

THE NON-ORDINARY REGGE BEHAVIOR OF THE $f_0(500)$ MESON*

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We review how the Regge trajectory of an elastic resonance can be obtained just from its pole position and coupling, by means of a dispersive formalism. This allows to deal correctly with the finite widths of resonances in Regge trajectories. For the $\rho(770)$ meson, this method leads to the ordinary linear Regge trajectory with a universal slope. In contrast, for the $f_0(500)$ meson, the resulting Regge trajectory is non-linear and with much smaller slope. This is another strong indication of the non-ordinary nature of the lightest scalar meson.

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1. Introduction

In a recent work [1], we used the analytic properties of amplitudes in the complex angular momentum plane to study the Regge trajectory associated to an elastic resonance. In principle, the form of these trajectories can be used to discriminate between the underlying QCD mechanisms that generate these resonances. Actually, linear (J, M^2) trajectories relating the angular momentum J and the mass squared are intuitively interpreted in terms of quark–antiquark states, since they are easily obtained from the rotation of a flux tube connecting a quark and an antiquark. Strong deviations from this linear behavior would suggest a rather different nature and the scale of the trajectory would also indicate the scale of the mechanism responsible for the presence of a resonance.

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In particular, we have studied in [1] the trajectories of the lightest resonances in elastic $\pi\pi$ scattering: the $\rho(770)$, which is a well established ordinary $\bar{q}q$ state, and the $f_0(500)$ or σ meson, whose nature is still under debate and whose resulting trajectory, as we will see, does not follow the ordinary linear (J, M^2) trajectories. Actually, scalar mesons, and particularly the sigma, are not included in those linear fits [2], or its huge width is used as the uncertainty in the mass, so that it could be accommodated easily. But as we will see, the width is part of the Regge trajectory, and considering the width as a mass uncertainty is not really justified.

2. Regge trajectory from a resonance pole

An elastic $\pi\pi$ partial wave near a Regge pole reads

$$t_l(s) = \beta(s)/(l - \alpha(s)) + f(l, s), \quad (1)$$

where $f(l, s)$ is a regular function of l , and the Regge trajectory $\alpha(s)$ and residue $\beta(s)$ are analytic functions, the former having a cut along the real axis for $s > 4m_\pi^2$.

The analytic properties of $\alpha(s)$ and $\beta(s)$ together with the elastic unitarity condition imply that they are constrained by the following system of coupled dispersion relations [3]:

$$\text{Re } \alpha(s) = \alpha_0 + \alpha' s + \frac{s}{\pi} \text{PV} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im } \alpha(s')}{s'(s' - s)}, \quad (2)$$

$$\text{Im } \alpha(s) = \frac{\rho(s)b_0 \hat{s}^{\alpha_0 + \alpha' s}}{|\Gamma(\alpha(s) + \frac{3}{2})|} \exp\left(-\alpha' s [1 - \log(\alpha' s_0)]\right) + \frac{s}{\pi} \text{PV} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im } \alpha(s') \log \frac{\hat{s}}{\hat{s}'} + \arg \Gamma(\alpha(s') + \frac{3}{2})}{s'(s' - s)}, \quad (3)$$

where PV denotes ‘‘principal value’’ and α_0, α' and b_0 are free parameters to be determined phenomenologically. For the $f_0(500)$ -meson, one can also make explicit in $\beta(s)$ the Adler-zero required by the chiral symmetry. In that case, b_0 will not be dimensionless.

3. $\rho(770)$ and $f_0(500)$ trajectories

For a given set of α_0, α' and b_0 parameters, we solve the system of Eqs. (2) and (3) iteratively. The value of the parameters is fixed by fitting only three inputs, namely, the real and imaginary parts of the resonance pole

position $s_M \simeq (M_R - i\Gamma/2)^2$, where M_R and Γ_R are the pole mass and width of the resonance, together with the absolute value of the pole residue $|g_M|$. Namely, we fit the resonance pole on the second Riemann sheet to: $\beta_M(s)/(l - \alpha_M(s)) \rightarrow |g_M^2|/(s - s_M)$, with $l = 0, 1$ for $M = \sigma, \rho$. The pole parameters are taken from a precise dispersive representation of $\pi\pi$ scattering data [4]. In Fig. 1 we compare, on the real axis, the partial waves obtained from data in [4] with the Regge amplitudes on the real axis obtained just from the pole fit. They do not need to overlap since they are only constrained to agree at the resonance pole. Nevertheless, we find a fair agreement in the resonant region. As expected, it deteriorates as we approach threshold or the inelastic region, particularly in the case of the S -wave due to the interference with the $f_0(980)$. But note that in the resonance region the Regge amplitude describes the empirical curve even better in the scalar than in the vector case.

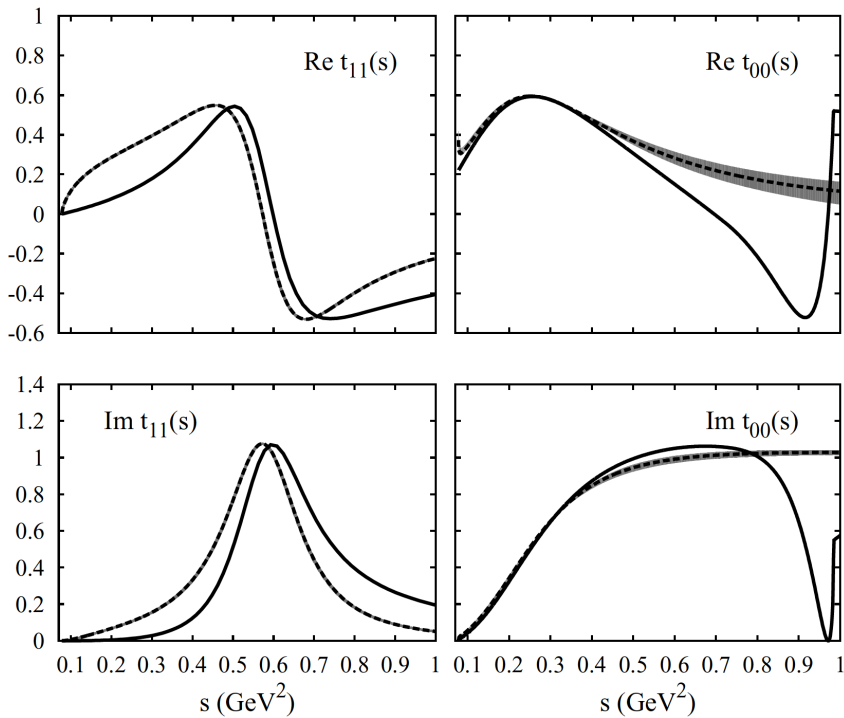


Fig. 1. Partial waves t_{lI} with $l = 1$ (left panels) and $l = 0$ (right panels). Solid lines represent the amplitudes from the dispersive analysis of data in [4]. Their corresponding resonance poles are the only input for the constrained Regge-pole amplitudes shown with dashed curves. The gray bands cover only the uncertainties due to the uncertainties in the inputs.

Therefore, once we have checked that fitting just the pole with a Regge-like amplitude still provides a fairly good description of the amplitude, we are ready to show their associated Regge trajectories calculated with our dispersive formalism. In particular, in the left panel of Fig. 2, we show the resulting Regge trajectories, whose parameters are given in Table I. The imaginary part of $\alpha_\rho(s)$ is much smaller than the real part, and the latter grows linearly with s . Taking into account our approximations, and that our error bands only reflect the uncertainty in the input pole parameters, the agreement with previous determinations is remarkable: $\alpha_\rho(0) = 0.52 \pm 0.02$ [5], $\alpha_\rho(0) = 0.450 \pm 0.005$ [6], $\alpha'_\rho \simeq 0.83 \text{ GeV}^{-2}$ [2], $\alpha'_\rho = 0.9 \text{ GeV}^{-2}$ [5], or $\alpha'_\rho \simeq 0.87 \pm 0.06 \text{ GeV}^{-2}$ [7].

TABLE I

Parameters of the $\rho(770)$ and $f_0(500)$ trajectories.

| | α_0 | $\alpha' [\text{GeV}^{-2}]$ | $b_0 [\text{GeV}^{-2}]$ |
|-------------|----------------------------|-----------------------------|-------------------------|
| $\rho(770)$ | 0.520 ± 0.002 | 0.902 ± 0.004 | 0.52 |
| $f_0(500)$ | $-0.090^{+0.004}_{-0.012}$ | $0.002^{+0.050}_{-0.001}$ | 0.12 |

In contrast, the $f_0(500)$ trajectory is not evidently linear and its slope is about two orders of magnitude smaller than that of the linear trajectories of ordinary quark–antiquark resonances, *e.g.*, ρ , a_2 , f_2 or π_2 . This provides strong support for a non-ordinary nature of the σ meson. Moreover, the resulting scale of tens of MeV or at most hundreds, for the slope, is more typical of meson physics than of quark–antiquark interactions. In addition, the tiny slope excludes the possibility that any of the known isoscalar resonances may lie on its trajectory. To test the robustness of this observation, we have checked that our results are very stable within the uncertainties of the pole parameters that we used as input. In addition, we have tried to impose a typical size linear trajectory on the σ , but that deteriorates the fit to the σ pole and particularly to the coupling, so the resulting amplitude in the physical region is qualitatively very different from the observations.

Note that with our formalism, we are dealing correctly with the huge $f_0(500)$ width, and, by no means should it be considered as an uncertainty in the mass.

Furthermore, in Fig. 2 we show the striking similarities between the $f_0(500)$ trajectory and those of Yukawa potentials in non-relativistic scattering [8]. From the Yukawa $G = 2$ curve in that plot, which lies closest to our result for the $f_0(500)$, we can estimate $a \simeq 0.5 \text{ GeV}^{-1}$, following [8]. This could be compared, for instance, to the S -wave $\pi\pi$ scattering length $\simeq 1.6 \text{ GeV}^{-1}$. Thus, it seems that the range of a Yukawa potential that would mimic our low energy results is comparable but smaller than the $\pi\pi$

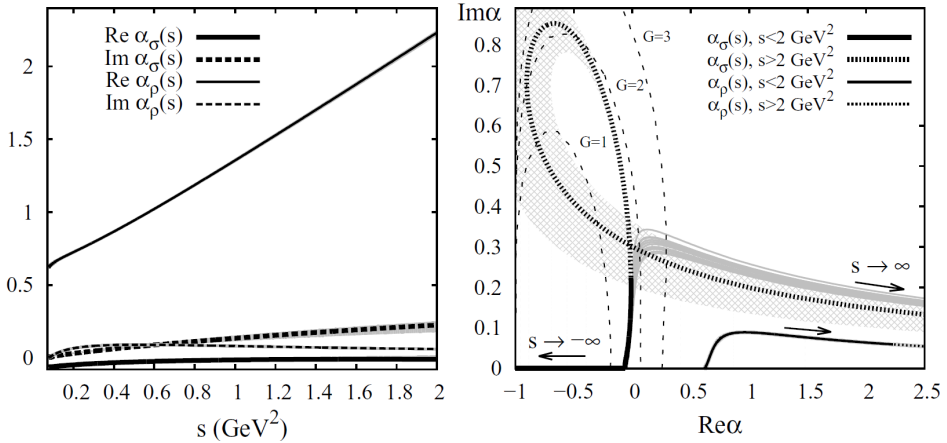


Fig. 2. (Left) $\alpha_\rho(s)$ and $\alpha_\sigma(s)$ Regge trajectories, from our constrained Regge-pole amplitudes. (Right) $\alpha_\sigma(s)$ and $\alpha_\rho(s)$ in the complex plane. At low and intermediate energies (thick continuous lines), the trajectory of the σ is similar to those of Yukawa potentials $V(r) = -Ga \exp(-r/a)/r$ [8] (thin dashed lines). Beyond 2 GeV^2 , we plot our results as thick discontinuous lines because they should be considered just as extrapolations.

scattering length in the scalar–isoscalar channel. Of course, our results are most reliable at low energies (thick continuous line) and the extrapolation should be interpreted cautiously. Nevertheless, our results suggest that the $f_0(500)$ looks more like a low-energy resonance of a short range potential, *e.g.* between pions, than a bound state of a confining force between a quark and an antiquark.

In summary, our formalism and the results for the $f_0(500)$ explain why the lightest scalar meson has to be excluded from the ordinary linear Regge fits of ordinary mesons.

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