UNDERSTANDING CONFINEMENT VIA INSTANTON-MONOPOLES*

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We discuss the role and importance of instanton-monopoles in QCD-like theories with and without matter. Most particularly we focus on SU(2) super QCD with heavy flavors.

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1. Introduction

Confinement is a problem which has puzzled physicists for many decades now. Although many models of QCD exist, analytically computable models of four-dimensional QCD-like theories are scarce and mostly supersymmetric, the best known of which is the Seiberg–Witten theory (SWT) [1]. The SWT, however, suffers of a severe drawback that it relies heavily on supersymmetries (SUSY), limiting its usability for non-SUSY theories.

The situation changes drastically if one spatial dimension is sacrificed globally, and instead of considering theories on $\mathbb{R}^4$, one considers theories on $\mathbb{R}^3 \times S^1$ with a stable center and a small compact circle $L \ll \Lambda^{-1}$, where Λ is the strong scale of the theory. Indeed, there exist both supersymmetric (SUSY) [2–5] and non-SUSY theories [6–8] over which one has complete theoretical control. In all of these theories, instanton-monopoles play an invaluable role. In addition, it has been suggested in [9] that instanton-monopoles have the potential to cure the divergences in the large orders of the perturbation theory and may help define the theory properly by doing a so-called resurgent series expansion. This has sparked some interest in the community [10–13].

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2. Super QCD

The instanton-monopoles are objects which appear in non-Abelian gauge theories on $\mathbb{R}^3 \times S^1$, as solutions of the YM equations with fixed non-zero asymptotic Polyakov loop $\mathrm{Tr}\,\Omega^\infty = \mathrm{Tr}\,e^{\int dx^4\, A_4(|\boldsymbol{x}|\to\infty)} \neq 0$. The Polyakov loop (or rather the A_4 component which acts like a compact Higgs field) breaks the SU(N) gauge symmetry to U$(1)^{N-1}$. In particular for SU(2) the gauge symmetry is broken down to U(1). As in the 3D Georgi–Glashow model, the theory has BPS monopoles with an action proportional to the Higgs VEV $S \propto v$, with $\sqrt{A_4^{a2}} = v$. In the 4D YM theory on $\mathbb{R}^3 \times S^1$, the Higgs field is compact and v is gauge equivalent to $v + 2\pi/L$. This distinct feature allows another set of solutions known as Kaluza–Klein, or KK monopoles with an action $S \propto (2\pi - vL)$. At the center symmetric point $v = \pi/L$, the two types of monopoles have the same action $S_0 = \frac{4\pi^2}{g^2}$ which is exactly half the instanton action. In fact, the BPS and the KK monopole together constitute an instanton [14–16]. It is useful to redefine $b' = \frac{4\pi}{g^2}vL - \frac{4\pi^2}{g^2}$ so that the BPS and KK monopole can be associated with the weights $e^{-S_0}e^{\mp b'}$ respectively. The $b' = 0$ corresponds to the center symmetric point. In addition, the effective 3D U(1) theory can be dualized to a theory with the Abelian dual field [17] σ (for our notations see *e.g.* [5, 18]), where (anti-)monopole charges couple as $e^{\pm i\sigma}$. The BPS and KK vertices become

$$[\mathrm{BPS}] \propto e^{-S_0}e^{-b' \pm i\sigma}\,, \qquad [\mathrm{KK}] \propto e^{-S_0}e^{b' \mp i\sigma}\,, \tag{1}$$

where the sign corresponds to self-dual and anti-self-dual solutions respectively.

In sQCD on $\mathbb{R}^3 \times S^1$, the center symmetry is not exact due to the presence of the fundamental multiplet and Polyakov loop screening is expected. In contrast to SYM, no mass gap can be generated by the instanton monopoles due to the presence of the fundamental zero modes attached to the monopole vertices [5]. To get somewhere, we consider fundamental multiplets with mass M as a continuos deformation of SYM which corresponds to infinite M limit.

A low energy effective theory is a theory of the scalar field b' and the dual field σ, both of which take values in the root space of the gauge group. These can combine to form a complex scalar $\phi = b' + i\sigma$, which, for the super YM (SYM) theory, is the lowest component of a chiral multiplet $\boldsymbol{B} = \phi + \sqrt{2}\theta\lambda + \ldots$ [1], where λ is the massless color component of the gaugino field.

[1] The relation between the chiral superfield and the field b' is actually not linear, due to the moduli space metric corrections. See the discussions in [5, 18].

The instanton-monopole resummation in SYM [2, 19] generates a superpotential of the form $\sim \cosh(\boldsymbol{B})$, which yields a bosonic potential

$$\sim \cosh(2b') - \cos(2\sigma)\,. \tag{2}$$

In [18] the second term was interpreted as a formation of the magnetic bions, objects important for IR physics in QCD(adj) [6] and Seiberg–Witten theory on $\mathbb{R}^3 \times S^1$ [3]. The first term was interpreted as a correlated pair contribution of a monopole and an anti-monopole, which belong to the sector of the perturbative vacuum. This term, having a minimum at $b' = 0$, is responsible for stabilizing the center and generating the confining potential in the Polyakov loop correlator.

In sQCD, however, center symmetry is broken by the presence of the fundamental multiplets and the identification of the lowest component of the superfield $\boldsymbol{B}$ is with $b' - \delta + i\sigma$, where δ is given by [5]

$$\delta \approx -\frac{2N_{\mathrm{f}}}{\pi}\sum_{n=1}^{\infty} \sin\left(\frac{nvL}{2}\right)\cos(n\phi)K_0(MLn)\,. \tag{3}$$

Above, ϕ is the periodicity phase of the fundamental multiplet, *i.e.* a constant Abelian holonomy. The effective potential (2) gets modified to $\sim \cosh(2(b' - \delta)) - \cos(2\sigma)$. The minimum is attained when $b' = \delta$, and, for large M the Polyakov loop average $\langle \mathrm{Tr}\,\Omega\rangle \sim N_{\mathrm{f}} e^{-ML}$. This is completely natural, as M can be interpreted as the cost of pulling a quark from the vacuum in order to screen one static quark. In addition, the expression (3) corresponds to the sum over objects with fundamental charge $e^{\pm in\frac{vL}{2}}$. The correlator

$$\langle \mathrm{Tr}\,\Omega(\boldsymbol{x})\,\mathrm{Tr}\,\Omega(\boldsymbol{y})\rangle \approx \langle \mathrm{Tr}\,\Omega\rangle^2 + (\dots)\frac{e^{-m_{\mathrm{el}}|\boldsymbol{x}-\boldsymbol{y}|}}{|\boldsymbol{x}-\boldsymbol{y}|L}\,, \tag{4}$$

where m_{el} is the mass of the b' field (the electric mass), and dots denote factors which depend on the renormalized coupling g^2 (for exact expressions, see [5]). The free energy of the heavy quark–anti-quark pair, which is the logarithm of the above expression, shows a linear raise until some distance $r_{\text{string breaking}} \propto M$, where the linear rise terminates and the free energy saturates to that of two independent static quarks.

3. Pure Yang–Mills

In the pure YM, no analytically tractable scenario exists from first principles. The center symmetry is stabilized by large fluctuations and no Abelianization can be argued. Nevertheless, one may try to employ the language of the Abelianized theory in order to gain some insight into the pure YM. In fact, in [18] a mechanism of confinement was proposed which is due to the formation of neutral bion term $\sim \cosh(2b')$. This term is second order in the semiclassical expansion, and it combats the term $-\cosh b' \cos\sigma$ coming from single-monopole terms, which contribute in pure YM as now there are no gaugino zero modes attached to them. The single monopole term is always minimized by $\sigma = 0$ and prefers the $b' \neq 0$, *i.e.* center symmetry broken. Moreover, the perturbative potential also prefers $b' \neq 0$. What is strange is that $\cos(2b')$ terms come with a relative negative sign compared to that of what would be naively expected [18]. Via analytical continuation in the coupling, the authors of this work were able to argue that the neutral bions have a center stabilizing contribution even in pure YM. In [20] it was suggested that this sign is due to the exclusion of the strongly overlapping monopole–anti-monopole pair, which belongs to the perturbative vacuum. This effectively introduces a negative fugacity to the neutral bions, so rather then viewing the YM vacuum as a liquid[2] of neutral bions, it is better viewed as a liquid of instanton-monopoles with a strong repulsing core. This core is negligible in dilute regimes, but it becomes vital in dense regimes. In addition, since depending on whether the monopole is a BPS monopole or a KK monopole, the core size of it grows or gets smaller as we move off-center. As this happens, the monopoles begin to push back on their anti-monopoles, trying to pack themselves as tightly as possible. The distance which has to be cut away is, however, not calculable and it was used as a parameter in [20]. The electric and magnetic masses and instanton-monopole densities were then compared to the lattice with order of magnitude agreement. We should warn, however, that the lattice measurements of instanton-monopole densities come with large uncertainties and it is unclear if they provide a decent test of phenomenological models such as the one proposed in [20]. The model is anyway difficult to justify at the scales $\sim \Lambda$ as fluctuations are strong, and mean field cannot be trusted, at least not from first principles.

Let us, nevertheless, give an argument why the instanton-monopoles are important even for low temperatures. To begin, let us construct an effective YM theory on $\mathbb{R}^3 \times S^1$ at any compact radius L for distances $\gg L$. At these distances, the theory must be three dimensional. In fact, the minimal gauge-invariant Lagrangian is[3]

$$\frac{L}{2g_{\text{eff}}^2}\mathrm{Tr}\, F_{ij}^2 + \Lambda^2 L \mathrm{Tr}\,(D_i\Omega)^\dagger(D_i\Omega) + LV(\Omega)\,, \tag{5}$$

[2] In the pure YM, the density of topological objects is not small, so a liquid rather then gas picture is more appropriate.

[3] We demand that the action scales properly with the compact radius of S^1, *i.e.* proportional to L.

where $\Omega(x)$ is the Polyakov loop, $V(\Omega)$ its potential, $g_{\rm eff}$ is a dimensionless effective coupling for the 3D components of the long wavelength YM fields and Λ is some dynamically generated scale required on dimensional grounds. The potential terms are required in order to have exponential decay of the Polyakov loop correlator of the form $e^{-\Sigma L r}$, where Σ is the string tension. Further, the potential has to have a minimum at $\mathrm{Tr}\,\Omega = 0$, so as to guarantee center symmetry. Because of this, the theory Abelianizes. Abelianization might be here misleading because, as oppose to the controllable theories discussed earlier, the Abelianization energy scale $1/L$ is much smaller then the mass gap $\propto e^{-\frac{(\dots)}{g^2(1/L)}}$ which is exponentially suppressed. In YM these two scales are expected to be the same, and no hierarchy of scales exists. Nevertheless, one can think of deforming YM in such a way so as to separate these two scales and then take the deformation to zero. The Higgs fields which get a mass from the kinetic Polyakov loop terms will anyway be irrelevant for area law, which is the salient feature of confinement. Once "Abelianization" sets in, the theory can be dualized, and since the UV theory has monopole solutions with proper asymptotic value of the Polyakov loop, these should appear as $\cos(n\sigma)$ (n-monopole contributions) terms in the dual theory, gapping the theory completely and inducing the area law for the Wilson loops.

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REFERENCES

[1] N. Seiberg, E. Witten, *Nucl. Phys.* **B426**, 19 (1994).

[2] N.M. Davies, T.J. Hollowood, V.V. Khoze, M.P. Mattis, *Nucl. Phys.* **B559**, 123 (1999).

[3] E. Poppitz, M. Ünsal, *J. High Energy Phys.* **1107**, 082 (2011).

[4] E. Poppitz, T. Schäfer, M. Ünsal, *J. High Energy Phys.* **1303**, 087 (2013).

[5] E. Poppitz, T. Sulejmanpasic, *J. High Energy Phys.* **1309**, 128 (2013).

[6] M. Ünsal, *Phys. Rev.* **D80**, 065001 (2009).

[7] M. Shifman, M. Ünsal, *Phys. Rev.* **D78**, 065004 (2008).

[8] M.M. Anber, E. Poppitz, M. Ünsal, *J. High Energy Phys.* **1204**, 040 (2012).

[9] P.C. Argyres, M. Ünsal, *J. High Energy Phys.* **1208**, 063 (2012).

[10] G.V. Dunne, M. Ünsal, *Phys. Rev.* **D87**, 025015 (2013).

[11] G.V. Dunne, M. Ünsal, *J. High Energy Phys.* **1211**, 170 (2012).

[12] A. Cherman, D. Dorigoni, G.V. Dunne, M. Ünsal, *Phys. Rev. Lett.* **112**, 021601 (2014).

[13] G. Basar, G.V. Dunne, M. Ünsal, *J. High Energy Phys.* **1310**, 041 (2013).

[14] K.-M. Lee, P. Yi, *Phys. Rev.* **D56**, 3711 (1997).

[15] K.-M. Lee, C.-h. Lu, *Phys. Rev.* **D58**, 025011 (1998).

[16] T.C. Kraan, P. van Baal, *Nucl. Phys.* **B533**, 627 (1998).

[17] A.M. Polyakov, *Nucl. Phys.* **B120**, 429 (1977).

[18] E. Poppitz, T. Schafer, M. Ünsal, *J. High Energy Phys.* **1210**, 115 (2012).

[19] N.M. Davies, T.J. Hollowood, V.V. Khoze, *J. Math. Phys.* **44**, 3640 (2003).

[20] E. Shuryak, T. Sulejmanpasic, *Phys. Lett.* **B726**, 257 (2013).