

HADRON SCATTERING FROM A LATTICE PERSPECTIVE*

C.B. LANG

Institute of Physics, University of Graz, 8010 Graz, Austria

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Recent years have seen considerable progress in *ab initio* QCD calculations of hadron scattering threshold parameters and scattering phase shifts in the (elastic) resonance region. The lattice approach is becoming powerful enough to even predict states in the heavy quark sector. Methods and recent results for light quarks and heavy-light quark hadrons are discussed.

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1. Introduction

Experiments measure cross sections and under certain assumptions on asymptotic behavior, crossing, unitarity and analyticity one derives from these scattering amplitudes and partial wave phase shifts. Often the outcome is not unique and there are several solutions. The resulting values of the scattering amplitude are located along the real s -axis (s denotes the CMS energy squared). Intricate further analyses then lead to estimates on resonance pole positions.

Based on concepts of QCD (Quantum Chromodynamics) and the mentioned analyticity structures, one may formulate phenomenological models for the scattering amplitude. The interaction (equivalent to the potential in the non-relativistic approximation) is parameterized *ad hoc* or systematically (like, *e.g.*, in unitarized chiral perturbation theory) and the parameters are adjusted such as to recover the experimental results.

We assume that QCD is the correct quantum field theory of quarks and gluons. The continuum theory, however, is not well-defined in the strict mathematical sense and the theory has to be defined by the limit from a regularized formulation. In the non-perturbative regime, the lattice regularization [1] turned out to be particularly efficient. It keeps manifest gauge

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invariance, is confining in the strong coupling limit (and presumably so for all couplings), and allows to implement the path integral quantisation on computers. Since the early success in the gluonic sector [2], we have learned how to compute hadron properties for full QCD, treating gluons and quarks dynamically and interacting.

The central results in Lattice QCD (LQCD) are correlation functions in Euclidean time and their asymptotic (large t) behavior. Due to the clustering property of quantum field theory, we can identify the energy values of a state by its exponential behavior $\sim \exp(-Et)$. The central difference to a continuum theory is the 4D lattice grid with a non-vanishing lattice spacing a and a finite size $N_s^3 \times N_t$ (in units of a). The finiteness of the 3D spatial volume implies discreteness of the energy levels with gaps of $\mathcal{O}(2\pi/L)$ for $L = aN_s$.

For ground state spectroscopy, one computes the lowest energy level; this is straightforward if one can go to large enough t (limited by statistics). For physical mass parameters only the proton and the pseudoscalar mesons are stable (considering only strong interactions) and thus ground states in the respective quantum channels. All other baryons and mesons may decay if the kinematics allows it. In the lattice universe, we may change masses by hand and have also further kinematic restrictions due to the finite volume, such that some decays may be impossible, *e.g.*, the meson–meson threshold might move to values above a resonance, which may become a bound state.

In general (if there is a mass gap), the correlation function will be a sum of exponential terms corresponding to a tower of energy levels. The spectral density becomes a sum of δ -functions

$$\langle X(t)X^\dagger(0) \rangle \equiv C(t) = \int d\omega \rho(\omega) e^{-\omega t} \rightarrow \sum_i w_i e^{-E_i t}. \quad (1)$$

The discrete energy levels have to be related to continuum QCD. One possibility is to use the above mentioned models and put them on a lattice, find energy levels and compare them with the LQCD derived values. Like general inverse scattering problems, this does not necessarily leads to unique and correct continuum results. An alternative approach relates the discrete spectrum directly to infinite volume continuum values of the phase shift at discrete points [3, 4]. This relationship needs further assumptions (*e.g.*, localized interaction regions and for single channel situations elasticity) and the obtained values are sparse.

This approach has originally been formulated in the rest frame for two interacting mesons of equal mass. As can be seen from Fig. 1, for typical lattice sizes of, *e.g.*, 3 fm and a pion mass of 250 MeV, we have $Lm_\pi \approx 3.8$. In a lattice simulation with such parameters only two energy level would be in the neighbourhood of the resonance leading to only two relevant data

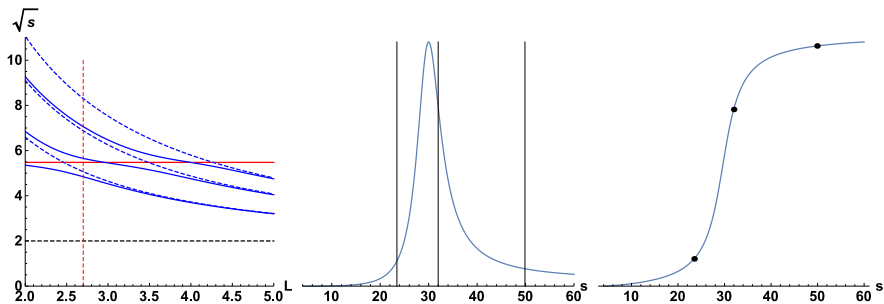


Fig. 1. The left-hand plot shows the energy levels for (spatial) lattice size L (in units of m ; dashed curves: non-interacting, full: interacting), the middle and the right-hand plots show the spectral density and phase shift of the $J = 1$ toy example resonance (s denotes the CMS energy squared in units of m^2). The vertical lines and the points indicate the expected results of a simulation for a lattice size $L = 2.7$.

points for the resonating phase shift. The generation of gauge field configurations with dynamical gluons and quarks is quite expensive. Therefore, one wants to exploit these configurations as much as possible. Working with meson–meson interpolators with total non-vanishing 3-momentum (in “moving frames”) leads to further energy levels [5, 6]. Also other approaches modifying boundary conditions have been proposed [7, 8]. Eventually, one cannot avoid working on lattices of different spatial volumes. The lattice geometry and the parameters of the simulation (and the QCD dynamics) determine the energy levels — one cannot fix the energy of the meson–meson system like in an experiment.

One should also point out a danger of misconception. The lattice interpolators X in (1) have to have the correct quantum numbers and total momentum (with components that are multiples of $2\pi/L$) and there are many choices obeying these conditions. They provide only the basis set for the “physical” states. A correlation matrix $C(t)$ of a set of such operators can be expressed in terms of eigenstates

$$C(t)_{ij} = \langle X_i(t) X_j^\dagger(0) \rangle = \sum_n \langle X_i | n \rangle e^{-E_n t} \langle n | X_j^\dagger \rangle. \quad (2)$$

Assuming that the set of interpolators is complete enough, the eigenstates will represent “physical” states of the finite volume system. They will be composed of combinations of lattice interpolators with overlap factors $\langle X_i | n \rangle$. As suggested [3, 9–11], solving a generalized eigenvalue problem for $C(t)$ leads via the asymptotic behavior of the eigenvalues $\lambda_n(t) \sim e^{-E_n t}$ to the energy levels.

The lattice interpolators are constructed as irreducible representations of the lattice rotation group O_h (for the rest frame) or corresponding little groups (for moving frames). Often one uses smeared quark sources in order to improve the signal. A particularly useful method is using a set of eigenvectors of the 3D lattice Laplacian in the source (and sink) time slices. The quark correlators, called in this context perambulators, are then constructed between these source vectors. This method has been called distillation [12] and is very versatile, allowing *a posteriori* construction of hadron interpolators; it is also efficient for computations involving backtracking quarks like in pair annihilation diagrams in (fully or partially) disconnected graphs.

2. Examples

I will discuss some examples for hadron–hadron scattering, assuming the process is, to good approximation, elastic in the studied energy region. Up to now, there are only very few numerical LQCD studies for coupled channel processes and the underlying theory is being developed [13–16].

2.1. The prototype resonance ρ

This is the most elegant¹ resonance: $\pi\pi$ in p -wave, essentially elastic, with experimental mass of 775 MeV and a width of 149 MeV. Traditionally, the ρ has been studied in the single hadron approximation, *i.e.*, using quark–antiquark interpolators only. This may be justified for large pion masses and on coarse lattices, such that the lowest $\pi\pi$ level (note that the pions have to have non-vanishing relative momentum in order to contribute to p -wave scattering) lies above the resonance region of the ρ , *i.e.*, at very small values of Lm_π in Fig. 1. In this approximation, no signals for the $\pi(1)\pi(-1)$ (the back-to-back momentum is given in units of $2\pi/L$) were found.

In Ref. [17], the system was studied including the 2-meson interpolators; altogether 18 interpolators (15 of quark–antiquark type and 3 $\pi\pi$ interpolators) were considered in the correlation matrix. Different numbers of distillation sources were used in the construction of the hadron operators. Since only the two lowest energy levels were considered statistically reliable, also moving frame operators were used. The gauge ensemble was generously provided by the authors of [18] and had two mass degenerate quarks at pion mass of 266 MeV, lattice spacing 0.1239 fm and lattice size $16^3 \times 32$. Figure 2 shows the resulting values for the phase shift and a Breit–Wigner fit to these points. The width comes out smaller than in experiment, due to the smaller phase space (larger pion mass), but the coupling $g_{\rho\pi\pi} \approx 5.61(12)$ is close to the experimental value 5.96.

¹ Ludwig Boltzmann: “Elegance should be left to shoemakers and tailors”.

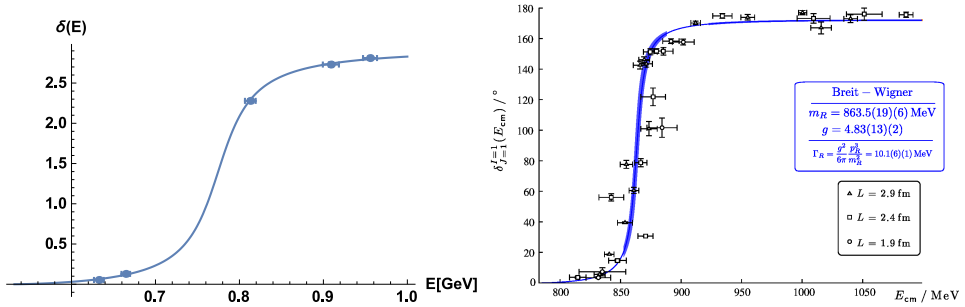


Fig. 2. Left-hand side: Plot of the $\pi\pi$ p -wave phase shift obtained for $m_\pi \approx 266$ MeV in [17]. Right-hand side: The same obtained by [19] for $m_\pi \approx 391$ MeV and several lattice sizes (figure from [19]).

Meanwhile, there has been considerable progress and the right-hand side of the plot shows recent results obtained at a higher pion mass for several lattice volumes [19].

2.2. $K\pi$ scattering

Moving frames for two mesons with different mass are more intricate. One needs to carefully choose the correct irreps of the corresponding little groups [22–24]. The first LQCD-based determination of the $I = \frac{1}{2}$ p -wave phase shift in the region of the resonance $K^*(892)$ used such tools [20, 21]. The lattice parameters were the same as discussed above for the $\pi\pi$ system. Figure 3 summarizes the results obtained for s - and p -waves in both isospin channels. In that study, the inelastic $K\eta$ channel was not considered. In more recent work [25], the coupled system was investigated.

2.3. Negative parity $N\pi$ s -wave scattering

In the channel with $J^P = \frac{1}{2}^-$, there are two low-lying resonances: $N^*(1535)$ and $N^*(1650)$ coupling mainly to $N\pi$ in s -wave. Above the 10% level, there are also further inelastic decays $N^*(1535) \rightarrow N\eta, N\pi\pi$ and $N^*(1650) \rightarrow N\eta, \Lambda K, N\pi\pi$. Earlier lattice simulations of this channel that have determined ground state energy levels and further excitations included only 3-quark interpolators [26–28]. In these studies, two low-lying energy levels have been identified and assigned to the two negative parity resonances. However, the lower of the two levels had a tendency to lie below the $N^*(1535)$ (see the middle plot of Fig. 4).

In a first LQCD study of this $N\pi$ scattering process [29], the inelastic contributions were neglected. The number of graphs (Wick contractions) in this $(4+1) \rightarrow (4+1)$ fermion system is considerably larger than for meson–

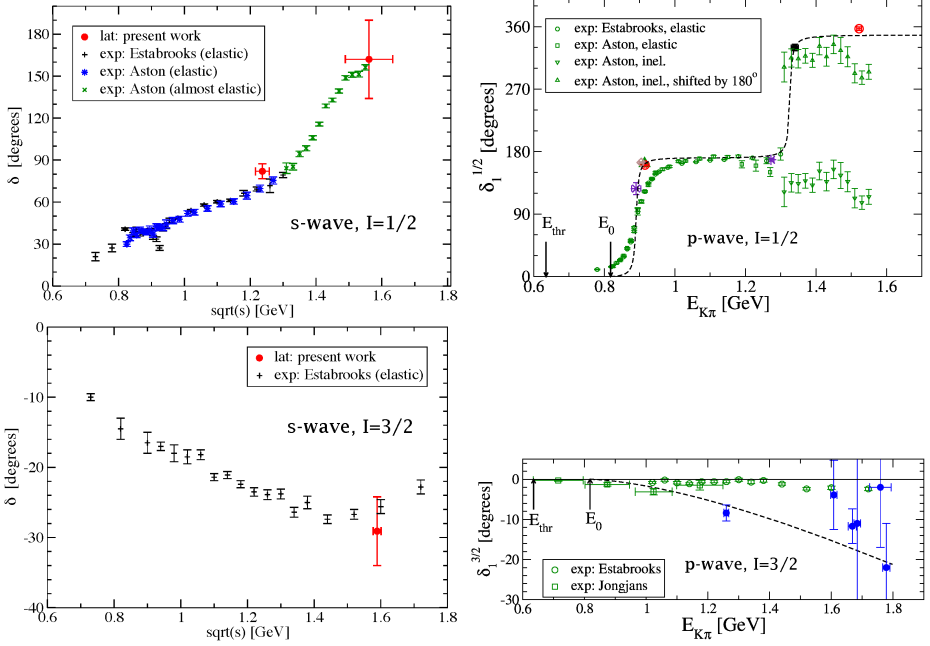


Fig. 3. $K\pi$ phase shift results for s - and p -waves and isospin $I = \frac{1}{2}$ and $\frac{3}{2}$ compared with experiments (figure from [20, 21], where there are also the references to the experiments).

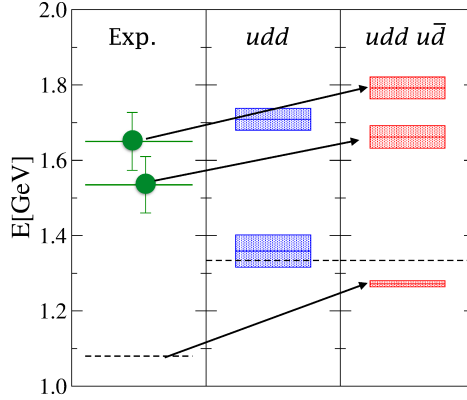


Fig. 4. The plot compares the experiment with the results obtained from the single hadron approximation (middle) and from the full meson-baryon simulation (right-hand column). The broken lines show the $N\pi$ threshold and the arrows indicate the expected mass shift due to the (compared to Nature) higher pion (and quark) mass.

meson scattering, including the notoriously demanding backtracking quark line contributions. There are 2 terms for $3 \rightarrow 3$ quarks (the nucleon), 4 terms for $3 \rightarrow 5$ and $5 \rightarrow 3$ fermions ($N \rightarrow N\pi$ and $N\pi \rightarrow N$) and 19 terms for $5 \rightarrow 5$ fermions ($N\pi \rightarrow N\pi$). We, therefore, used the distillation method [12] for determining the cross-correlation matrix for up to nine interpolators. The configurations were the same as discussed for $\pi\pi$ scattering, with $m_N \approx 1068$ MeV and a pion mass of 266 MeV (for the uncertainties, see [29]).

We found a significant change of the energy spectrum (see Fig. 4). The lowest level now is the expected energy level closely below threshold (which approaches the threshold from below in the infinite volume limit) and the two higher lying levels we associated with $N^*(1535)$ and $N^*(1650)$.

2.4. The heavy-light meson D_s

We studied the three D_s quantum channels $J^P = 0^+, 1^+$ and 2^+ where experiments have identified the charm-strange states $D_{s0}^*(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$ near the DK and D^*K thresholds, and $D_{s2}^*(2573)$. This behavior was not reproduced in quark models or in earlier LQCD calculations. In both approaches, the bound states moved above the threshold becoming resonances [30–36]. These calculations were in the “single hadron” approach, *i.e.*, without considering the two-meson channels. As had been pointed out already earlier [37], threshold effects may be critical, though.

We, therefore, considered correlation functions for sets of $\bar{q}q$ operators and, for $J^P = 0^+, 1^+$, also the DK and D^*K meson–meson interpolators and determined for these cases values of the elastic scattering amplitude [38, 39].

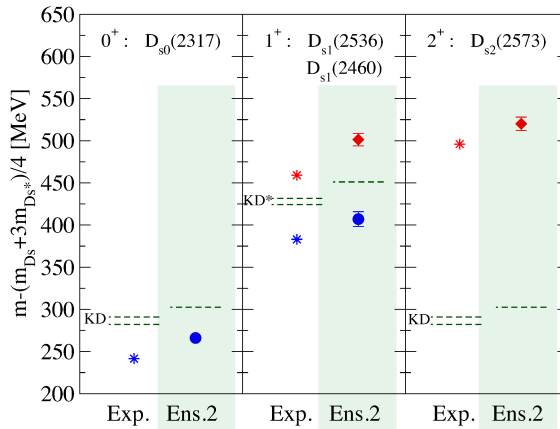


Fig. 5. Comparison of the experiments with lattice results for $m_\pi \approx 156$ MeV. The residual differences are due to finite volume and discretisation effects.

For this study also another ensemble of gauge configurations with $N_f = 2 + 1$ dynamical quarks and generated by the PACS-CS Collaboration [40] was used. It has lattice spacing 0.0907 fm, size $32^3 \times 64$ and a pion mass of 156 MeV. Sea and valence quarks are non-perturbatively improved Wilson fermions. Here, we used the stochastic distillation method [41]. The light and strange quarks were dynamical, the charm quark treated as a valence quark. For the K , we used the relativistic dispersion relation and for D , D^* the Fermilab method [42, 43] like in [44]. We determined bound state parameters and found good agreement with experiments, see Fig. 5.

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