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MESON MASS SPLITTINGS IN UNQUENCHED QUARK MODELS*

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General results are obtained for meson mass splittings and mixings in unquenched (coupled-channel) quark models. Theorems derived previously in perturbation theory are generalised to the full coupled-channel system. A new formula is obtained for the mass splittings of physical states in terms of the splittings of the valence states. The S-wave hyperfine splitting decreases due to unquenching, but its relation to the vector e^+e^- width is unchanged; this yields a prediction for the missing $\eta_b(3S)$. In the ordinary (quenched) quark model, the P-wave hyperfine splitting vanishes: this result also survives in the unquenched quark model, despite large mass shifts across the P-wave multiplet. A ratio of mass splittings used to discriminate quarkonium potential models is scarcely affected by unquenching.

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1. Angular momentum coefficients

Unquenched quark models for meson spectroscopy incorporate $q\overline{q}$ pair creation via the transition $Q\overline{Q} \to (Q\overline{q})(q\overline{Q})$. Most models have an operator with the same basic structure, and so share the same general solution [1, 2]: this applies to $^3P_0$ models, flux tube models ($^3P_0$ and $^3S_1$), pseudoscalar–meson emission models, the Cornell model with Lorentz vector confinement and, in the heavy-quark limit, more general microscopic models with Lorentz scalar confinement and one-gluon exchange.

These "non-flip, triplet" models are characterised by the assumptions that the initial Q and $\overline{Q}$ spins are conserved, and that the created $q\overline{q}$ pair is coupled to spin triplet. The operator is a scalar product $\boldsymbol{\chi} \cdot \boldsymbol{O}$ of a spin

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triplet wavefunction $\boldsymbol{\chi}$ (common to all models) and a spatial operator $\boldsymbol{O}$ (which differs from model to model). The predictions of such models are consistent with lattice QCD [3, 4].

In Refs. [1, 2], a general expression is obtained for the transition matrix element valid for all non-flip, triplet models. The initial $Q\overline{Q}$ state is characterised by a radial quantum number n, and spin, orbital and total angular momenta S, L and J. Similarly, the $Q\overline{q}$ and $q\overline{Q}$ mesons have quantum numbers $n_1S_1L_1J_1$ and $n_2S_2L_2J_2$, are coupled to angular momentum j, and are in a partial wave l. The matrix element factorises

$$M_{jl}\begin{bmatrix} n\ S\ L\ J \\ n_1S_1L_1J_1 \\ n_2S_2L_2J_2 \end{bmatrix} = \boldsymbol{\xi}_{jl}\begin{bmatrix} S\ L\ J \\ S_1L_1J_1 \\ S_2L_2J_2 \end{bmatrix} \cdot \boldsymbol{A}_l\begin{bmatrix} n\ L \\ n_1L_1 \\ n_2L_2 \end{bmatrix}, \tag{1}$$

where $\boldsymbol{\xi}$ and $\boldsymbol{A}$ are the matrix elements of $\boldsymbol{\chi}$ and $\boldsymbol{O}$ respectively, along with some angular momentum factors. The dependence of the matrix element on the relative momenta of the meson pair is contained in $\boldsymbol{A}$.

The angular momentum coefficients $\boldsymbol{\xi}$ are model independent and are discussed in detail in Refs. [1, 2]. For present purposes, we need only exploit their orthogonality, which leads to the closure relation

$$\sum_{\substack{S_1J_1\\S_2J_2\\j}} M_{jl}\begin{bmatrix} \widehat{n}\ \widehat{S}\ \widehat{L}\ J \\ n_1S_1L_1J_1 \\ n_2S_2L_2J_2 \end{bmatrix}^* M_{jl}\begin{bmatrix} n\ S\ L\ J \\ n_1S_1L_1J_1 \\ n_2S_2L_2J_2 \end{bmatrix} = \delta_{\widehat{S}S}\delta_{\widehat{L}L}\boldsymbol{A}_l^*\begin{bmatrix} \widehat{n}\ \widehat{L} \\ n_1L_1 \\ n_2L_2 \end{bmatrix} \cdot \boldsymbol{A}_l\begin{bmatrix} n\ L \\ n_1L_1 \\ n_2L_2 \end{bmatrix}. \tag{2}$$

2. The coupled-channel problem

The eigenstates i of the coupled-channel problem are admixtures of valence states $Q\overline{Q}$ and meson–meson continua $(Q\overline{q})(q\overline{Q})$. In solving for the eigenvalues E_i, the key quantity is the following matrix element

$$\left\langle \widehat{n}\widehat{S}\widehat{L}J \| \varOmega(E_i) \| nSLJ \right\rangle = \sum_{\substack{n_1S_1L_1J_1\\n_2S_2L_2J_2\\jl}} \int dpp^2 \frac{M_{jl}\begin{bmatrix} \widehat{n}\ \widehat{S}\ \widehat{L}\ J \\ n_1S_1L_1J_1 \\ n_2S_2L_2J_2 \end{bmatrix}^* M_{jl}\begin{bmatrix} n\ S\ L\ J \\ n_1S_1L_1J_1 \\ n_2S_2L_2J_2 \end{bmatrix}}{E_{12}(p) - E_i}, \tag{3}$$

where p and E_{12} are the momenta and energy of the continuum mesons.

If there are no spin splittings among the continua, the closure relation (2) can be exploited. (This is otherwise not possible since E_{12} depends on S_1, J_1, S_2 and J_2 through the continuum meson masses.) This gives

$$\left\langle \widehat{n}\widehat{S}\widehat{L}J\|\Omega(E_i)\|nSLJ\right\rangle = \delta_{\widehat{S}S}\delta_{\widehat{L}L}\langle\widehat{n}L\|\Omega(E_i)\|nL\rangle, \text{ with} \tag{4}$$

$$\langle\widehat{n}L\|\Omega(E_i)\|nL\rangle = \sum_{\substack{n_1L_1\\n_2L_2l}}\int dpp^2\frac{\boldsymbol{A}_l^*\begin{bmatrix}\widehat{n}\ L\\n_1L_1\\n_2L_2\end{bmatrix}\cdot\boldsymbol{A}_l\begin{bmatrix}n\ L\\n_1L_1\\n_2L_2\end{bmatrix}}{E_{12}(p)-E_i}\,. \tag{5}$$

In this approximation,s there is no mixing due to unquenching among states with different S or L. This is a generalisation of a theorem obtained in perturbation theory [5] to the full coupled-channel problem.

If we further assume (and this assumption will be relaxed shortly) that there are no spin splittings among the valence masses, then since the mixing matrix is independent of S and J, the physical masses are also independent of S and J (another generalisation of Ref. [5]), as is the configuration mixing of different radial states (a new result).

We return now to the more general case with splittings among the valence and (consequently) physical masses, but not among the continua. (Small continuum splittings can be dealt with, and do not modify the results below.) Ignoring mixing among different radial states, the physical mass E_{nSLJ} of a state below threshold is related to its valence mass M_{nSLJ},

$$E_{nSLJ} = M_{nSLJ} - \langle\Omega(E_{nSLJ})\rangle_{nSLJ} \tag{6}$$

and the squared amplitude that the state is in the valence configuration is

$$Z_{nSLJ} = \frac{1}{1+\langle\omega(E_{nSLJ})\rangle_{nSLJ}}\,, \quad \text{with} \quad \omega(E_{nSLJ}) = \frac{\partial\Omega(E_{nSLJ})}{\partial E_{nSLJ}}\,. \tag{7}$$

With the following parametrisation of masses

$$M_{nSLJ} = M_{nL} + \delta M_{nSLJ}\,, \qquad E_{nSLJ} = E_{nL} + \delta E_{nSLJ}\,, \tag{8}$$

the Taylor expansion of the mass shift about E_{nL} can be written as

$$\langle\Omega(E_{nSLJ})\rangle_{nSLJ} \approx \langle\Omega(E_{nL})\rangle_{nL} + \delta E_{nSLJ}\langle\omega(E_{nL})\rangle_{nL} \tag{9}$$

which leads to a relation between the physical and valence spin splittings

$$\delta E_{nSLJ} = Z_{nL}\delta M_{nSLJ}\,, \quad \text{with} \quad Z_{nL} = \frac{1}{1+\langle\omega(E_{nL})\rangle_{nL}}\,. \tag{10}$$

This is the main result of this work. Unquenching reduces spin splittings, such that the physical splittings are suppressed with respect to the valence splittings by the (spin-averaged) valence component Z_{nL}.

The validity of the formula can be checked by comparing its predictions to existing model calculations in the literature. Table I gives a typical example for the hyperfine splitting of S-wave charmonia and bottomonia. A forthcoming paper will discuss the result in more detail, testing its predictions more widely against the literature (including for orbitally excited states). The rest of this paper is instead devoted to some applications.

TABLE I

Some model calculations (from Refs. [6, 7]) for the spin-averaged mass shifts $\langle\Omega\rangle_{nS}$, the bare and physical hyperfine splittings δM and δE, and the splittings $\delta E^{\mathrm{pred.}}$ predicted by equation (10). All quantities are in MeV. The entry (*) uses the author's own calculation for Z_{nS}, which disagrees with Ref. [7].

	$\langle\Omega\rangle_{nS}$	δM	δE	$\delta E^{\mathrm{pred.}}$
$c\bar{c}$ [6]				
$1S$	174	129	117	116.4
$2S$	212	64	48	48.4
$b\bar{b}$ [7]				
$1S$	57.41	71.39	68.50	*68.44
$2S$	67.58	23.12	21.30	21.36
$3S$	67.74	15.73	14.00	14.06

3. Some applications

3.1. S-wave hyperfine splitting and e^+e^- widths

In the quenched quark model, meson hyperfine splittings and e^+e^- widths are both proportional to the square of the $Q\overline{Q}$ wavefunction at the origin, which leads to the model-independent relation

$$\delta M_{2S}/\delta M_{1S} = \Gamma_{e^+e^-\to 2^3S_1}/\Gamma_{e^+e^-\to 1^3S_1}\,. \tag{11}$$

The relation is satisfied by the data for charmonia and bottomonia, so it is important to establish that it survives the effects of unquenching [8]. Unquenching suppresses the physical mass splittings by a factor Z_{nS}, but at the same time, suppresses the e^+e^- widths by $Z_{n^3S_1}$ (assuming that they are dominated by the $Q\overline{Q}$ component). To a very good approximation $Z_{n^3S_1} \approx Z_{nS}$, so the relation survives with physical masses. The corresponding relation between the $1S$ and $3S$ levels yields a mass prediction 10334.6 ± 2.2 MeV for the $\eta_b(3S)$.

3.2. P-wave hyperfine splitting

The quenched quark model result for the P-wave hyperfine splitting

$$\tfrac{1}{9}\left(M_{^3P_0} + 3M_{^3P_1} + 5M_{^3P_2}\right) - M_{^1P_1} = 0 \tag{12}$$

is satisfied by charmonia and bottomonia. Since each of the states in the multiplet is subject to large, and different, mass shifts, *a priori* the result could be spoiled by the effects of unquenching. Remarkably, across most models these shifts conspire to make very little contribution to the hyperfine splittings; for example, the shifts (MeV) of $1P$ bottomonia from Ref. [7] give

$$\tfrac{1}{9}\left(80.777 + 3 \times 84.823 + 5 \times 87.388\right) - 85.785 = 0.013\,. \tag{13}$$

The vanishing hyperfine splitting is protected by a mechanism observed and explained in Ref. [9, 10], but can also be seen as a simple consequence of equation (10): if the bare states have zero hyperfine splitting, so too do the physical states. Corrections to (10) due to different continuum masses turn out to affect only the spin–orbit splittings at leading order. The vanishing D-wave splitting (also protected by this mechanism) leads to a prediction for the missing $^1D_2$ bottomonium [11].

3.3. The $Q\overline{Q}$ potential

The ratio of mass splittings $R = (M_{^3P_2} - M_{^3P_1})/(M_{^3P_1} - M_{^3P_0})$ has been used by many authors to discriminate among models for the $Q\overline{Q}$ potential, which raises the question of whether such conclusions should be modified due to unquenching. According to equation (10), the ratio R is invariant under unquenching, and so conclusions based on quenched quark models survive. In practice, there are corrections to equation (10) which lead to a decrease of R due to unquenching, but the effect is not substantial.

3.4. Leptonic width inequalities

Expanding $\omega(E_{nSLJ})$ in a similar way to $\Omega(E_{nSLJ})$ leads to inequalities among the Z-factors, for example $Z_{^1S_0} > Z_{^3S_1}$ and $Z_{^3P_0} > Z_{^3P_1} > Z_{^3P_2}$. These modify relations among leptonic widths which arise due to eliminating common factors of the wavefunction at the origin, such as

$$\frac{\Gamma_{^1S_0\to\gamma\gamma}}{\Gamma_{^3S_1\to e^+e^-}} > \frac{4}{3}\left(1 + 1.96\frac{\alpha_\mathrm{s}}{\pi}\right)\,, \qquad \frac{\Gamma_{^3P_2\to\gamma\gamma}}{\Gamma_{^3P_0\to\gamma\gamma}} < \frac{4}{15}\left(1 - 5.51\frac{\alpha_\mathrm{s}}{\pi}\right)\,. \tag{14}$$

It will be difficult to identify such effects in practice, as leptonic width relations are subject to large theoretical and experimental uncertainties; nevertheless, it is instructive to know the direction in which unquenching modifies such relations.

3.5. Splittings in lattice QCD

In unquenched lattice QCD, spin splittings vary with dynamical quark masses. On the basis of equation (10), one might expect that decreasing the quark mass leads necessarily to a decrease in spin splittings. (Due to a decrease in binding energy and an enhancement in the coupling matrix element, the valence component would decrease.) The situation is not so simple, though, since the quark mass influences $\alpha_{\rm s}$, and moreover there is considerable evidence that unquenched lattice QCD without explicit $(Q\overline{q})(q\overline{Q})$ operators is not sensitive to the coupling $Q\overline{Q} \to (Q\overline{q})(q\overline{Q})$ [12].

Nevertheless future lattice calculations with (complete multiplets of) $(Q\overline{q})(q\overline{Q})$ operators could, in principle, offer a direct test of equation (10), if spin splittings and Z-factors are measured at various quark masses. There is already some work in this direction; Bali *et al.* [13] have measured splittings and Z-factors for several charmonia, but at one quark mass and with one $(Q\overline{q})(q\overline{Q})$ operator per channel.

4. Conclusion

General results have been obtained for unquenched quark models based on the non-flip, triplet operator. Previous results from perturbation theory, valid in the absence of spin splittings, have been generalised to the full coupled-channel problem, and extended. The more realistic scenario, incorporating spin splittings among the valence and physical masses, involves a simple mass formula, some of whose implications have been discussed here. The formula ensures that several empirically successful results of quenched quark models survive the effects of unquenching. The formula should be testable in future lattice QCD calculations. Although it was not discussed here, the formula is also useful for practical calculations of mass splittings in unquenched quark models.

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