

ASPECTS OF GLUON PROPAGATION IN LANDAU GAUGE: SPECTRAL DENSITIES, AND MASS SCALES AT FINITE TEMPERATURE*

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We discuss a method to extract the Källén–Lehmann spectral density of a particle (be it elementary or bound state) propagator and apply it to compute gluon spectral densities from lattice data. Furthermore, we also consider the interpretation of the Landau-gauge gluon propagator at finite temperature as a massive-type bosonic propagator.

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1. Spectral densities

In general, an Euclidean momentum-space propagator $\mathcal{G}(p^2) \equiv \langle \mathcal{O}(p) \mathcal{O}(-p) \rangle$ of a (scalar) physical degree of freedom ought to have a Källén–Lehmann spectral representation

$$\mathcal{G}(p^2) = \int_0^\infty d\mu \frac{\rho(\mu)}{p^2 + \mu}. \quad (1)$$

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The knowledge of the spectral function $\rho(\mu)$ is useful for, amongst other things, getting the masses of the physical states described by the operator $\mathcal{O}$.

Here, we describe a method to compute the spectral density given a numerical estimate of the propagator, computed using *e.g.* lattice techniques. Note that Eq. (1) is equivalent to a double Laplace transform $\mathcal{G} = \mathcal{L}^2 \hat{\rho} = \mathcal{L} \mathcal{L}^* \hat{\rho}$, where $(\mathcal{L}f)(t) \equiv \int_0^\infty ds e^{-st} f(s)$; the (double) inversion is then a notorious ill-posed problem, due to the exponential dampening.

For positive spectral functions, a popular approach is the maximum entropy method [1]. An alternative approach, aiming to compute spectral densities not necessarily positive, has been developed by some of us [2], based on the Tikhonov regularization supplemented with the Morozov discrepancy principle.

Specifically, setting $D_i \equiv D(p_i^2)$ and assuming we have N data points, we minimize

$$\mathcal{J}_\lambda = \sum_{i=1}^N \left[\int_{\mu_0}^{+\infty} d\mu \frac{\rho(\mu)}{p_i^2 + \mu} - D_i \right]^2 + \lambda \int_{\mu_0}^{+\infty} d\mu \rho^2(\mu), \quad (2)$$

where we use lattice data in momentum space for the gluon propagator computed in a 80^4 volume, with $\beta = 6.0$ (Wilson gauge action) [3]. In Eq. (2), $\lambda > 0$ is a regularization parameter designed to overcome the ill-posed nature of the inversion. We choose λ by means of the Morozov principle: the optimal value $\bar{\lambda}$ is such that the quality of the inversion is equal to the error on the data, *i.e.* $\|D^{\text{reconstructed}} - D^{\text{data}}\| = \delta$, where δ is the total noise on the input data. The IR regulator (threshold) μ_0 will be determined self-consistently by means of the optimal (Morozov) regulator $\bar{\lambda}$: we take the minimal value for $\bar{\lambda}(\mu_0)$ that can be reached by varying μ_0 .

The minimization of (2) proceeds through a linear perturbation of ρ and imposing the vanishing of the variation of $\mathcal{J}_\lambda$

$$\underbrace{\sum_{i=1}^N \left[\int_{\mu_0}^{+\infty} d\nu \frac{\rho(\nu)}{p_i^2 + \nu} - D_i \right]}_{\equiv c_i} \frac{1}{p_i^2 + \mu} + \lambda \rho(\mu) = 0 \quad (\mu \geq \mu_0). \quad (3)$$

The Källén–Lehmann inverse can be computed explicitly

$$\rho_\lambda(\mu) = -\frac{1}{\lambda} \sum_{i=1}^N \frac{c_i}{p_i^2 + \mu} \Theta(\mu - \mu_0), \quad (4)$$

where $\Theta(\cdot)$ is the Heaviside step function. We get a linear system for the coefficients c_i

$$\lambda^{-1} \mathcal{M} c + c = -D \quad (5)$$

with

$$\mathcal{M}_{ij} = \int_{\mu_0}^{+\infty} d\nu \frac{1}{p_i^2 + \nu} \frac{1}{p_j^2 + \nu} = \frac{\ln \frac{p_j^2 + \mu_0}{p_i^2 + \mu_0}}{p_j^2 - p_i^2}. \quad (6)$$

In Fig. 1, we plot the spectral density and, as a check, the reconstructed propagator which can be easily computed combining Eqs. (4) and (1). In this case, we have two minima for $\bar{\lambda}(\mu_0)$, at $\mu_0 \approx 0.03 \text{ GeV}^2$ and $\mu_0 \approx 0.16 \text{ GeV}^2$. We display the results for both values. We conclude that the gluon spectral density is indeed a nonpositive quantity. This is not surprising, since the gluons are not part of the physical spectrum [4]. In the near future, we plan to apply this method to glueballs¹ and other physical degrees of freedom.

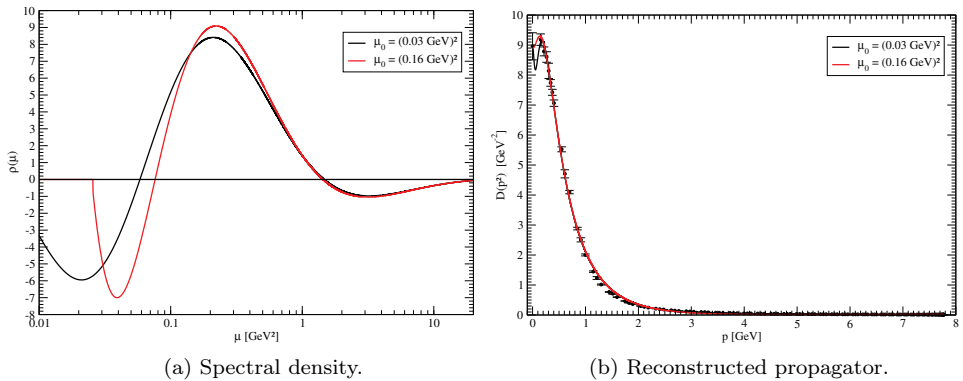


Fig. 1. Results for the gluon spectral function and the reconstructed propagator *vs.* the input data. We refer to the main text and [2] for additional details.

2. Gluon mass at finite temperature

In this section, we briefly describe a recent investigation by some of us [6], where we address the interpretation of the Landau gauge gluon propagator at finite temperature as a massive-type bosonic propagator. For such a goal, we consider a Yukawa-type propagator

$$D(p) = \frac{Z}{p^2 + m^2}, \quad (7)$$

where m is the gluon mass and $Z^{\frac{1}{2}}$ the overlap between the gluon state and the quasi-particle massive state.

¹ See [5] for a preliminary study of glueball spectral densities.

At finite temperature, the Landau gauge gluon propagator is splitted into two components

$$D_{\mu\nu}^{ab}(\hat{q}) = \delta^{ab} (P_{\mu\nu}^T D_T(q_4, \vec{q}) + P_{\mu\nu}^L D_L(q_4, \vec{q})) , \quad (8)$$

where D_T and D_L are the transverse and longitudinal propagators respectively.

The lattice setup for the simulations at finite temperature considered here is described in Table I. The surface plots of the two form factors can be seen in Fig. 2. For further details, see [6].

TABLE I

Lattice setup used for the computation of the gluon propagator at finite temperature. Simulations used the Wilson gauge action; β was adjusted to have a constant physical volume, $L_s a \simeq 6.5$ fm. For the generation of gauge configurations and Landau gauge fixing, we used Chroma [7] and PFFT [8] libraries.

Temp. [MeV]	β	L_s	L_t	a [fm]	$1/a$ [GeV]
121	6.0000	64	16	0.1016	1.9426
162	6.0000	64	12	0.1016	1.9426
194	6.0000	64	10	0.1016	1.9426
243	6.0000	64	8	0.1016	1.9426
260	6.0347	68	8	0.09502	2.0767
265	5.8876	52	6	0.1243	1.5881
275	6.0684	72	8	0.08974	2.1989
285	5.9266	56	6	0.1154	1.7103
290	6.1009	76	8	0.08502	2.3211
305	6.1326	80	8	0.08077	2.4432
324	6.0000	64	6	0.1016	1.9426
366	6.0684	72	6	0.08974	2.1989
397	5.8876	52	4	0.1243	1.5881
428	5.9266	56	4	0.1154	1.7103
458	5.9640	60	4	0.1077	1.8324
486	6.0000	64	4	0.1016	1.9426

A simple definition for a mass scale associated with the gluon propagator can be given by

$$m = 1/\sqrt{D(p^2 = 0; T)} . \quad (9)$$

Our results for such a mass scale are shown in Fig. 3.

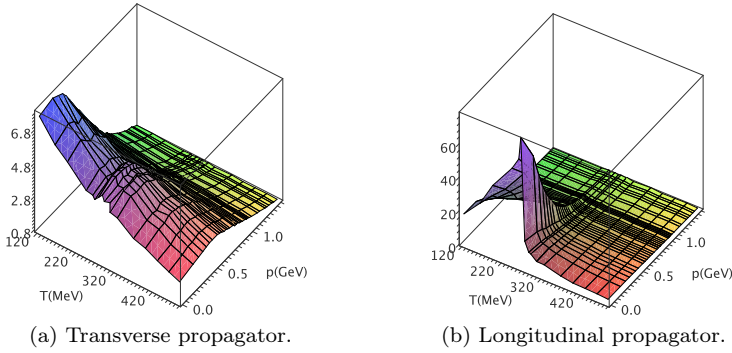


Fig. 2. Components of the gluon propagator as a function of momentum and temperature.

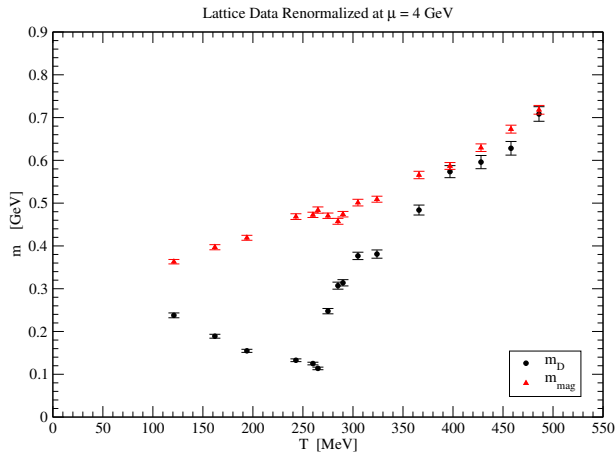


Fig. 3. Electric and magnetic mass defined from zero momentum propagators.

A more realistic value for the gluon mass can be obtained by a fit to the lattice data in the infrared region using the ansatz described in Eq. (7). It turns out that the transverse form factor is not described by a Yukawa-type propagator. Therefore, one concludes that D_T does not behave as quasi-particle massive boson for $T < 500$ MeV. In what concerns the longitudinal form factor, we report the values of $Z(T)$ and $m_g(T)$ in Fig. 4.

We observe that both $m_g(T)$ and $Z(T)$ are sensitive to the confinement–deconfinement phase transition; the data suggests that the phase transition is of first order. Below T_c , the gluon mass is a decreasing function of T , whereas it increases for $T > T_c$. Furthermore, the gluon mass follows the expected perturbative behaviour for $T > 400$ MeV.

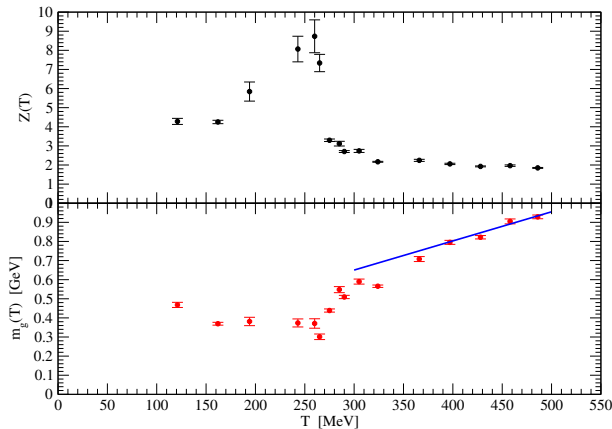


Fig. 4. $Z(T)$ and $m_g(T)$ from fitting the longitudinal gluon propagator to a Yukawa form. The curve in the lower plot is the fit of m_g to the functional form predicted by perturbation theory.

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