WHAT HAPPENED WITH THE $f_0(500)/\sigma$ MESON? THEORY AND EXPERIMENT*

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(Received December 5, 2014)

Presented are the most interesting and spectacular events in history of the σ meson and its almost rediscovery two years ago. Also proof of the uniqueness and correctness of the dispersion method used to precise determination of its parameters is shortly discussed. Example of a successful application of this method in modification of coupled channel $\pi\pi$, $K\bar{K}$ and $\eta\eta$ amplitudes fitted in past only to experimental data and not fulfilling crossing symmetry condition is mentioned.

DOI:10.5506/APhysPolBSupp.8.103
PACS numbers: 11.55.Fv, 11.55.–m, 11.80.Et, 13.75.Lb

1. Introduction

The $f_0(500)$ resonance, known also as σ, is the lightest scalar–isoscalar meson and has recently been re-parameterized and became subject of a number of innovative works in QCD physics [1–5]. After many years of problems with experimental determination of its parameters, this meson has been finally precisely described by means of theoretical dispersion relations with imposed crossing symmetry condition. Fundamental importance of this meson for low energy QCD opened up the possibility of, for example, constructive examination of its internal structure, study of delicate phenomena like CP violation in heavy meson decays via final strong pion–pion state interactions and quark condensate masses.

Its history started before year 1976 when it was called ϵ or σ. In years 1978–1992, it disappeared from Particle Data Tables and appeared again as $f_0(400–1200)$ in 1994. In years 2002–2010, it had the name $f_0(600)$ and finally since 2012, it is known as $f_0(500)$ [6]. The most striking are large differences between its parameters in those years. For example till 2010, the

* Talk presented at the EEF70 Workshop on Unquenched Hadron Spectroscopy: Non-Perturbative Models and Methods of QCD *vs.* Experiment, Coimbra, Portugal, September 1–5, 2014.

mass and width were known with very poor accuracy, M : 400–1200 MeV and Γ : 500–1000 MeV [7] but now are M : 400–550 MeV and Γ : 400–700 MeV.

2. Precise determination of the $\pi\pi$ amplitudes

Precise determination of the $\pi\pi$ amplitudes was possible after combined analyses of the experimental data and dispersion relation were performed [1, 8]. Two kinds of dispersion relations were used: with two subtractions (so-called Roy's equations) and with one subtraction (so-called GKPY equations). Both were derived with imposed crossing symmetry condition.

Much more demanding (more precise) are the GKPY equations and their general structure has a form

$$\begin{aligned}\mathrm{Re}\ t_\ell^{I(\mathrm{OUT})}(s) &= \sum_{I'=0}^{2} C^{II'} t_0^{I'(\mathrm{IN})}\left(4m_\pi^2\right) \\ &+ \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \fint_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}\left(s,s'\right) \mathrm{Im}\ t_{\ell'}^{I'(\mathrm{IN})}\left(s'\right), \end{aligned} \tag{1}$$

where the $C^{II'}$ is the constant crossing matrix and $K_{\ell\ell'}^{II'}(s,s')$ are kernels constructed for partial wave projected amplitudes with imposed $s \leftrightarrow t$ crossing symmetry condition. Given $t_J^I(s)$ amplitude fulfills this symmetry when the output amplitude $\mathrm{Re}\ t_J^{I(\mathrm{OUT})}(s)$ is equal to the input one $\mathrm{Re}\ t_J^{I(\mathrm{IN})}(s)$. Summation over all partial waves (in practice, up to F-wave) and integration over whole energy range provides unique opportunity to relate all these waves together for each energy.

Similar but twice subtracted dispersion relation were used in past to test the $\pi\pi$ S- and P-wave amplitudes. In result, the long-standing up–down ambiguity has been eliminated [9, 10]. Figure 1 presents the S-wave phase shifts for the both up and down solution.

In Table I there are presented differences between the twice subtracted Roy's equations and once subtracted GKPY ones. Due to higher power of energy in denominator of kernels, the former ones converge faster but linear term in energy squared in subtracting term leads to propagation of errors of the scattering lengths.

Figure 1 presents also experimental data for the phase shifts of the $S0$ wave together with central values and band allowed by the GKPY equations. As is seen, due to these equations uncertainty of the phase shifts decreased by factor about 6 in vicinity of 800 MeV. It eliminated ambiguities between various kinds of data and made possible construction of much more precise and reliable $\pi\pi$ amplitudes.

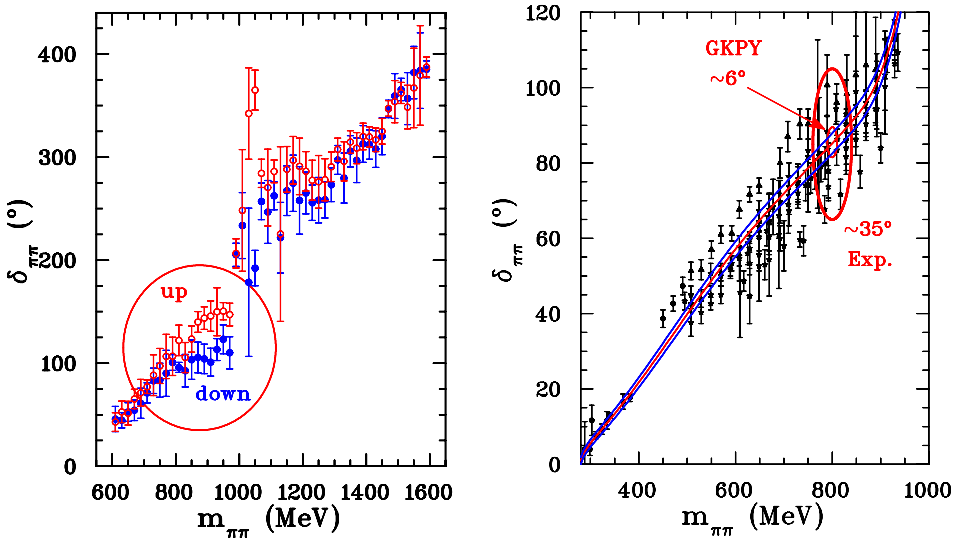

Fig. 1. Left hand-side figure: the $\pi\pi$ S-wave phase shifts with up–down ambiguity; right-hand side figure: experimental and given by the GKPY equations uncertainties of the phase shifts below 1 GeV. The later are given by three lines representing central values and band allowed by the GKPY equations. Numbers 35 and 6 are estimated errors of the experimental data and GKPY equations respectively.

TABLE I

Comparison of the Roy's and GKPY equations. The a_ℓ^I are $\pi\pi$ scattering lengths in the $I\ell$ wave and ST_0^0 are subtracting terms in the $S0$ wave.

Roy' 1971	GKPY' 2011
two subtractions	one subtraction
$K_{\ell\ell'}^{II'}(s,s') \sim s'^{-3}$ — fast convergence	$K_{\ell\ell'}^{II'}(s,s') \sim s'^{-2}$
$ST_0^0 = a_0^0 + \left(2a_0^0 - 5a_0^2\right)(s-4)$	$ST_0^0 = a_0^0 + 5a_0^2$ — no error propagation!

3. Precise determination of the σ pole position

Precise determination of the σ pole position was something like artefact of analyses done by the Bern and Madrid–Kraków groups [1, 8]. Particle Data Tables still present the mass and width of this resonance with estimated uncertainties higher that 100 MeV although these have been determined with much higher accuracy of only several MeV [2, 3]. Obtained precision of the mass and width (in fact, of the real and imaginary part of the σ pole) is directly related to the precision of the calculated S-wave amplitude.

Figure 2 presents differences in values estimated for the position of the σ pole (real and imaginary parts) before 2012 and after. As is seen, the Roy's and GKPY equations led to dramatic changes in these estimations. Following these new results the S $\pi\pi$ amplitudes can be parameterized with much higher precision.

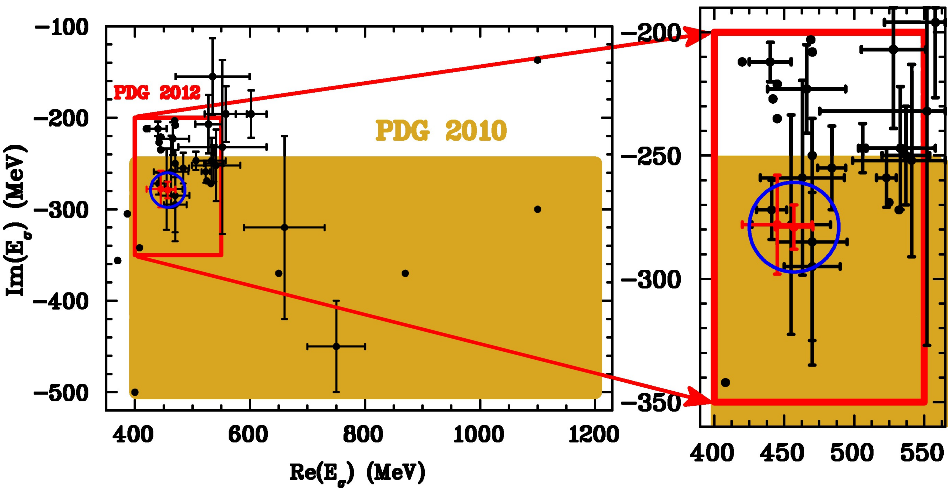

Fig. 2. Present and previous ranges of the real and imaginary parts of the σ pole estimated by the Particle Data Group (PDG2012 and PDG2010 respectively). In the middle of the circle, there are positions of this pole calculated by the Madrid–Kraków group [2] using the Roy's (left cross) and GKPY equations (the right one).

To check how the GKPY equations can modify amplitudes determined previously without crossing symmetry condition, these dispersion relations have been applied to the S- and P-wave amplitudes from analysis presented in [11]. The mass and the width of the σ meson in this analysis had values of about several hundred MeV what significantly differed from those obtained by the Bern and Madrid–Kraków groups. After application of the GKPY equations the σ pole shifted and placed in vicinity of the position found by both these groups. Results of this practical application of the GKPY equations were presented in [12].

Question of uniqueness of results obtained by the Bern and Madrid–Kraków groups and confirmed later in [12] can be easily proved using simple trigonometric arguments. As presented in [12], due to some trigonometric relations fulfilled by the $\pi\pi$ phase shifts in the S-wave, the crossing symmetry can be fulfilled only when the real and imaginary parts of the σ pole are in vicinity of those indicated by both groups. This proof can definitely finish discussions and eliminate doubts about the uniqueness of the results.

4. Conclusions

Presented were most important modifications of the $\pi\pi$ amplitudes and related with them positions of the σ pole. The described method and results, their uniqueness and precision should facilitate modification of often used incorrect $\pi\pi$ amplitudes. This should significantly increase the reliability of obtained results (*e.g.* in decays of heavy mesons) and accelerate research on other light mesons — candidates for being the lightest non-quark–antiquark states (*e.g.* $f_0(980)$ and $f_0(1500)$).

The authors are grateful to Yu.S. Surovtsev for useful discussions. This work has been funded by the Polish National Science Center (NCN) grant DEC-2013/09/B/ST2/04382.

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