

WHY $f_0(500)$ MUST BE NARROWER?*

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Proof of correctness of the results obtained in the recent analysis of the $\pi\pi$ interactions using new dispersion relations with imposed crossing symmetry condition is presented. The proof concerns position of the $f_0(500)$ (former σ) pole and is based on a purely mathematical relations and properties of analytic functions. It is shown that the mere analysis of amplitudes expressed by the trigonometric functions and their derivatives clearly define the area in which mass of the σ and its width must be located. These results require also a knowledge of integrals of amplitudes over the physical region.

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1. Introduction

The recent precise determination of the σ parameters was done in analysis of the $\pi\pi$ interactions using new dispersion relations — so-called GKPY equations, with imposed crossing symmetry condition [1, 2]. Although these results have changed parameters of that meson in the Particle Data Tables [3] and are widely accepted, one can still find analyses which use old amplitudes with significantly heavier and wider σ resonance. Therefore, it is worth to construct a new, simple and convincing proof of the results found in the dispersive analysis of the data.

Before the presentation of that proof let us, however, first respond to some frequently asked questions and raised doubts about the correctness of the dispersive analysis and results presented in [1] and [2].

One of these questions concerns uniqueness of those results. They have been obtained without any assumptions about the energy dependence of the $\pi\pi$ amplitudes. In contrast to this approach, another but similar analysis

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of the Roy equations [4] has been performed using two assumptions for the S -wave amplitude *i.e.* its values at 800 MeV and at the $\pi\pi$ threshold. Due to these two boundary conditions, it was possible to find unique analytical solution of the Roy equations below 800 MeV in accordance with the method described in [5]. The position of the σ pole obtained in [4] differs by less than one standard deviation from that received in [1, 2]. Therefore, one can be sure that this solution is also unique.

Another question concerns parameterization of the amplitudes used in [1, 2]. Although they were purely mathematical, *i.e.* without any physical bias, one can easily prove that, independently on the used parameterizations, the σ becomes lighter and narrower when the amplitudes are fitted to dispersion relations with imposed crossing symmetry. For example, in [6] was shown that using completely different parameterization of the so-called “old” amplitudes [7] supplemented by simple threshold expansion and fitted to experimental data and to the GKPY equations, one obtains the “new” amplitudes which give almost the same position of the σ pole as in [1, 2]. Its movement from that given by “old” amplitudes to the “new” one is presented in figure 1.

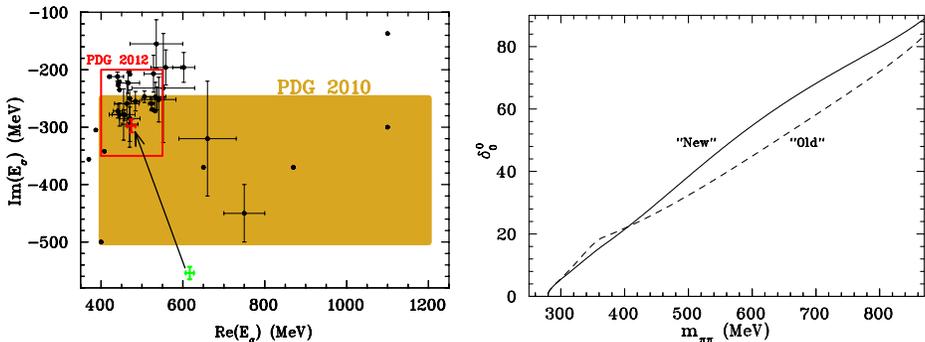


Fig. 1. Left panel: shift of the σ pole after fitting to the GKPY equations. Cross on the bottom denotes its position for the “old” amplitude, while cross indicated by the arrow its position for the “new” one. The big and smaller rectangle show area allowed by the Particle Data Tables in 2010 and 2012 respectively. Black points are positions of the poles listed in the Particle Data Tables published in 2010. Right panel: energy dependence of the phase shifts corresponding to the “new” and “old” amplitudes.

One of the objections to the Roy-like equations is that they can be applied only to the $\pi\pi \rightarrow \pi\pi$ scattering so they may ignore information from other coupled channels. Looking, however, at the full form of the *e.g.* GKPY equations (1), one sees that all other channels appear in the right-hand side of that equation and are fitted indirectly. Moreover, due to unitarity, below

about 1100 MeV *i.e.* below the $\eta\eta$ threshold, the inelasticity of the $\pi\pi \rightarrow K\bar{K}$ channel is equal to that of the $\pi\pi \rightarrow \pi\pi$ one. Therefore, the inelasticity of the $\pi\pi \rightarrow K\bar{K}$ channel is also fitted to the GKPY equations, moreover fitted directly.

2. Simple trigonometric proof

Full expression for the GKPY equations reads

$$\begin{aligned} \text{Re } t_\ell^{I(\text{OUT})}(s) &= \sum_{I'=0}^2 C^{II'} t_0^{I'(\text{IN})} (4m_\pi^2) \\ &+ \sum_{I'=0}^2 \sum_{\ell'=0}^4 \int_{4m_\pi^2}^\infty ds' K_{\ell\ell'}^{II'}(s, s') \text{Im } t_{\ell'}^{I'(\text{IN})}(s'), \end{aligned} \quad (1)$$

where the $C^{II'}$ is the crossing matrix constant and $K_{\ell\ell'}^{II'}(s, s')$ are kernels constructed for partial wave projected amplitudes with imposed $s \leftrightarrow t$ crossing symmetry condition. Given $t_J^I(s)$ amplitude fulfills this symmetry when the output amplitude $\text{Re } t_J^{I(\text{OUT})}(s)$ is equal to the input one $\text{Re } t_J^{I(\text{IN})}(s)$.

Figure 2 presents effective two pion mass $m_{\pi\pi}$ dependence of the input and output amplitudes before and after fitting to the GKPY equations. In Fig. 3 one can see that the real part of the amplitude increases with phase shifts below $\delta_0^0 = 45^\circ$ and decreases above this value. Knowing, from Fig. 2, the energy dependence of the input and output amplitudes, one could intuitively think that the former one should decrease below ≈ 650 MeV and increase above this energy. It corresponds to decrease of the phase shifts in both regions. Simultaneously, however, the OUT real part also changes due

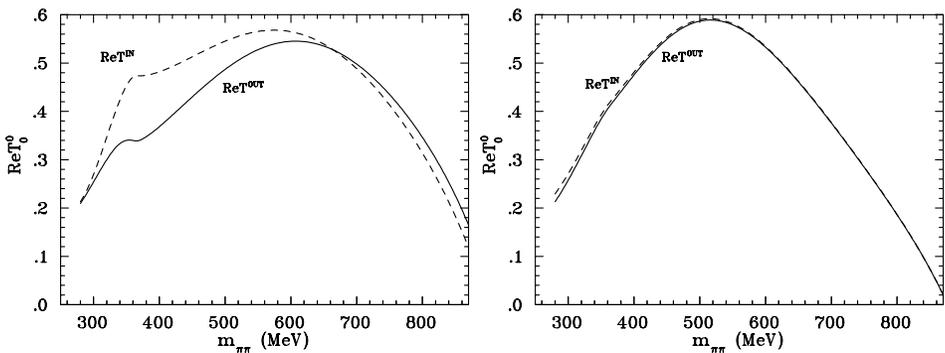


Fig. 2. Input (dashed line) and output (solid line) real parts of the amplitudes before fitting (left panel) and after fitting (the right panel) to the GKPY equations.

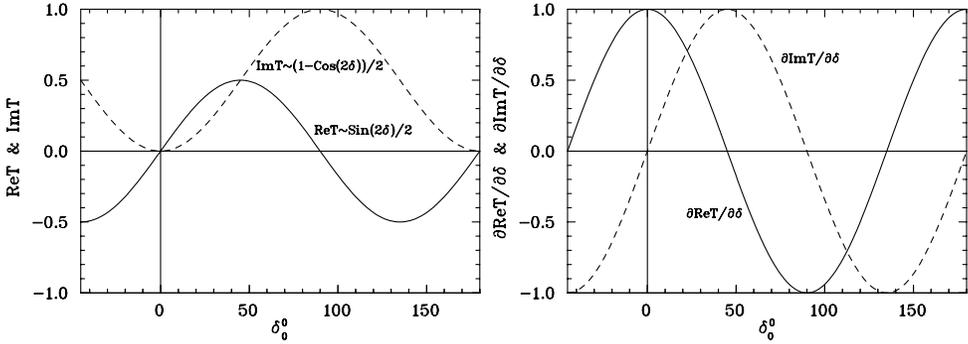


Fig. 3. Left panel: dependence of the real and imaginary part of the amplitude on the phase shifts; Right panel: gradient of the real and imaginary part of the amplitude in function of the phase shifts.

to variation of the IN amplitude in Eq. (1). As one can see in Fig. 3, below $\delta_0^0 = 90^\circ$, the IN part decreases together with decreasing phase shifts. The integral of the product $K_{00}^{00}(s, s')\text{Im}t_0^0(s')$ over s' (hereafter KT_{00}^{00} part) is presented in Fig. 4. Due to positive value of the $\text{Im}t_0^0(s')$ and its smooth variation below about 900 MeV the s -dependence of this integral is mostly given by the kernel part *i.e.* does not depend noticeably on parameterization of the phase shifts. Therefore, decrease of the phase shifts leads to decrease of the KT_{00}^{00} (*i.e.* the OUT amplitude) below about 650 MeV and to increase above this energy. Bigger gradient of the IN amplitude than that of the OUT seen in Fig. 3 in the energy range corresponding to δ_0^0 between 20° and 110° does not allow, however, for a reduction of the distance between the IN and OUT amplitudes. Therefore, the only possibility to fit the IN and OUT parts is to increase the phase shifts in the whole region below about 900 MeV. Using the same arguments as above, one can easily show that in this case the OUT amplitude will go up (down) below (above) about 650 MeV and will quickly catch up the IN amplitude. This situation one can see in Fig. 2 where the IN and OUT amplitudes after fitting are presented.

In Fig. 4 one can see decomposition of the KT_{00}^{00} into parts being integrals along the right and left cut. Dominant is the former part what contradicts opinion that introduction of the left cut into any amplitude can sufficiently mimic fulfillment of the crossing symmetry condition and thereby can be used instead of the fitting to the Roy-like equations.

Figure 5 presents an example of the phase shifts corresponding to the “old” amplitude [7] supplemented by near threshold expansion polynomial (extended amplitude) and next fitted to the GKPY equations (re-fitted amplitude). Clearly is seen a rise of the phase shifts related with movement of the σ pole towards the smaller mass and width.

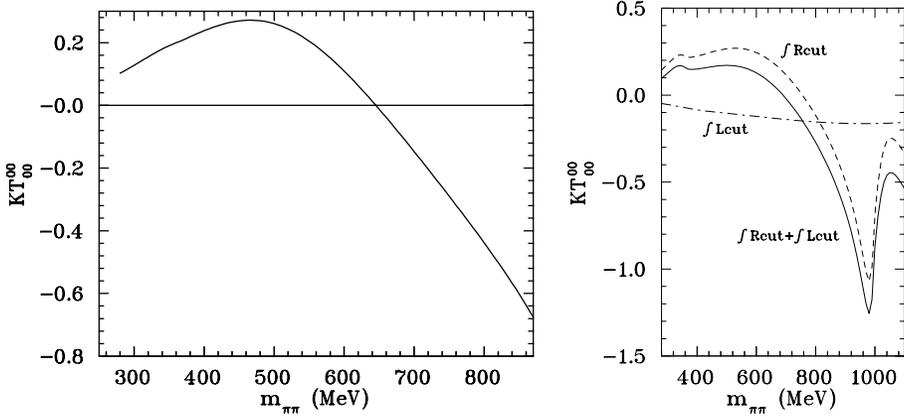


Fig. 4. Effective two pion mass dependence of the KT_{00}^{00} term (left panel) and of its left cut part (f_{Lcut}), right cut part (f_{Rcut}) and of their sum (right panel).

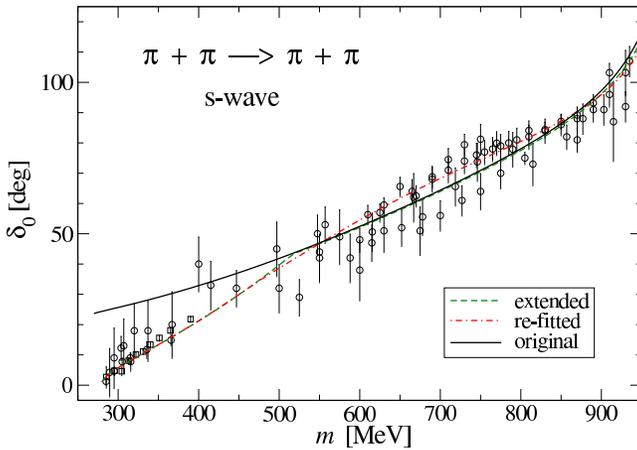


Fig. 5. Phase shifts for the S_0 wave as a function of effective two pion mass m for the three amplitudes considered in the text. Data are taken from [7].

3. Conclusions

Simple proof of correctness of the results obtained in the recent dispersive analysis of the data [1, 2, 4] has been presented. These results concern position of the σ pole which, due to this analysis, significantly moved in the complex energy plane towards the smaller mass and width. In the proof only purely mathematical properties of the tested amplitudes have been used. It has been shown that the shift of the σ pole is entirely due to the crossing symmetry condition imposed on the dispersion relations used in fitting of the S - and P -wave $\pi\pi$ amplitudes. Also answers on the most frequently

asked questions about the dispersive analysis of the data have been given. One can expect that this simple proof will eliminate still existing doubts about the parameters of σ meson.

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