}^2} \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{24}^2}{\vec{r}_{04}^2 \vec{r}_{12}^2} \right) , \quad (18)$$

$$\tilde{L}_{12}^C = \tilde{L}_{12} + \frac{\vec{r}_{12}^2}{4\vec{r}_{01}^2 \vec{r}_{04}^2 \vec{r}_{24}^2} \ln \left(\frac{\vec{r}_{02}^2 \vec{r}_{14}^2}{\vec{r}_{04}^2 \vec{r}_{12}^2} \right) - \frac{\vec{r}_{12}^2}{4\vec{r}_{02}^2 \vec{r}_{04}^2 \vec{r}_{14}^2} \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{24}^2}{\vec{r}_{04}^2 \vec{r}_{12}^2} \right) , \quad (19)$$

$$\begin{aligned}
M_{12}^C &= \frac{\vec{r}_{12}^2}{16\vec{r}_{02}^2 \vec{r}_{04}^2 \vec{r}_{14}^2} \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{02}^2 \vec{r}_{34}^4}{\vec{r}_{03}^4 \vec{r}_{14}^2 \vec{r}_{24}^2} \right) + \frac{\vec{r}_{13}^2}{16\vec{r}_{01}^2 \vec{r}_{04}^2 \vec{r}_{34}^2} \ln \left(\frac{\vec{r}_{02}^4 \vec{r}_{14}^2 \vec{r}_{34}^2}{\vec{r}_{01}^2 \vec{r}_{03}^2 \vec{r}_{24}^4} \right) \\
&+ \frac{\vec{r}_{23}^2}{16\vec{r}_{02}^2 \vec{r}_{04}^2 \vec{r}_{34}^2} \ln \left(\frac{\vec{r}_{01}^4 \vec{r}_{03}^2 \vec{r}_{24}^6 \vec{r}_{34}^2}{\vec{r}_{02}^2 \vec{r}_{04}^4 \vec{r}_{14}^4 \vec{r}_{23}^4} \right) + \frac{\vec{r}_{23}^2}{16\vec{r}_{03}^2 \vec{r}_{04}^2 \vec{r}_{24}^2} \ln \left(\frac{\vec{r}_{02}^2 \vec{r}_{03}^2 \vec{r}_{14}^4}{\vec{r}_{01}^4 \vec{r}_{24}^2 \vec{r}_{34}^2} \right) \\
&+ \frac{\vec{r}_{13}^2}{16\vec{r}_{03}^2 \vec{r}_{04}^2 \vec{r}_{14}^2} \ln \left(\frac{\vec{r}_{02}^4 \vec{r}_{14}^2 \vec{r}_{34}^2}{\vec{r}_{01}^2 \vec{r}_{03}^2 \vec{r}_{24}^4} \right) + \frac{\vec{r}_{12}^2}{16\vec{r}_{01}^2 \vec{r}_{04}^2 \vec{r}_{24}^2} \ln \left(\frac{\vec{r}_{03}^4 \vec{r}_{04}^4 \vec{r}_{12}^4 \vec{r}_{24}^2}{\vec{r}_{01}^2 \vec{r}_{02}^6 \vec{r}_{14}^2 \vec{r}_{34}^4} \right) \\
&+ \frac{\vec{r}_{03}^2 \vec{r}_{12}^2}{8\vec{r}_{01}^2 \vec{r}_{02}^2 \vec{r}_{04}^2 \vec{r}_{34}^2} \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{03}^2 \vec{r}_{24}^4}{\vec{r}_{02}^2 \vec{r}_{04}^2 \vec{r}_{12}^2 \vec{r}_{34}^2} \right) + \frac{\vec{r}_{23}^2 \vec{r}_{12}^2}{8\vec{r}_{01}^2 \vec{r}_{02}^2 \vec{r}_{24}^2 \vec{r}_{34}^2} \\
&\times \ln \left(\frac{\vec{r}_{02}^2 \vec{r}_{12}^2 \vec{r}_{34}^2}{\vec{r}_{01}^2 \vec{r}_{23}^2 \vec{r}_{24}^2} \right) + \frac{\vec{r}_{14}^2 \vec{r}_{23}^2}{8\vec{r}_{01}^2 \vec{r}_{04}^2 \vec{r}_{24}^2 \vec{r}_{34}^2} \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{04}^2 \vec{r}_{23}^2 \vec{r}_{24}^2}{\vec{r}_{02}^4 \vec{r}_{14}^2 \vec{r}_{34}^2} \right) . \quad (20)
\end{aligned}$$

This equation has the correct dipole limit [4] and it matches the BFKL kernel [16].

3. Conclusion

We presented the NLO evolution equation for the 3-quark Wilson loop operator $\varepsilon^{i'j'h'}\varepsilon_{ijh}U_{1i'}^iU_{2j'}^jU_{3h'}^h$ based on the study in progress [15]. Kernel of this equation (7) has nonconformal terms not related to renormalization. We found composite 3QWL operator (15) obeying the NLO evolution equation with quasi-conformal kernel (16). Our results have correct dipole limit.

The 3QWL operator is a natural SU(3) model for a baryon Green function in the Regge limit. It is also the irreducible operator describing C-odd exchange. The evolution equation for the C-odd part of the 3QWL operator is the generalization of the BKP equation for odderon exchange to the saturation regime. However, it is valid for the colorless object. The linear approximation of the equation for the C-odd part of the 3QWL should be equivalent to the NLO BKP for odderon exchange acting in the space of functions describing colorless objects. One may try to restore the full NLO BKP kernel from our result via the technique similar to the one developed for the 2-point operators in [17].

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