SU(2) LANDAU GLUON PROPAGATOR AROUND CRITICALITY*

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We present a status report of our study of Landau-gauge gluon propagators for SU(2) gauge theory at finite temperature.

1. Introduction

An expected phenomenon in Yang–Mills theories at nonzero temperature is Debye screening of the color charge. In particular, at high temperatures, deconfinement should be felt in the longitudinal (i.e. electric) gluon propagator as an exponential fall-off at long distances, defining a screening length and conversely a screening mass [1]. Of course, it is not clear if and how such a mass would show up around the critical temperature T_c. At the same time, studies of the gluon propagator at zero temperature have shown a (dynamical) mass (see *e.g.* [2]). One can try to use this knowledge to define temperature-dependent masses for the region around T_c. Conversely, the dimensional-reduction picture (based on the 3D-adjoint-Higgs model) suggests a confined magnetic gluon, associated to a nontrivial magnetic mass. This mass should, in turn, be obtained from the infrared behavior of the transverse gluon propagator.

Lattice studies of the Landau-gauge gluon propagator around the deconfinement phase transition in pure SU(2) and SU(3) theory, as well as considering dynamical quarks, have been presented in [3–10]. Our SU(2) study has been reported in [11–15]. Let us discuss general findings of these studies.

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In the transverse (*i.e.* magnetic) sector, one sees strong infrared suppression of the propagator, with a turning point of the curve described by the momentum-space magnetic propagator $D_T(p^2)$ for momenta p around 400 MeV. This suppression seems even more pronounced than in the zero-temperature case discussed in Introduction. Also, $D_T(p^2)$ shows considerable finite-physical-size effects in the infrared limit, as observed for $T = 0$. Furthermore, just as for $T = 0$, the magnetic propagator displays a clear violation of reflection positivity in real space. Essentially the same features are seen for $D_T(p^2)$ at all nonzero temperatures considered.

The longitudinal propagator $D_L(p^2)$, on the other hand, shows significantly different behavior for different temperatures. As soon as a nonzero temperature is introduced in the system, $D_L(p^2)$ increases considerably (whereas $D_T(p^2)$ decreases monotonically). More precisely, for all fixed temperatures, the curve described by $D_L(p^2)$ seems to reach a plateau in the low-momentum region (see *e.g.* [12]). As the temperature is increased, this plateau increases slightly until, approaching the phase transition from below, it has been observed to rise further and then, just above the transition temperature, to drop sharply. This has been interpreted as a sign of singular behavior of the longitudinal gluon propagator around T_c and, in fact, it has been related to several proposals of a new order parameter for the deconfinement transition. (Of course, a relevant question is then, whether this singularity survives the inclusion of dynamical quarks in the theory [8, 9].)

Let us mention that, at all investigated temperatures, the infrared plateau just described is not long enough to justify a fit to the Yukawa form

$$D_L\left(p^2\right) = C\,\frac{1}{p^2 + m^2}\,, \tag{1}$$

predicted at high temperatures. If this were the case, $D_L(0)^{-1/2}$ would provide a natural (temperature-dependent) mass scale. Note that this value depends also on the global constant C. On the other hand, the so-called Gribov–Stingl forms involve complex-conjugate poles, defining real and imaginary masses (independently of C). Here we do not show data (or fits) for $D_L(p^2)$. Such curves and (preliminary) fits can be seen *e.g.* in [13]. Instead, we will look at the value of $D_L(0)$ (after normalization by C) as a function of T. This quantity has the disadvantage that it does not contain information on the length of the plateau and it also has large errors, but it is very sensitive to the temperature.

Concerning the longitudinal propagator in real space (see *e.g.* [13]), positivity violation is observed unequivocally only at zero temperature and for a few cases around the critical region, in association with the severe systematic errors discussed below. For all other cases, there is no violation

within errors. Also, we always observe an oscillatory behavior, indicative of a complex-mass pole. In the next section, we present our new results for the infrared values of $D_{\mathrm{L}}(p^2)$.

2. Results

Our large-lattice study was done considering the pure SU(2) case, with a standard Wilson action and lattice sizes $N_{\mathrm{s}}^3 \times N_{\mathrm{t}}$ ranging from $48^3 \times 4$ to $192^3 \times 16$. For our runs, we employ a cold start, performing a projection on positive-Polyakov-loop configurations. Also, gauge fixing is implemented using stochastic overrelaxation. The gluon dressing functions are normalized to 1 at 2 GeV. We considered several values of the lattice parameter β, allowing a broad range of temperatures. Our procedure for determining the physical temperature T is described in [12]. The momentum-space expressions for the transverse and longitudinal gluon propagators $D_{\mathrm{T}}(p^2)$ and $D_{\mathrm{L}}(p^2)$ can be found *e.g.* in [3].

As can be seen from the data in [13], the longitudinal (electric) propagator $D_{\mathrm{L}}(p^2)$ displays severe systematic effects around T_{c} for the smaller values of N_{t}. These effects are strongest at temporal extent $N_{\mathrm{t}} = 4$ and large values of N_{s}. We note that the systematic errors for small N_{t} come from two different sources: "pure" small-N_{t} effects (associated with discretization errors) and strong dependence on the spatial lattice size N_{s} at fixed N_{t}, for the cases in which the value of N_{t} is smaller than 16. The latter effect was observed only at temperatures slightly below T_{c}, whereas the former is present in a wider range of temperatures around T_{c}. In particular, the finite-spatial-volume effects for $D_{\mathrm{L}}(p^2)$ at $N_{\mathrm{t}} = 4$ are strongest at T_{c}, but are still very large at $T = 0.98T_{\mathrm{c}}$ and are much less pronounced for $T = 1.01T_{\mathrm{c}}$.

In Fig. 1 we show data for $D_{\mathrm{L}}(0)$ as a function of temperature T, for temperatures around the critical value T_{c}. We show such values as obtained from all our runs, grouping together (with the same symbol) the runs performed at the same temporal extent N_{t}. We remark that, as said above, not all curves of $D_{\mathrm{L}}(p^2)$ reach a clear plateau in the infrared limit. Nevertheless, looking at the value of $D_{\mathrm{L}}(0)$ gives us an indication of what this plateau might be, and is useful to expose the strong systematic effects discussed here.

We can see that the very suggestive sharp peak at T_{c} seen for $N_{\mathrm{t}} = 4$ (corresponding to the stars in Fig. 1) turns into a finite maximum around $0.9\,T_{\mathrm{c}}$ for $N_{\mathrm{t}} = 16$ (diamonds). In other words, the observed singularity at smaller values of N_{t} seems to disappear. The only indication of a possible singular behavior is a finite maximum close to (but not *at*) the critical point, somewhat reminiscent of a pseudo-critical point as observed for the magnetic susceptibility of spin models in an external magnetic field (see *e.g.* [16, 17]).
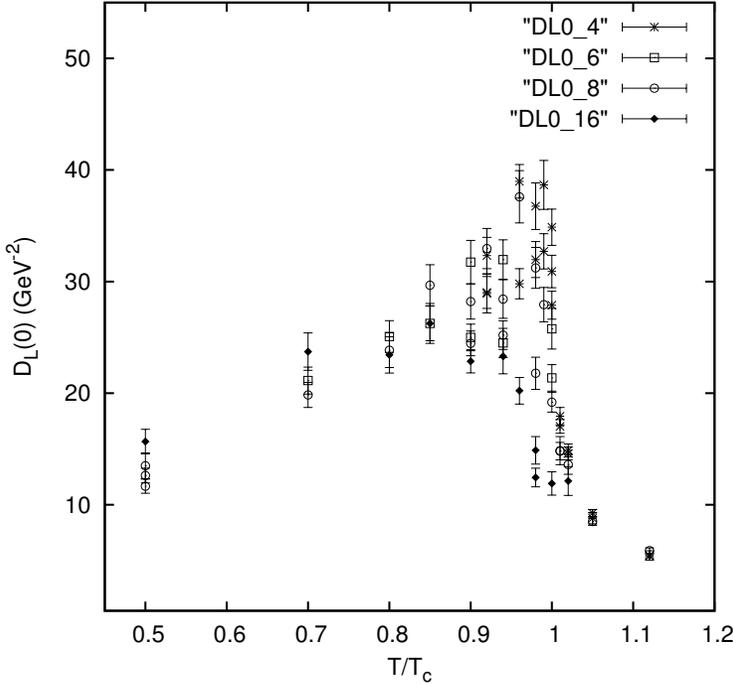
Fig. 1. Infrared-plateau value for the longitudinal gluon propagator [estimated by $D_{\mathrm{L}}(0)$] as a function of temperature for the values of T/T_{c} around criticality. Data points from runs at the same value of N_{t} are grouped together and indicated by the label "DL0_N_{t}".

Let us mention that, as reported in [13], good fits are obtained (in the transverse and longitudinal cases) to several generalized Gribov–Stingl forms, indicating the presence of comparable real and imaginary parts of pole masses. These masses are smooth functions of T around the transition, and the imaginary part of the electric mass seems to get smaller at higher T, as expected.

3. Conclusions

We have performed numerical simulations of the longitudinal (electric) and transverse (magnetic) Landau-gauge gluon propagator at nonzero temperature for pure SU(2) lattice gauge theory. We employ the largest lattices to date, especially for temperatures around the deconfinement phase transition. We are currently completing our study of fitting forms for describing the massive behavior of the propagator [18]. From our data for the longitudinal gluon propagator $D_{\mathrm{L}}(p^2)$, we have uncovered quite severe systematic effects.

Our results point to unusually large systematic errors around criticality. In particular, very strong effects related to small values of the temporal extent N_t of the lattice are seen on the lower side of the transition temperature and are practically absent just above T_c. Strong finite-size effects are certainly not unexpected around a second-order phase transition, such as the deconfinement transition in the SU(2) theory. On the other hand, we note that our data show a nontrivial dependence on the finite *temporal* size of the lattice and on the distance from the critical point, not easily interpreted as a finite-size or a discretization effect.

After removing these systematic effects, i.e. considering the data obtained with the largest value of N_t in Fig. 1, we see that the sharp peak suggested by the stars (and even the empty symbols) turns into a smooth maximum, at around $0.9\,T_c$. In agreement with several observations that the gluon mass scale is a smooth function of the temperature, this suggests that there is no specific signature of deconfinement associated with $D_L(p^2)$. In fact, the only definite qualitative feature of a deconfined phase we observe is the lack of violation of reflection positivity for the real-space electric propagator, which holds however for all $T \neq 0$ considered.

Finally, let us mention the similarity between our smaller-lattice results for the SU(2) case and existing results for SU(3), calling into question the possibility that the inverse of the zero-momentum value of the gluon propagator might provide an order parameter for the deconfinement phase transition.

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