

NEUTRINOS AND THE AVERAGE THERMAL EVOLUTION OF THE UNIVERSE*

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The cosmological effect of sterile neutrinos is estimated in a study of the average behavior of the Λ CDM universe. Dark matter is assumed to be composed of neutralinos and it is argued that dark energy may result from conformal variations of the metric. The eventual presence of sterile neutrinos does not noticeably change the evolution of the universe and we find that their density at primordial nucleosynthesis is consistent with predictions when the sterile neutrinos are in temperature equilibrium with the rest of the universe at the end of inflation.

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1. Introduction

We investigate the evolution of neutrinos and the effect of the neutrino character (Majorana or Dirac) on the global evolution of the standard homogeneous and isotropic perfect fluid universe. Our calculation starts at the end of the epoch of inflation which left the universe in a homogeneously thermalized state. As we are interested in average properties, neither the impact of structure formation nor the details of the primordial nucleosynthesis are taken into account. Dark matter of only one type is assumed and the chemical potentials of the various particle species can be considered negligible [1]. Observations [2] indicate that the universe contains about 72% dark energy, 23% dark matter and 5% baryonic matter.

The question whether neutrinos are their own antiparticles (of Majorana type) or not (of Dirac type) is not settled yet and we concentrate on the consequences of these two hypotheses. When the temperature of the universe decreases during expansion, Majorana neutrinos decouple at a temperature

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$T_{D\nu}$ when their interaction rate with electrons becomes too low for equilibrium to be maintained [3]. However, they continue sharing the temperature of radiation until the temperature drops to the e^+e^- annihilation threshold, when only the photons absorb the e^+e^- annihilation energy and not the neutrinos. Thereafter, the photon temperature is higher than the neutrino temperature.

In the case of Dirac neutrinos, one needs to take into account that only left-handed (“active”) neutrinos are observed in experiments. The additional right-handed neutrino states should be considered sterile. Whereas the active neutrinos have the same number of degrees of freedom as in the Majorana case (denoted by the index ν in Table I below), the sterile neutrinos (index s) are not coupled to the rest of the universe at all, exhibiting a mere $T_s \propto q^{-1}$ dependence, with q the scale factor (see below).

Turning our attention to dark energy next, one possible theoretical approach to explain the origin of the observed [2, 4–7] dark energy component (cosmological constant) considers quantum fluctuations of the metric as [8, 9] $g_{\mu\nu} = (1 + \varphi)^2 \bar{g}_{\mu\nu}$, where $\bar{g}_{\mu\nu}$ is the classical metric about which the fluctuations represented by the scalar field φ occur. The Einstein equation derived from the Hilbert action *without*¹ a cosmological constant changes due to the presence of the fluctuations and does contain now a cosmological constant term [9] with a value consistent with observations, (units $\hbar = c = 1$)

$$\Lambda = - \left(\bar{R} + 8\pi G T_{\lambda\mu} \bar{g}^{\lambda\mu} \right) / 4, \quad (1)$$

where R is the scalar curvature and the bars refer to the classical expressions without fluctuations. The stress-energy tensor for a perfect fluid universe is

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu - P g^{\mu\nu}, \quad (2)$$

where ρ is the energy density and P the pressure. With (2), we verify that the cosmological term (1) is indeed a constant [11].

In our calculation [12] of the thermal evolution of the universe, we use the usual Friedmann–Lemaître–Robertson–Walker metric

$$ds^2 = dt^2 - q(t)^2 \left[dr^2 / (1 - kr^2) + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (3)$$

where $q(t)$ is the scale factor and k the curvature of space. The expanding universe is described by Friedmann’s equation

$$3(\dot{q}^2 + k) / q^2 - \Lambda = 8\pi G \rho. \quad (4)$$

¹ The same Λ is obtained even if starting from a Hilbert action *with* a cosmological constant, which then cancels in the procedure [10].

It is customary to divide the density by the critical density $\rho_c = 3H_0^2/8\pi G$ (H_0 being the Hubble constant) and the density parameter $\Omega = \rho/\rho_c$ is composed of matter Ω_m (baryonic Ω_b and cold dark matter Ω_c) and radiation Ω_r containing all ultrarelativistic species. Similarly, the cosmological constant and curvature can be expressed as density parameters $\Omega_\Lambda = \Lambda/3H_0^2$ and $\Omega_k = -k/q(t)^2H_0^2$. Energy conservation

$$\dot{\rho}_i = -3(\rho_i + P_i)\dot{q}/q \quad (5)$$

can be written independently for each component i in the time intervals when no significant conversion between components takes place. We can then use the equations of state [1] $\omega_i = P_i/\rho_i$ to integrate the energy conservation equation and relate the density parameters with the scale factor for radiation (index r) and for non-relativistic matter (dust, index m):

$$\omega_r = 1/3, \quad \Omega_r \propto q^{-4}, \quad T_r \propto q^{-1}; \quad \omega_m = 0, \quad \Omega_m \propto q^{-3}, \quad T_m \propto q^{-2}. \quad (6)$$

2. Modelling the densities and temperatures

The density of ultrarelativistic species is [13]

$$\rho = \frac{\pi^2}{30} \left(\sum_B g_B + \frac{7}{8} \sum_F g_F \right) T^4 \equiv \frac{\pi^2}{30} N(T) T^4. \quad (7)$$

The total number of effective degrees of freedom $N(T)$ results from summing the contribution of all bosonic (B) and fermionic (F) species with their respective degeneracy factors g_B and g_F . When the temperature drops during the expansion of the universe, particles of a species annihilate with their antiparticles when the temperature falls below their rest mass, liberating thermal energy of annihilation in the process and ceasing to contribute to $N(T)$. As the temperature decreases with time, more and more particle species annihilate until only photons and neutrinos remain. Reading from bottom to top, Table I lists the values of $N(T)$ as Standard Model particles are annihilated. The Majorana (or active) neutrino component N_ν appears at decoupling T_{D_ν} and the sterile Dirac neutrino component N_s is always present, if it exists.

The entropy transfer in an annihilation process from state b to a implies [1]

$$N_b(qT_b)^3 = N_a(qT_a)^3 \implies T_a = (N_b/N_a)^{\frac{1}{3}} T_b. \quad (8)$$

For the change of the density parameter in a threshold process, we have

$$\Omega_a/\Omega_b = (N_a/N_b) (T_a/T_b)^4 \implies \Omega_a = (N_b/N_a)^{\frac{1}{3}} \Omega_b. \quad (9)$$

TABLE I

Effective relativistic degrees of freedom with increasing temperature (adapted from PDG [13]).

		4N		
T_r	New particles	$4N_r(T_r)$	$4N_\nu(T_\nu)$	$4N_s(T_s)$
$T_r < m_e$	γs	8	21	21
$m_e < T_r < T_{D_\nu}$	e^\pm	22		
$T_{D_\nu} < T_r < m_\mu$	νs	43		
$m_\mu < T_r < m_\pi$	μ^\pm	57		
$m_\pi < T_r < T_c$	πs	69		
$T_c < T_r < m_s$	$u\bar{u}, d\bar{d}, g s - \pi s$	205		
$m_s < T_r < m_c$	$s\bar{s}$	247		
$m_c < T_r < m_\tau$	$c\bar{c}$	289		
$m_\tau < T_r < m_b$	τ^\pm	303		
$m_b < T_r < m_{W,Z}$	$b\bar{b}$	345		
$m_{W,Z} < T_r < m_H$	W^\pm, Z^0	381		
$m_H < T_r < m_t$	H^0	385		
$m_t < T_r$	$t\bar{t}$	427		

We model the annihilation stage by assuming that the energy liberated in the annihilations keeps the temperature constant at $T_r(t) = m_i$ until all annihilations cease, by requiring that during the annihilation process [11, 12]

$$N(t) = \zeta \Omega_r(t_i) / (q(t_i) q(t)^3 m_i^4) \quad \text{with } \zeta \equiv 90 H_0^2 / (8\pi^3 G). \quad (10)$$

To model dark matter, we consider one of the proposed candidates, the neutralino, since the thermal decoupling of neutralinos has been quantitatively studied [14, 15]. Neutralinos drop from thermal chemical equilibrium with radiation when the expansion of the universe dilutes their density and thus reduces the annihilation probability. However, by collisions with fermions, neutralinos maintain kinetic equilibrium and continue sharing the radiation temperature. The temperature at which neutralinos kinetically decouple is [15] $T_{\text{kd}} = (1.2 \times 10^{-2} m_{\text{Pl}} M_{\tilde{\chi}}^{-1} (M_{\tilde{L}}^2 - M_{\tilde{\chi}}^2)^{-2})^{-1/4}$, where m_{Pl} is the Planck mass, $M_{\tilde{L}}$ the mass of the sfermions (assumed here to be the same for all sfermions) and $M_{\tilde{\chi}}$ the mass of the neutralino. At this point, they are already not relativistic and evolve as non-relativistic matter (6).

The equation of motion for the scale factor is obtained from Friedmann's equation (4) and by writing the density parameter of each species as a function of the scale factor one obtains

$$\dot{q}^2 = [(\Omega_r(t_i) + \Omega_s(t_1))/q^2 + [\Omega_b(t_0) + \Omega_c(t_0)]/q + \Omega_k(t_0) + \Omega_\Lambda q^2], \quad (11)$$

where the sterile neutrinos are included if $\Omega_s(t_1)$ is not set to zero. The initial time t_1 and scale factor values correspond to the estimates of the end of inflation ($t_1 \sim 10^{-32}$ s and $q(t_1) \sim 10^{-27}$) [1]. Solving (11) numerically yields $q(t)$ and we can calculate the temperatures and densities.

3. Results and conclusions

Our calculation shows that the time evolution of the scale factor is almost unaffected by the presence or absence of a sterile neutrino component. The age of the universe is in both cases $t_0 \simeq 0.9983 H_0^{-1}$, equivalent to 13.87 Gyrs. Majorana neutrinos (or active neutrinos in the Dirac case) decouple shortly before electrons and positrons annihilate and do not absorb the e^+e^- annihilation energy as the photons do. If neutrinos are of Dirac type and since the scale factor does not change noticeably when including sterile neutrinos, the active neutrino temperature is practically indistinguishable from the Majorana neutrino temperature T_ν . The sterile neutrino temperature T_s decreases monotonically, unaffected by any of the threshold processes (Fig. 1).

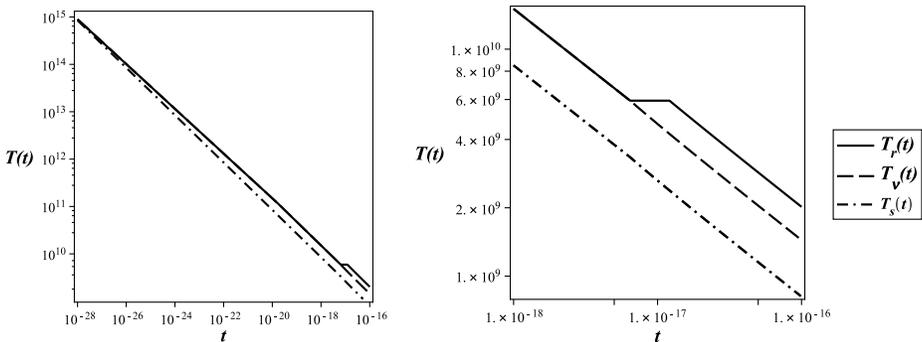


Fig. 1. Radiation and Majorana neutrino temperatures T_r , T_ν and sterile neutrino temperature T_s , in Kelvin, as functions of time t , in units H_0^{-1} . On the right-hand side, we focus on the most conspicuous event, the e^+e^- annihilation threshold.

The number of neutrino species affects primordial nucleosynthesis and sterile neutrinos should be suppressed by a factor $\simeq 0.05$ at that epoch, when $T = 1$ MeV [16]. It has been argued also that they can never have

been in thermal equilibrium if they are light (below 30 eV) [16]. We note that our calculation, which starts at the end of inflation assuming a *common* initial temperature for all species, predicts $\rho_s/\rho_\nu = 0.047$ at the epoch of nucleosynthesis. The suppression factor stems from the lower temperature of the sterile neutrinos as they do not acquire any of the annihilation energies.

Considering dark matter, with the mass estimates $m_1 = 10$ GeV, $m_2 = 100$ GeV and $m_3 = 1$ TeV for the neutralinos, their present epoch average temperature is, respectively, $T(m_1) \simeq 1.79 \times 10^{-11}$ K, $T(m_2) \simeq 1.16 \times 10^{-11}$ K and $T(m_3) \simeq 4.70 \times 10^{-13}$ K. As for the baryons, they are created at quark–hadron confinement temperature and can already be considered non-relativistic from the time they form. They remain in thermal equilibrium with radiation until decoupling at $T_D \simeq 3000$ K [13], corresponding in the present work to a universe aged 373 000 years, consistent with observations [7]. The present average baryonic temperature is calculated as 2.475×10^{-3} K. The present epoch temperatures of baryonic and dark matter are idealized averages, since clustering into structures has affected their temperature evolution.

To summarize, we modelled the universe as a perfect fluid, interpreting dark energy based on fluctuations of the metric and assuming dark matter to consist of neutralinos. The overall development of the universe is not noticeably affected by the extra degrees of freedom of sterile neutrinos. We find that at the epoch of primordial nucleosynthesis the sterile neutrino density is suppressed by a factor which is in agreement with estimates in the literature, when assuming the sterile neutrinos to start out, at the end of inflation, with the same temperature as the rest of the universe.

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REFERENCES

- [1] E. Kolb, M. Turner, *The Early Universe*, Addison-Wesley, 1989.
- [2] E. Komatsu *et al.*, *Astrophys. J. Suppl.* **192**, 18 (2011).
- [3] S. Dodelson, G. Feinberg, *Phys. Rev.* **D43**, 913 (1991).
- [4] S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999).
- [5] A. Riess *et al.*, *Astron. J.* **116**, 1009 (1999).
- [6] P.A.R. Ade *et al.* [Planck Collaboration], *Astron. Astroph.* **566**, A54 (2014).
- [7] G. Hinshaw *et al.*, *Astrophys. J. Suppl.* **208**, 19 (2013).

- [8] J. Narlikar, T. Padmanabhan, *Gravity, Gauge Theories and Quantum Cosmology*, D. Reidel Publishing Company, 1986.
- [9] A.H. Blin, *Af. J. Math. Phys.* **3**, 121 (2006).
- [10] A.H. Blin, *Int. J. Theor. Math. Phys.* **2**, 61 (2012).
- [11] N. Leite, Master's Dissertation, University of Coimbra, 2013.
- [12] N. Leite, A.H. Blin, [arXiv:1404.1833](#) [[hep-ph](#)].
- [13] J. Beringer *et al.* [Particle Data Group], *Phys. Rev.* **D86**, 010001 (2012).
- [14] T. Bringmann, S. Hofmann, *J. Cosmol. Astropart. Phys.* **04**, 016 (2007).
- [15] S. Hofmann *et al.*, *Phys. Rev.* **D64**, 083507 (2001).
- [16] A.D. Dolgov, *Phys. Rep.* **370**, 333 (2002).