CALCULATION OF REGGE TRAJECTORIES FROM ELASTIC SCATTERING POLES AND THE NON-ORDINARY $f_0(500)$ MESON*

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We show how Regge trajectories of resonances appearing in elastic two-meson scattering can be obtained just from their pole position and coupling, using a dispersive formalism. In this way, the finite widths of resonances can be taken into account in Regge trajectories. For the $\rho(770)$, $f_2(1270)$ and $f_2'(1525)$, this method leads to ordinary linear Regge trajectories with the universal slope, as expected for ordinary $\bar{q}q$ resonances. In contrast, for the $f_0(500)$ meson, the resulting Regge trajectory is non-linear and with much smaller slope, which is another strong indication of its non-ordinary nature.

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1. Introduction

This workshop was a celebration of Eef van Beveren's long and fruitful career at the occasion of his 70$^{\text{th}}$ birthday. I gladly joined this celebration with a work which, starting from a rather unrelated framework, is finally related to some of his best known works [1, 2]. At a time when the σ meson (nowadays the $f_0(500)$) had even been removed from the Review of Particle Properties, the lightest scalar nonet was identified, including a σ around 500 MeV with a large width and a large meson–meson admixture [1]. A pure quark model description was shown not to be enough to obtain the correct mass and width, but an interaction with the meson–meson state was needed.

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The σ physical mass was a consequence of such interactions, since if they were tuned weaker, the mass would move towards 1 GeV, *i.e.* light scalars "behave in a completely different fashion to regular $q\bar{q}$ nonets" [2].

Actually, in a recent work [3] and in some ongoing research project [4], we have used the analytic properties of amplitudes in the complex angular momentum plane to study Regge trajectories of resonances decaying predominantly to one channel (almost elastic resonances). One of the main features of our calculation is that we take the widths into account, *i.e.*, our Regge trajectories are complex functions, as they should be in principle. Note that widths have been traditionally ignored in Regge descriptions, but that does not seem appropriate for resonances as wide as the $f_0(500)$. The interest of our results stems from the form of these trajectories, which can be used to discriminate between different underlying QCD structures. As it is well known, linear (J, M^2) trajectories relating the angular momentum J and the mass squared can be interpreted in terms of quark–antiquark states (they can be obtained from the rotation of a flux tube connecting a quark and an antiquark). Significant deviations from this linear behavior would support a different nature of a resonance and the scale of the trajectory slope would indicate the scale of the mechanism responsible for its existence.

In particular, we have studied in [3] the trajectories of the lightest resonances in elastic $\pi\pi$ scattering: the $\rho(770)$, which is a well established ordinary $\bar{q}q$ state, and the $f_0(500)$ or σ meson, whose nature is still under debate and whose resulting trajectory, as we will see, does not follow the ordinary linear (J, M^2) trajectories. Frequently, the sigma is not included in those linear fits [5] (which, as we will see, is the correct thing to do), or its huge width is used as the uncertainty in the mass, so that it could be accommodated easily. But as we will see, the width is part of the Regge trajectory, and considering it as just a mass uncertainty is not really justified.

2. Regge trajectories from a resonance pole and residue

Near a Regge pole, the partial wave for the scattering of two particles with equal mass m reads

$$t_l(s) = \beta(s)/(l - \alpha(s)) + f(l, s)\,, \tag{1}$$

where $f(l, s)$ is a regular function of the generalized angular momentum l and the Regge trajectory $\alpha(s)$ and residue $\beta(s)$ are analytic functions, the former having a cut along the real axis for $s > 4m^2$.

The analytic properties of $\alpha(s)$ and $\beta(s)$ and the elastic unitarity condition imply the following system of coupled dispersion relations [6]

$$\mathrm{Re}\,\alpha(s) \;=\; \alpha_0 + \alpha' s + \frac{s}{\pi}\mathrm{PV}\int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\,\alpha(s')}{s'(s'-s)}\,, \tag{2}$$

$$\operatorname{Im}\alpha(s) = \frac{\rho(s) b_0 \hat{s}^{\alpha_0+\alpha' s}}{\left|\Gamma\left(\alpha(s)+\frac{3}{2}\right)\right|} \exp\left(-\alpha' s\left[1-\log\left(\alpha' s_0\right)\right] + \frac{s}{\pi}\mathrm{PV}\int_{4m^2}^{\infty} ds' \frac{\operatorname{Im}\alpha\left(s'\right)\log\frac{\hat{s}}{\hat{s}'} + \arg\Gamma\left(\alpha\left(s'\right)+\frac{3}{2}\right)}{s'\left(s'-s\right)}\right), \quad (3)$$

where PV denotes "principal value" and α_0, α' and b_0 are free parameters to be determined by forcing the resulting amplitude to have a specified pole and residue. In particular, for a given set of α_0, α' and b_0 parameters, we solve the system of Eqs. (2) and (3) iteratively. The value of the parameters is fixed by fitting only three inputs, namely, the real and imaginary parts of the resonance pole position $s_M \simeq (M_{\rm R} - i\Gamma_{\rm R}/2)^2$, where $M_{\rm R}$ and $\Gamma_{\rm R}$ are the pole mass and width of the resonance, together with the absolute value of the pole residue $|g_M|$. Namely, we fit the resonance pole on the second Riemann sheet to: $\beta_M(s)/(l - \alpha_M(s)) \to |g_M^2|/(s - s_M)$, with $l = 0, 1$ for $M = \sigma, \rho$. The pole parameters of the $f_0(500)$ and the $\rho(770)$ are taken from a precise dispersive representation of $\pi\pi$ scattering data [7, 8]. *Note that we are just fitting the scattering pole parameters and never the Regge trajectory, which is therefore a prediction of our approach.*

Thus, in the left panel of Fig. 1, we show the resulting Regge trajectories found in [3]. The resulting trajectory parameters are given in Table I. The imaginary part of $\alpha_\rho(s)$ is much smaller than the real part, and the latter

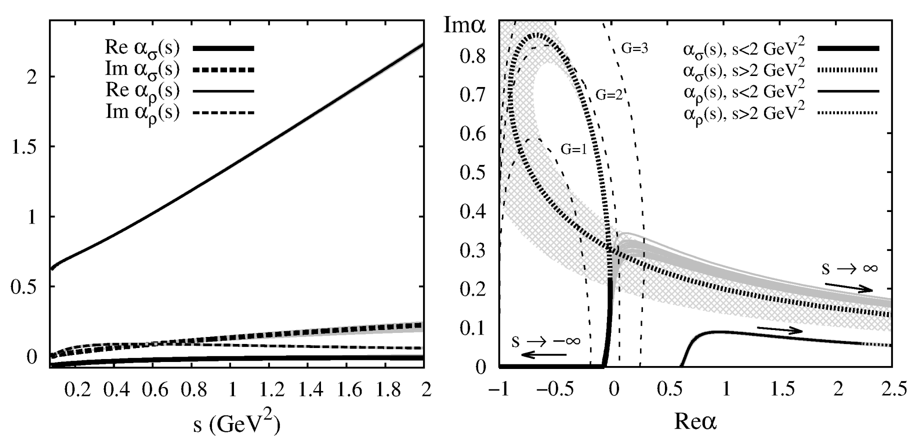

Fig. 1. (Left) $\alpha_\rho(s)$ and $\alpha_\sigma(s)$ Regge trajectories, from our constrained Regge-pole amplitudes. (Right) $\alpha_\sigma(s)$ and $\alpha_\rho(s)$ in the complex plane. At low and intermediate energies (thick continuous lines), the trajectory of the σ is similar to those of Yukawa potentials $V(r) = -Ga\exp(-r/a)/r$ [12] (thin dashed lines). Beyond 2 GeV^2, we plot our results as thick discontinuous lines because they should be considered just as extrapolations.

grows linearly with s. In view of our approximations, and taking into account the fact that our error bands only reflect the uncertainty in the input pole parameters, the agreement with previous Regge trajectory determinations is remarkable when comparing with $\alpha_\rho(0) = 0.52 \pm 0.02$ [9], $\alpha_\rho(0) = 0.450 \pm 0.005$ [10], $\alpha'_\rho \simeq 0.83$ GeV^{-2} [5], $\alpha'_\rho = 0.9$ GeV^{-2} [9], or $\alpha'_\rho \simeq 0.87 \pm 0.06$ GeV^{-2} [11].

TABLE I

Parameters of the $\rho(770)$ and $f_0(500)$ Regge trajectories calculated from their poles in scattering. For the $f_0(500)$, b_0 is not dimensionless because we have factorized explicitly in $\beta(s)$ the Adler zero required by chiral symmetry.

	α_0	α' [GeV^{-2}]	b_0
$\rho(770)$	0.520 ± 0.002	0.902 ± 0.004	0.52
$f_0(500)$	$-0.090\,^{+\,0.004}_{-\,0.012}$	$0.002^{+0.050}_{-0.001}$	0.12 GeV^{-2}

In contrast, the $f_0(500)$ trajectory is definitely not linear and the slope of the resulting curve at the physical mass is about two orders of magnitude smaller than that of ordinary mesons, like that of the $\rho(770)$ calculated above, or those of the $f_2(1270)$ or $f'_2(1525)$ that we will show below. This provides strong support for a non-ordinary nature of the $f_0(500)$ resonance. Furthermore, the resulting slope, smaller than 1 GeV^{-2} by two orders of magnitude or more is more typical of meson physics than of quark–antiquark interactions. Moreover, the tiny slope excludes that any of the known isoscalar resonances may lie on the $f_0(500)$ trajectory. To test how robust this observation is, we have checked that our results are very stable within the uncertainties of the pole parameters. In addition, we have tried to force a typical size linear trajectory on the σ, but that deteriorates the fit to the σ pole and, particularly, to the coupling [3], and thus the resulting amplitude is qualitatively very different from the observations in the physical region.

Note that with our formalism we are dealing correctly with the huge $f_0(500)$ width, which by no means should be considered as an uncertainty in the mass.

Furthermore, in Fig. 1 we show the striking similarities between the $f_0(500)$ trajectory and those of Yukawa potentials in non-relativistic scattering [12]. From the Yukawa $G = 2$ curve in that plot, which lies closest to our result for the $f_0(500)$, we can estimate $a \simeq 0.5$ GeV^{-1}, following [12]. This could be compared, for instance, to the S-wave $\pi\pi$ scattering length $\simeq 1.6$ GeV^{-1}. Thus, it seems that the range of a Yukawa potential that would mimic our low energy results is comparable but smaller than the $\pi\pi$ scattering length in the scalar isoscalar channel. Of course, our results are most reliable at low energies (thick continuous line) and the extrapolation should be interpreted cautiously. Nevertheless, our results suggest that the $f_0(500)$ looks more like a low-energy resonance of a short range potential, *e.g.* between pions, than a bound state of a confining force between a quark and an antiquark.

Let us also report on our preliminary results for the $f_2(1270)$ and $f_2'(1525)$ resonances, which appear in D-wave $\pi\pi$ and $K\bar{K}$ scattering, respectively. Both of them are almost elastic: the $f_2(1270)$ has a branching ratio to $\pi\pi$ of approximately 85%, whereas the $f_2'(1525)$ has a branching ratio to $K\bar{K}$ of 88% and thus we have treated them both within a purely elastic formalism. Still, we have considered a relative systematic uncertainty of the size of one minus their respective branching ratio. To determine the pole and residue of the $f_2(1270)$, we have used the phenomenological parametrizations in [8]. For the $f_2'(1525)$, we have just assumed a Breit–Wigner resonance shape whose pole and residue have a straightforward relation to the observed mass and the decay width to two kaons. Thus, in Fig. 2 we show the preliminary resulting Regge trajectories [4], which come out to be almost real, linear and with slopes quite consistent with the universal one, as it happened for the $\rho(770)$. The preliminary $f_2(1270)$ slope is 0.7 GeV^{-2} and the intercept is

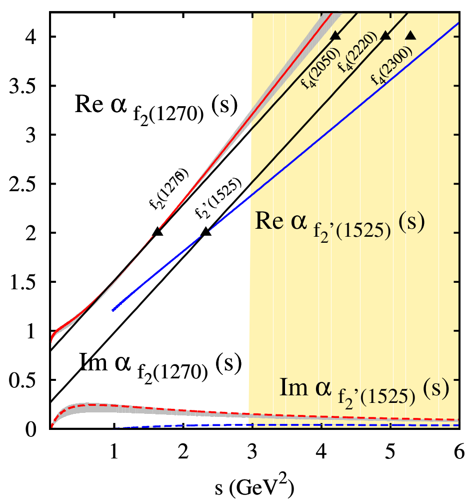

Fig. 2. $f_2(1270)$ (red) and $f_2'(1525)$ (blue) Regge trajectories calculated from their poles in $\pi\pi$ and $K\bar{K}$ scattering respectively. The continuous lines correspond to the real part of the trajectory (to be identified with spin at integer values), whereas the dashed lines stand for the imaginary parts. The gray bands cover the uncertainties in our calculation, mostly due to using the elastic approximation. The black straight lines are the Regge trajectories obtained by fitting the mass and spin of two resonances per trajectory. The light gray/yellow area is the mass region where our elastic approach should be considered cautiously as a mere extrapolation.

$\alpha_0 = 0.9$, whereas for the $f_2'(1525)$ we obtain 0.63 GeV^{-2} and $\alpha_0 = 0.53$, fairly consistent with expectations given the uncertainties of our approach. Detailed error estimates will be evaluated in [4].

In summary, our formalism is able to predict the Regge trajectory of a meson from its associated pole in elastic meson–meson scattering. We have been able to calculate the Regge trajectories of the $\rho(770)$, $f_2(1270)$ and $f_2'(1525)$, which come out almost real and linear, with the typical slope of ordinary $q\bar{q}$ resonances. However, this is not the case for the $f_0(500)$, which explains why the lightest scalar meson has to be excluded from the linear Regge fits of ordinary mesons.

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