

LARGE- $N_c$  REGGE SPECTROSCOPY\*

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This paper, dedicated to Eef van Beveren on the occasion of his birthday, reviews some of our results concerning the hadron spectroscopy, Regge trajectories, and the large- $N_c$  meson-dominance of hadronic form factors.

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The paper is based on several of our recent papers [1–5] devoted to hadron spectroscopy involving the large- $N_c$  arguments and Regge trajectories. We begin with a brief review of the old Hagedorn [6] idea, applied to the fundamental question of understanding the spectrum of QCD, as well as its applications to thermodynamic properties which find practical use in understanding the lattice data and modelling the relativistic heavy-ion collisions. The excitation function of QCD is presented in Fig. 1, where we plot

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the density of states (left panel) represented via the Breit–Wigner functions (for plotting purposes, the stable states were attributed some finite width), as well as the cumulative number of states with mass below  $m$ . We include all light hadron states as described in [7, 8]. We note that the cumulative number of states increases exponentially up to  $m \sim 2$  GeV, in accordance to the Hagedorn hypothesis. Above, no states have been identified, which may be a feature of QCD (the states are wider as  $m$  grows), or result from no experimental “coverage”. The mentioned growth of the width of resonances with the mass is visualized in Fig. 2. Except for some unusual states, such as the  $\sigma$  meson, all states follow this trend. Thus, as  $m$  increases, we have more and more states which become wider. This leads to the expectation that the region indicated with a question mark in the left panel of Fig. 1 is indeed filled with strength coming from such states.

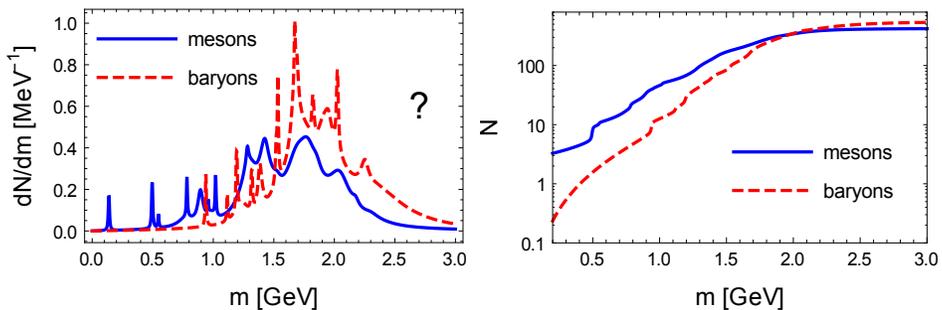


Fig. 1. Left: excitation function of QCD for light ( $u, d, s$ ) hadrons, where the states are represented as Breit–Wigner distributions. Right: the corresponding number of states below mass  $m$ . The question mark indicates the experimentally unexplored region.

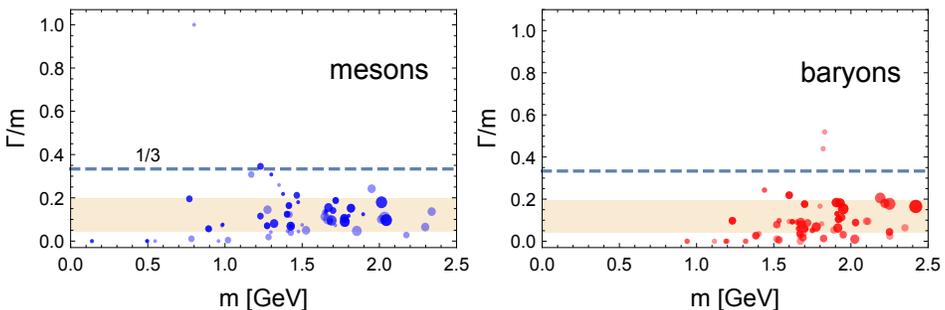


Fig. 2. Ratio of the width to mass for the known ( $u, d, s$ ) meson (left) and baryon (right) states. The size of the dots is proportional to the spin degeneracy, and the intensity to the isospin degeneracy. The band indicates the mean  $\pm$  standard deviation.

One may use the resonances to evaluate thermodynamic properties of QCD in the hadronic phase (see, *e.g.*, [9] and references therein). An example of the trace anomaly  $\Theta_\mu^\mu$  is shown in Fig. 3, where we can enjoy the nice display of the parton–hadron duality in the agreement of the hadron resonance gas and the lattice data.

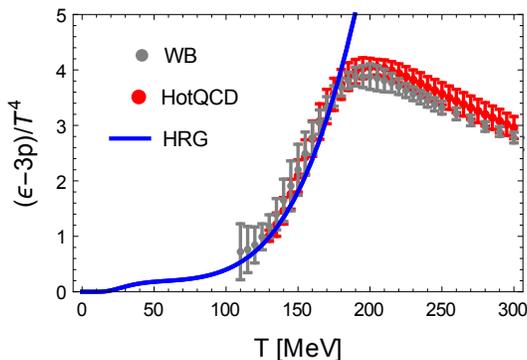


Fig. 3. Trace anomaly of QCD from the hadron resonance gas (HRG) compared to the lattice data of the Wuppertal–Budapest (WB) [10] and HotQCD [11] collaborations.

As discussed in [12] (see also references therein), the Hagedorn growth of states may be explained with string models which result in large degeneracy of the daughter Regge trajectories. These models also explain the faster growth for baryons than for mesons, as realized in the data. Thus, the question of the Regge spectra is immanently related to the issues discussed here.

A comprehensive review of the Regge spectroscopy in both the angular and radial quantum numbers is presented in [3], following the compilation of [13]. An example should be the case of scalar–isoscalar states. It turns out that these states may be arranged along two Regge trajectories with the standard slope, or, equivalently, on one with half the slope. Then, it becomes degenerate with the pion trajectory, as seen from Fig. 4, *cf.* [1].

As hadrons may be grouped in angular and radial Regge trajectories, both may be combined into *Regge planes*, where the principal and daughter planes are nearly parallel. An example of two such cases is shown in Fig. 5. A fundamental question here is whether the slopes of the radial and angular Regge trajectories are universal or not. Our study presented in Ref. [3] shows that at the statistical level of 4.5 standard deviations there is no universality. In this study, we have fitted the formula

$$M^2 = an + bJ + c = 1.38(4)n + 1.12(4)J - 1.25(4). \quad (1)$$

The result of the fit for the individual trajectories is presented in Fig. 6.

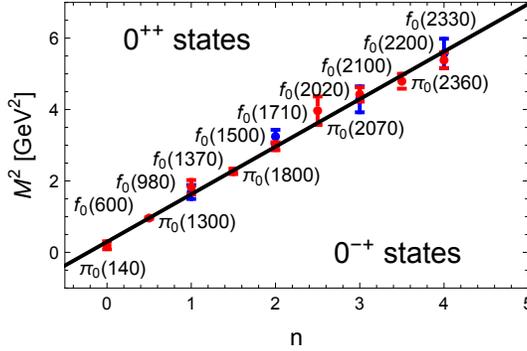


Fig. 4. Radial Regge trajectories for the  $f_0$  and pion states. The two  $f_0$  trajectories were overlaid, yielding a single trajectory with half the slope. The universality with the pion trajectory is vivid.

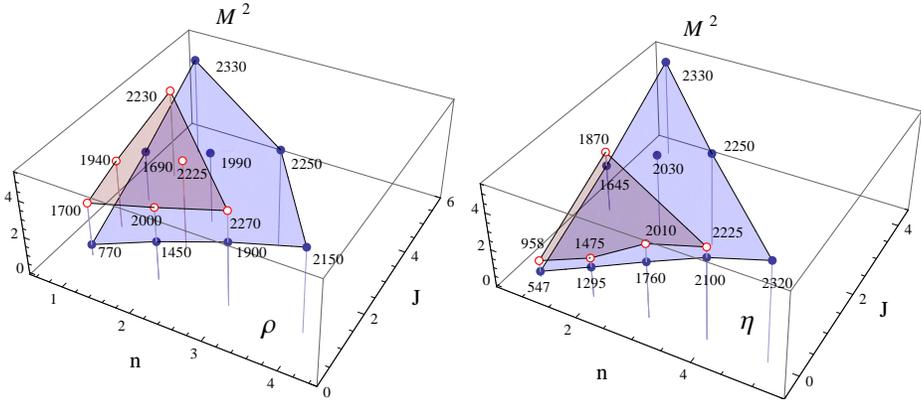


Fig. 5. Regge planes based on the angular and radial  $\rho$  and  $\eta$  Regge trajectories.

An interesting application of meson spectroscopy is the explanation of various hadronic form factors with the meson dominance principle, which generalizes the well-known vector–meson dominance. In the large- $N_c$  limit, where all diagrams are trees, one proceeds by coupling the mesons with appropriate quantum numbers to currents, and then to the probed hadron. The masses of the mesons are taken as their physical values, while the relevant coupling constants are fitted to the form factor data. The QCD short-distance constraints may be satisfied when a sufficient number of mesons is included [5, 14]. Of course, one may always include more states, even infinitely many.

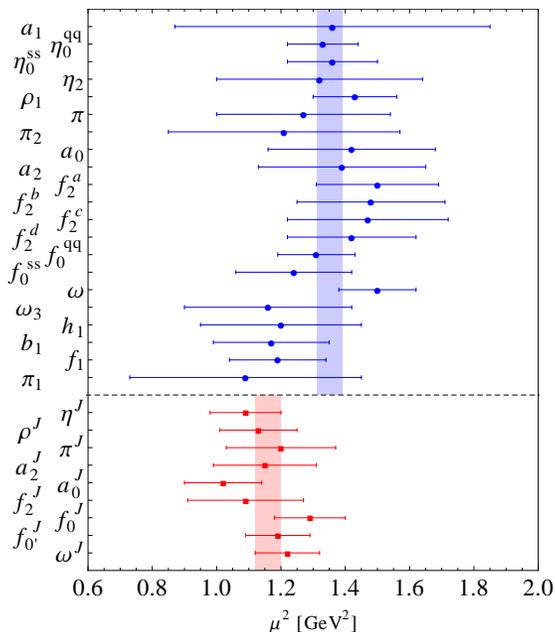


Fig. 6. Slope parameters  $\mu^2 = a$  for the radial and  $\mu^2 = b$  for the angular Regge trajectories from Ref. [3]. The bands indicate the mean  $\pm$  standard deviation. The lack of universality of the radial and angular slopes is evident.

An important issue in such studies is the error analysis, which allows for an assessment how well the approach works. In our studies, based on the large- $N_c$  limit, we have used either the width of the state as its uncertainty of mass, or mass divided by  $N_c$ . We call them, correspondingly, the  $\Gamma$  rule or the  $1/N_c$  rule. In Fig. 7, we show two examples of such calculations: the

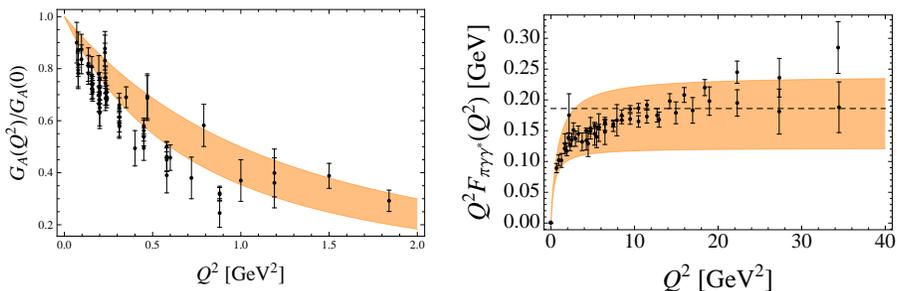


Fig. 7. Examples of the meson-dominance principle: (left) the axial-vector form factor of the nucleon compared to the lattice data [15] and (right) the pion-photon transition form factor multiplied by  $Q^2$ , compared to the experimental data [16–19]. The model uncertainty bands are obtained with the  $1/N_c$  rule.

axial–vector isovector form factor of the nucleon and the pion–photon transition form factor. The bands are obtained from the  $1/N_c$  rule. We note that the data are compatible with the experimental data. A similar agreement is found for other form factors of the nucleon and the pion (electromagnetic, gravitational) [5], indicating that the large- $N_c$  meson-dominance principle is capable of reproducing the data.

Happy birthday, Eef!

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