THE 3 FLAVOR NAMBU–JONA-LASINIO WITH EXPLICIT SYMMETRY BREAKING INTERACTIONS: SCALAR AND PSEUDOSCALAR SPECTRA AND DECAYS* **

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The effective quark interactions that break explicitly the chiral $\mathrm{SU}(3)_\mathrm{L}\times \mathrm{SU}(3)_\mathrm{R}$ and $\mathrm{U}_\mathrm{A}(1)$ symmetries by current-quark mass source terms are considered in NLO in N_c counting. They are of the same order as the 't Hooft flavor determinant and the eight quark interactions that extend the LO Nambu–Jona-Lasinio Lagrangian, and complete the set of non-derivative and spin 0 interactions relevant for the N_c scheme. The bosonized Lagrangian at meson tree level describes accurately the empirical ordering and magnitude of the splitting of states in the low-lying pseudoscalar and scalar meson nonets, for which the explicit symmetry breaking terms turn out to be essential. The strong interaction and radiative decays of the scalar mesons are understood in terms of the underlying microscopic multi-quark states, which are probed differently by the strong and the electromagnetic interactions. We also obtain that the anomalous two photon decays of the pseudoscalars are in very good agreement with data.

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Effective low energy Lagrangians of QCD are operational at the scale of spontaneous breaking of chiral symmetry, of the order of $\Lambda_{\chi\mathrm{SB}} \sim 4\pi f_\pi$ [1]. In the Nambu–Jona-Lasinio (NJL) model [2], this scale is also related to

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the gap equation and given by the ultra-violet cutoff Λ of the one-loop quark integral, above which one expects non-perturbative effects to be of less importance. We consider in our Lagrangian [3, 4] generic vertices L_i of non-derivative type that contribute to the effective potential as $\Lambda \to \infty$

$$L_i \sim \frac{\bar{g}_i}{\Lambda^\gamma} \chi^\alpha \Sigma^\beta \,, \tag{1}$$

where powers of Λ give the correct dimensionality of the interactions (below we use also unbarred couplings, $g_i = \frac{\bar{g}_i}{\Lambda^\gamma}$); the L_i are C, P, T and chiral $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}$ invariant blocks, built of powers of the sources χ which at the end give origin to the explicit symmetry breaking and have the same transformation properties as the U(3)Lie-algebra valued field $\Sigma = (s_a - ip_a)\frac{1}{2}\lambda_a$; here $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a i\gamma_5 q$, and $a = 0, 1, \ldots, 8$, $\lambda_0 = \sqrt{2/3} \times 1$, λ_a being the standard SU(3) Gell-Mann matrices for $1 \le a \le 8$.

The interaction Lagrangian without external sources χ is well known,

$$\begin{aligned} L_{\mathrm{int}} &= \frac{\bar{G}}{\Lambda^2} \mathrm{tr}\left(\Sigma^\dagger \Sigma\right) + \frac{\bar{\kappa}}{\Lambda^5}\left(\det \Sigma + \det \Sigma^\dagger\right) \\ &\quad + \frac{\bar{g}_1}{\Lambda^8}\left(\mathrm{tr}\, \Sigma^\dagger \Sigma\right)^2 + \frac{\bar{g}_2}{\Lambda^8} \mathrm{tr}\left(\Sigma^\dagger \Sigma \Sigma^\dagger \Sigma\right) . \end{aligned} \tag{2}$$

The second term is the 't Hooft determinant [5–13], the last two the 8 quark (q) interactions [14] which complete the number of relevant vertices in 4D for dynamical chiral symmetry breaking [15]. The interactions dependent on the sources χ contain eleven terms [3, 4],

$$L_\chi = \sum_{i=0}^{10} L_i \,, \tag{3}$$

$$\begin{aligned}
L_0 &= -\mathrm{tr}\left(\Sigma^\dagger \chi + \chi^\dagger \Sigma\right) , & L_1 &= -\frac{\bar{\kappa}_1}{\Lambda} e_{ijk} e_{mnl} \Sigma_{im} \chi_{jn} \chi_{kl} + \mathrm{h.c.} , \\
L_2 &= \frac{\bar{\kappa}_2}{\Lambda^3} e_{ijk} e_{mnl} \chi_{im} \Sigma_{jn} \Sigma_{kl} + \mathrm{h.c.} , & L_3 &= \frac{\bar{g}_3}{\Lambda^6} \mathrm{tr}\left(\Sigma^\dagger \Sigma \Sigma^\dagger \chi\right) + \mathrm{h.c.} , \\
L_4 &= \frac{\bar{g}_4}{\Lambda^6} \mathrm{tr}\left(\Sigma^\dagger \Sigma\right) \mathrm{tr}\left(\Sigma^\dagger \chi\right) + \mathrm{h.c.} , & L_5 &= \frac{\bar{g}_5}{\Lambda^4} \mathrm{tr}\left(\Sigma^\dagger \chi \Sigma^\dagger \chi\right) + \mathrm{h.c.} , \\
L_6 &= \frac{\bar{g}_6}{\Lambda^4} \mathrm{tr}\left(\Sigma \Sigma^\dagger \chi \chi^\dagger + \Sigma^\dagger \Sigma \chi^\dagger \chi\right) , & L_7 &= \frac{\bar{g}_7}{\Lambda^4}\left(\mathrm{tr} \Sigma^\dagger \chi + \mathrm{h.c.}\right)^2 , \\
L_8 &= \frac{\bar{g}_8}{\Lambda^4}\left(\mathrm{tr} \Sigma^\dagger \chi - \mathrm{h.c.}\right)^2 , & L_9 &= -\frac{\bar{g}_9}{\Lambda^2} \mathrm{tr}\left(\Sigma^\dagger \chi \chi^\dagger \chi\right) + \mathrm{h.c.} , \\
L_{10} &= -\frac{\bar{g}_{10}}{\Lambda^2} \mathrm{tr}\left(\chi^\dagger \chi\right) \mathrm{tr}\left(\chi^\dagger \Sigma\right) + \mathrm{h.c.} & &
\end{aligned} \tag{4}$$

The N_c assignments are $\Sigma \sim N_c$; $\Lambda \sim N_c^0 \sim 1$; $\chi \sim N_c^0 \sim 1$ [1]. We get that exactly the diagrams which survive as $\Lambda \to \infty$ also survive as $N_c \to \infty$ and comply with the usual requirements.

At LO in $1/N_c$ only the $4q$ interactions ($\sim G$) in Eq. (2) and L_0 contribute. The Zweig's rule violating vertices are always of the order of $\frac{1}{N_c}$ with respect to the leading contribution. Non-OZI-violating Lagrangian pieces scaling as N_c^0 represent NLO contributions with one internal quark loop in N_c counting; their couplings encode the admixture of a four quark component $\bar{q}q\bar{q}q$ to the leading $\bar{q}q$ at $N_c \to \infty$. Diagrams tracing Zweig's rule violation are: $\kappa, \kappa_1, \kappa_2, g_1, g_4, g_7, g_8, g_{10}$; Diagrams with admixture of 4-quark and 2-quark states are: g_2, g_3, g_5, g_6, g_9.

With all the building blocks in conformity with the symmetry content of the model, one is free to choose the external source χ. Putting $\chi = \frac{1}{2}\mathrm{diag}(\mu_u, \mu_d, \mu_s)$, we obtain a consistent set of explicitly breaking chiral symmetry terms.

From the 18 model parameters, 3 of them ($\bar{\kappa}_1, \bar{g}_9, \bar{g}_{10}$) contribute to the current quark masses $m_i, i = u, d, s$ and express the Kaplan–Manohar ambiguity [16]. They can be set to 0 without loss of generality. One ends up with 5 parameters needed to describe the LO contributions (the scale Λ, the coupling G, and the m_i) and 10 in NLO ($\bar{\kappa}, \bar{\kappa}_2, \bar{g}_1, \ldots, \bar{g}_8$). They are controlled on the theoretical side through the symmetries of the Lagrangian and on the phenomenological side through the low energy characteristics of the pseudoscalar and the scalar mesons.

The details of bosonization in the framework of functional integrals, which lead finally from $L = \bar{q}i\gamma^\mu\partial_\mu q + L_{\mathrm{int}} + L_\chi$ to the long distance effective mesonic Lagrangian $\mathcal{L}_{\mathrm{bos}}$, can be found in [3, 4, 17, 18]

$$
\begin{aligned}
&\mathcal{L}_{\mathrm{bos}} = \mathcal{L}_{\mathrm{st}} + \mathcal{L}_{\mathrm{hk}}\,, \\
&\mathcal{L}_{\mathrm{st}} = h_a\sigma_a + \frac{h^{(1)}_{ab}}{2}\sigma_a\sigma_b + \frac{h^{(2)}_{ab}}{2}\phi_a\phi_b + \sigma_a\left(\frac{1}{3} + h^{(1)}_{abc}\sigma_b\sigma_c + h^{(2)}_{abc}\phi_b\phi_c\right) + \ldots \\
&W_{\mathrm{hk}}(\sigma,\phi) = \frac{1}{2}\ln\left|\det D^\dagger_{\mathrm{E}} D_{\mathrm{E}}\right| = -\int\frac{d^4x_{\mathrm{E}}}{32\pi^2}\sum_{i=0}^{\infty} I_{i-1}\mathrm{tr}(b_i) = \int d^4x_{\mathrm{E}}\mathcal{L}_{\mathrm{hk}}\,, \\
&b_0 = 1\,, \quad b_1 = -Y\,, \quad b_2 = \frac{Y^2}{2} + \frac{\lambda_3}{2}\Delta_{ud}Y + \frac{\lambda_8}{2\sqrt{3}}(\Delta_{us} + \Delta_{ds})Y\,, \quad \ldots\,, \\
&Y = i\gamma_\alpha\left(\partial_\alpha\sigma + i\gamma_5\partial_\alpha\phi\right) + \sigma^2 + \{\mathcal{M},\sigma\} + \phi^2 + i\gamma_5[\sigma + \mathcal{M},\phi]
\end{aligned}
\tag{5}
$$

with $\Delta_{ij} = M_i^2 - M_j^2$. Here, $\sigma = \lambda_a\sigma_a$ and $\phi = \lambda_a\phi_a$ are nonet valued scalar and pseudoscalar fields. The $\mathcal{L}_{\mathrm{st}}$ is the result of the stationary phase integration at leading order, over the auxiliary bosonic variables s_a, p_a, shown

[1] The counting for Λ is a direct consequence of the gap equation $1 \sim N_c G\Lambda^2$.

in (5) as a series in growing powers of σ_a and ϕ_a. The coefficients $h_{ab\dots}$ in $\mathcal{L}_{\rm st}$ are obtained recursively from h_a (which are related to the condensates). The result of the remaining Gaussian integration over the quark fields is given by $W_{\rm hk}$, in the heat kernel approach. The Laplacian in Euclidean space-time $D_{\rm E}^\dagger D_{\rm E} = \mathcal{M}^2 - \partial_\alpha^2 + Y$ is associated with the Euclidean Dirac operator $D_{\rm E} = i\gamma_\alpha\partial_\alpha - \mathcal{M} - \sigma - i\gamma_5\phi$. The constituent quark mass matrix is denoted by $\mathcal{M} = \mathrm{diag}(M_u, M_d, M_s)$ (fields σ_a, ϕ_a have vanishing vacuum expectation values in the spontaneously broken phase). The quantities I_i are the arithmetic averages $I_i = \frac{1}{3}\sum_{f=u,d,s} J_i(M_f^2)$ over the 1-loop Euclidean momentum integrals J_i with $i+1$ vertices ($i = 0, 1, \dots$)

$$J_i\left(M^2\right) = 16\pi^2\Gamma(i+1)\int \frac{d^4p_{\rm E}}{(2\pi)^4}\,\hat{\rho}_\Lambda \frac{1}{\left(p_{\rm E}^2 + M^2\right)^{i+1}}\,, \tag{6}$$

evaluated with a Pauli–Villars regulator $\hat{\rho}_\Lambda$ with two subtractions in the integrand. Note that the integrals I_i do not depend on external momenta, and thus are free from $q\bar{q}$ thresholds [19]. The possible external momentum dependence of an amplitude is converted to terms involving derivative interactions in $\mathcal{L}_{\rm hk}$. We consider only the dominant contributions to the heat kernel series, up to b_2 for the meson spectra and strong decays. These involve the quadratic and logarithmic in Λ quark loop integrals I_0 and I_1 respectively. We stress that all symmetries are respected in the process of truncation, as the heat kernel series remains an invariant order by order.

In the following, we consider the isospin limit $\hat{m} = m_u = m_d \neq m_s$. The low-lying characteristics of the spin 0 mesons in Table I and m_i in Table II are used as input (marked by *) to obtain the parameters indicated in Tables II, III (for other sets, related to slightly different values of $m_\sigma(500)$, $\theta_{\rm P}$ and $\theta_{\rm S}$ see [4]). The calculated values of quark condensates are: $-\langle\bar{u}u\rangle^{\frac{1}{3}} = 232$ MeV, and $-\langle\bar{s}s\rangle^{\frac{1}{3}} = 206$ MeV. We stress that without the new explicit symmetry breaking terms the high accuracy achieved for the observables had not been possible. We find that the couplings g_8 and κ_2 are crucial for the high precision within the pseudoscalar sector. Furthermore, the low-lying scalar nonet mesons can be obtained according to the empirical ordering: $m_\kappa < m_{a_0} \simeq m_{f_0}$, in contrast to the $m_\sigma < m_{a_0} < m_\kappa < m_{f_0}$ sequence obtained otherwise in the framework of the NJL models, *e.g.* [8, 14, 18, 20, 21]. The main parameter responsible for the lower mass of $\kappa(800)$ as compared to the mass of $a_0(980)$ is g_3; g_6 allows for fine tuning. We understand the empirical masses inside the light scalar nonet as a consequence of some predominance of the explicit chiral symmetry breaking terms over the dynamical chiral symmetry breaking ones for certain states. Note that the couplings g_3 and g_6 encode $\bar{q}q\bar{q}q$ admixtures to the $\bar{q}q$ states. This establishes a link between the asymptotic meson states obtained from the effective multiquark interactions considered to the successful approaches which support $\bar{q}q$ states with a meson–meson admixture [22] or mixing of $q\bar{q}$-states with $q^2\bar{q}^2$ [23].

TABLE I

The pseudoscalar and scalar mass spectra, the weak decay constants (all in MeV) and the mixing angles $\theta_{\rm P} = -12^{\circ *}$ and $\theta_{\rm S} = 27.5^{\circ *}$.

m_π	m_K	m_η	$m_{\eta'}$	f_π	f_K	m_σ	m_κ	m_{a_0}	m_{f_0}
138*	494*	547*	958*	92*	113*	550	850*	980*	980*

TABLE II

The model parameters $\hat{m} = m_u = m_d, m_s$, and Λ are given in MeV. The couplings have the following units: $[G] = \mathrm{GeV}^{-2}$, $[\kappa] = \mathrm{GeV}^{-5}$, $[g_1] = [g_2] = \mathrm{GeV}^{-8}$. We also show here the values of constituent quark masses $\hat{M}$ and M_s in MeV.

$\hat{m}$	m_s	$\hat{M}$	M_s	Λ	G	$-\kappa$	g_1	g_2
S4.0*	100*	373	544	828	10.48	122.0	3284	173*

TABLE III

Explicit symmetry breaking interaction couplings. The couplings have the following units: $[\kappa_1] = \mathrm{GeV}^{-1}$, $[\kappa_2] = \mathrm{GeV}^{-3}$, $[g_3] = [g_4] = \mathrm{GeV}^{-6}$, $[g_5] = [g_6] = [g_7] = [g_8] = \mathrm{GeV}^{-4}$, $[g_9] = [g_{10}] = \mathrm{GeV}^{-2}$.

κ_2	$-g_3$	$-g_4$	g_5	$-g_6$	$-g_7$	g_8
6.17	6497	1235	213	1642	13.3	−64

In Table IV there are shown the strong decay widths of the scalars, which are within the current expectations. The widths of the $a_0(980) \to \pi\eta$ and $f_0(980) \to \pi\pi$ decays are well accommodated within a Flatté description. We corroborate other model calculations in which the coupling to the $K\bar{K}$ channel is needed for the description of these decays. We obtain however that although the $a_0(980)$ meson couples with a large strength of the multi-quark components to the two-kaon channel in its strong decay to two pions, it evidences a dominant $q\bar{q}$ component in its radiative decay. The latter is thus fairly well described by a quark 1-loop triangle diagram, $\Gamma_{a_0\gamma\gamma} = 0.38$ KeV. As opposed to this, the σ and $f_0(980)$ mesons do not display an enhanced $q\bar{q}$ component neither in their two-photon decays nor strong decays. The quark 1-loop contributions $\Gamma_{f_0\gamma\gamma} = 0.08$ KeV, $\Gamma_{\sigma\gamma\gamma} = 0.21$ KeV, fall thus short of describing the data. Finally, the anomalous 2 photon decays of the pseudoscalars are in very good agreement with data, see Table V. For a full discussion, see [4].

TABLE IV

Strong decays of the scalar mesons, m_R is the resonance mass in MeV, Γ^{BW} and Γ^{Fl} are the Breit–Wigner width and the Flatté distribution width in MeV, $R^S = \frac{\bar{g}_K^S}{\bar{g}_\beta}$. The couplings $\bar{g}_\beta, \bar{g}_K^S$ are dimensionless and correspond to the shown transitions $S \to PP$ and to $S \to \bar{K}K$ respectively [4].

Decays	m_R	Γ^{BW}	Γ^{Fl}	$\bar{g}_\beta$	$\bar{g}_K^S$	R^S
$\sigma \to \pi\pi$	550	461	—	1.94	0.63	0.33
$f_0 \to \pi\pi$	980	62	30	0.23	0.30	3.90
$\kappa \to K\pi$	850	310	—	1.2	0	—
$a_0 \to \eta\pi$	980	420	46	1.32	2.73	2.07

TABLE V

Anomalous decays $\Gamma_{P\gamma\gamma}$ in KeV, corresponding to $\theta_P = -12°$, m_R is the particle mass in MeV.

Decays	m_R	$\Gamma_{P\gamma\gamma}$	$\Gamma_{P\gamma\gamma}^{exp}$ [24]
$\pi^0 \to \gamma\gamma$	136	0.00798	$0.00774637 \div 0.00810933$
$\eta \to \gamma\gamma$	547	0.5239	$(39.31 \pm 0.2)\% \, \Gamma_{tot} = 0.508 \div 0.569$
$\eta' \to \gamma\gamma$	958	5.225	$(2.18 \pm 0.08)\% \, \Gamma_{tot} = 3.99 \div 4.70$

The response to the external parameters T, μ has been recently addressed in [25], with implications on strange quark matter formation.

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