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SCALAR MESONS IN $\tau \to K\pi\nu$ AND $\tau \to \eta\pi\nu$ DECAYS*

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The τ decay modes $\tau \to P_1P_2\nu$ provide clean probes of the couplings of the flavoured scalar mesons to the $\bar{u}s$ or $\bar{u}d$ scalar currents. We review the theoretical constraints which relate the P_1P_2 scalar form factors with P_1P_2 scattering for $P_1P_2 = K\pi, \eta\pi$ and their applications.

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1. Introduction

The lightest glueball in QCD is expected to be a scalar resonance and it is still an open problem to properly identify the corresponding state(s) in the physical spectrum (see *e.g.* [1] for a review). There has been significant progress, recently, in clarifying the status of the lightest physical scalar resonances. In particular, two "broad" resonances, the σ (or $f_0(500)$) and κ (or $K_0^*(800)$) have now been accepted into the PDG. The most reliable determinations concern the σ [2, 3]; they make use of theoretical tools combined with accurate experimental data on low energy $\pi\pi$ scattering (in particular, the recent results from the NA48/2 [4] and DIRAC [5] collaborations). Reaching a similar level of confidence for the κ would be important since its existence would disfavour the interpretation proposed in [6] of the σ as a glueball-related state. New experimental measurements related to πK scattering have been performed, notably a determination of S-wave phase-shifts from D_{l4} decays [7] and a first measurement of the πK atom lifetime [8], but better precision is required.

In this paper, I discuss the relevance of τ decays, in particular the decay modes into two pseudo-scalar mesons, P_1P_2, for probing the scalar resonances which couple to P_1P_2. In particular, these amplitudes provide a

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means of measuring experimentally the coupling of scalar resonances to the scalar operators $\bar{u}s$ (for the $S = 1$ scalar resonances) and $\bar{u}d$ for the $I = 1$ resonances. These couplings are important for confirming the nonet assignments of resonances [9] and provide a quantitative measure of their degree of exoticity.

2. $\tau \to K\pi\nu$ decays and πK scattering

The decay amplitudes of the τ lepton into two pseudo-scalar mesons, $\tau \to P_1P_2\nu$ are particularly simple objects (depending on a single variable) which are strongly tied to P_1P_2 scattering. Let us first recall some results on $P_1P_2 = K\pi$ and we will next consider[1] $P_1P_2 = \eta\pi$. The decay amplitude involves the matrix element of the vector current which is expressed in terms of two form factors, *e.g.* for $K^+\pi^0$

$$\left\langle K^+(p_K)\pi^0(p_\pi)|\bar{u}\gamma^\mu s|0\right\rangle = \frac{1}{\sqrt{2}}\left[f_+^{K\pi}(t)(p_K - p_\pi)^\mu + f_-^{K\pi}(t)(p_K + p_\pi)^\mu\right] \tag{1}$$

(with $t = (p_K + p_\pi)^2$). The scalar form factor is defined as the combination $f_0^{K\pi}(t) = f_+^{K\pi}(t) + t/(m_K^2 - m_\pi^2)f_-^{K\pi}(t)$. Making use of the Ward identity for the vector current, $i\partial_\mu\bar{u}\gamma^\mu s = (m_s - m_u)\bar{u}s$, one sees that the scalar form factor describes the matrix element of the scalar current $\bar{u}s$

$$(m_s - m_u)\left\langle K^+(p_K)\pi^0(p_\pi)|\bar{u}s|0\right\rangle = -\frac{1}{\sqrt{2}}\left(m_K^2 - m_\pi^2\right)\, f_0^{K\pi}(t)\,. \tag{2}$$

Watson's theorem implies that the phases of the form factors of $f_+^{K\pi}$ and $f_0^{K\pi}$ must be identical to the $I = 1/2$ phase shifts with $J = 1$ and $J = 0$ respectively, in the energy region where πK scattering is elastic.

One can actually evaluate the form factor phases in a somewhat larger energy region, because the onset of inelasticity for P_1P_2 scattering is strongly dominated by two-body channels. This was first used for the $\pi\pi$ scalar form factors in Ref. [10]. In the case of $K\pi$ with $J = 0$, the main inelastic channel is $K\eta'$ [11] and neglecting further inelastic channels, the Cauchy representation takes the form of a set of coupled singular integral equations

$$\begin{pmatrix} f_0^{K\pi}(t) \\ f_0^{K\eta'}(t) \end{pmatrix} = \frac{1}{\pi}\int\limits_{(m_K+m_\pi)^2}^{\infty} \frac{dt'}{t'-t}\boldsymbol{T}_0^*\left(t'\right)\Sigma\left(t'\right)\begin{pmatrix} f_0^{K\pi}(t') \\ f_0^{K\eta'}(t') \end{pmatrix}, \tag{3}$$

where Σ is a diagonal matrix of kinematical factors and $\boldsymbol{T}_0$ is the 2×2 partial-wave T-matrix. Equations (3) were first studied in Ref. [12]. Solving

[1] In the case of $P_1P_2 = \pi\pi$, the form factors are related to $I = 1$, $J = 1$ and the non-resonant $I = 2$, $J = 0$ scattering.

the equations[2] gives, in particular, the phase of $f_0^{K\pi}$, which should be reliable in the region where inelasticity is effectively dominated by $K\eta'$. Figure 1 shows the result and illustrates the generic feature that the sharp onset of inelasticity is associated with a sharp dip of the form-factor phase. This dip corresponds to a minimum in the modulus of the form factor which implies some suppression of the coupling of the $K_0^*(1430)$ resonance to the $\bar{u}s$ operator.

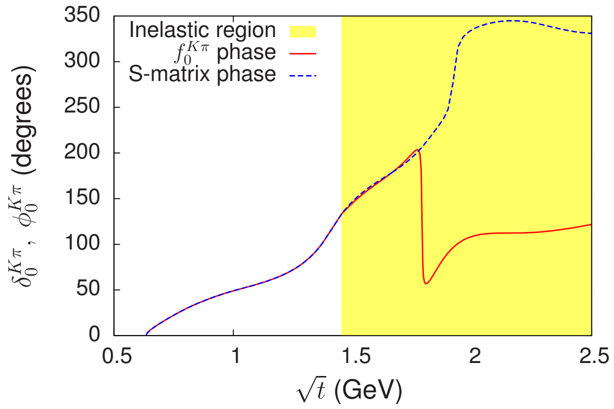

Fig. 1. Phase of the $K\pi$ scalar form factor generated by solving Eqs. (3).

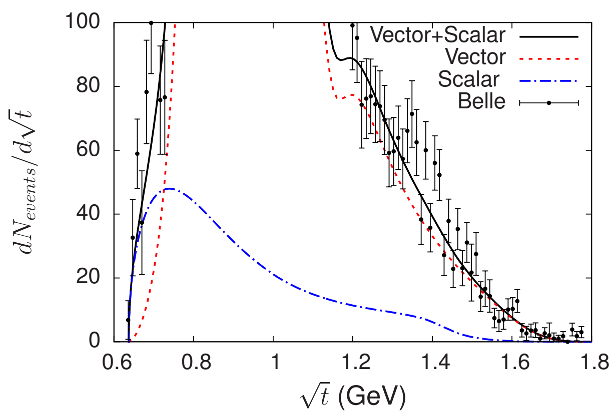

Fig. 2. The $\tau \to K\pi\nu$ decay width as a function of the $K\pi$ energy. The contributions from the vector (scalar) form factors correspond to the dotted/red (dash-dotted/blue) curves.

[2] Proper asymptotic conditions must be imposed on the T-matrix which ensure a unique solution compatible with QCD asymptotic behaviour.

An analogous description can be developed for the vector form factor $f_+^{K\pi}$ [13]. The onset of inelasticity, in this case, is dominated by the quasi-two-body channels: $K^*\pi$, $K\rho$. The experimental measurement of the energy distribution $d\Gamma_{\tau\to K\pi\nu}/dE_{K\pi}$ [14] may be used to improve the determination of the $J=1$ πK scattering phase shift (see Ref. [15]). Figure 2 illustrates the vector and scalar form factor contributions to the energy distribution. The scalar contribution dominates over the vector one in the region $\sqrt{t} <$ 0.8 GeV and its size is confirmed by experiment. It displays a low energy enhancement which may be interpreted as reflecting the effect of the $K_0^*(800)$ resonance and favours a non-exotic nature.

3. The $\eta\pi$ scalar form factor and the isospin violating $\tau \to \eta\pi\nu$ amplitude

The amplitude for $\tau \to \eta\pi\nu$ involves two form factors $f_+^{\eta\pi}$, $f_0^{\eta\pi}$ exactly as in Eq. (1). Weinberg's G-parity argument [16] indicates that these form factors are isospin violating. This is also seen in the case of the scalar form factor from the Ward identity: $i\partial_\mu \bar{u}\gamma^\mu d = (m_d - m_u)\bar{u}d - eA_\mu \bar{u}\gamma^\mu d$. We will revisit here the estimate of the scalar form factor making use of analyticity, chiral symmetry constraints, and analogies with the $K\pi$ scalar form factor.

Concerning analyticity, one might be concerned about anomalous thresholds, since the η meson is not stable in QCD. This was investigated in detail in Ref. [17] based on Mandelstam's approach: one starts from an unphysical situation where the mass of the η is small $m_\eta < 3m_\pi$ and one follows the motion of the endpoint singularities, while varying the mass towards its physical value. The result is that no anomalous threshold appears.

Near $t=0$, the form factors are constrained by chiral symmetry: they were computed up to NLO in ChPT in Refs. [18, 19]. At this order, the value at $t=0$ obeys a parameter-free relation with an isospin violating combination of $K\pi$ form factors which can be evaluated rather precisely using experimental inputs on K_{l3} decays from K factories (see [20]),

$$f_+^{\eta\pi}(0) = f_0^{\eta\pi}(0) = \frac{1}{\sqrt{3}}\left(\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^+}(0)} - 1\right) = (1.49 \pm 0.23)\times 10^{-2}\,. \qquad (4)$$

A further constraint is associated with the derivative of the form factor

$$\begin{aligned}\dot{f}_0^{\eta\pi}(0) \;=\; & f_0^{\eta\pi}(0)\left\{\frac{1}{12F_\pi^2}\left[48\,L_5^r - \frac{1}{16\pi^2}\left(9\,\log\frac{m_K^2}{\mu^2} + 11\right) + 4m_\pi^2\,\dot{J}_{\eta\pi}(0)\right]\right. \\ & \left. + \frac{\sqrt{3}e^2}{18\Delta_{\eta\pi}\,\epsilon}\left[-2(2S_2 + S_3) + \frac{11}{16\pi^2}Z\right]\right\}. \qquad (5)\end{aligned}$$

Numerically, this corresponds to a value for the η–π scalar radius

$$\langle r^2 \rangle_S^{\eta\pi} \equiv \frac{6\dot{f}_0^{\eta\pi}(0)}{f_0^{\eta\pi}(0)} = (0.083 \pm 0.006)\,\mathrm{fm}^2 \qquad (6)$$

which is remarkably small: it is twice smaller than the analogous $K\pi$ scalar radius (interestingly, the two radii should be equal in the large N_c limit).

We can now estimate $f_0^{\eta\pi}$, writing a phase dispersive representation,

$$f_0^{\eta\pi}(t) = f_0^{\eta\pi}(0) \exp\left[\zeta\, t + \frac{t^2}{\pi} \int\limits_{(m_\eta+m_\pi)^2}^{\infty} dt' \, \frac{\phi^{\eta\pi}(t')}{(t')^2\,(t'-t)}\right], \qquad (7)$$

where $\zeta = \langle r^2 \rangle_S^{\eta\pi}/6$. Finiteness when $t \to \infty$ yields the sum rule,

$$\zeta = \frac{1}{\pi} \int\limits_{(m_\eta+m_\pi)^2}^{\infty} dt' \, \frac{\phi^{\eta\pi}(t')}{(t')^2}\,. \qquad (8)$$

Further properties of $\phi^{\eta\pi}$ is that it should obey Watson's theorem below the $K\overline{K}$ threshold and that it should display a dip at some point t_1 above. The sum rule (8) proves rather constraining. Using the $\eta\pi$ phase-shift model of Ref. [21], the position of the dip is restricted to the range of $2m_K < \sqrt{t_1} < 1.2$ GeV. In the dispersive approach, the exotic nature of one of the a_0 resonances corresponds to the dip lying close to its mass. Figure 3 illustrates the phase $\phi^{\eta\pi}$ generated in this approach. The corresponding

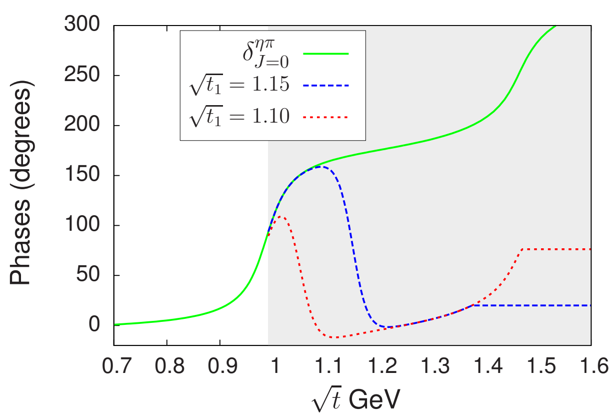

Fig. 3. Model for the scalar form factor phase satisfying the sum rule (8) for several values of the dip position t_1.

shape of scalar component of the energy distribution of the $\tau \to \eta\pi\nu$ decay as a function of the dip position t_1 is shown in Fig. 4 (the vector component, computed in Ref. [17] is also shown). This could be probed experimentally at the future super-B or charm-tau factory.

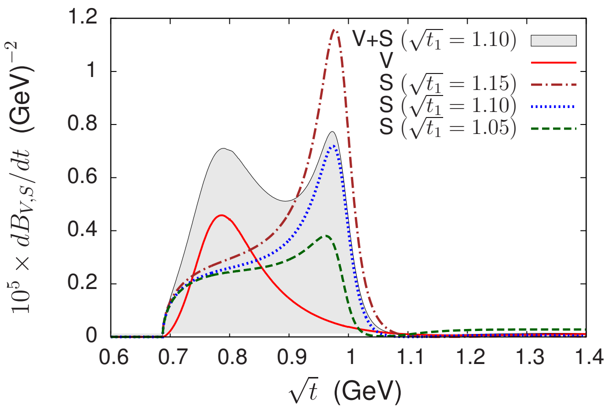

Fig. 4. Contribution to the energy distribution of the $\tau \to \eta\pi\nu$ branching fraction from the scalar form factor for several values of the dip position.

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REFERENCES

[1] W. Ochs, *J. Phys. G* **40**, 043001 (2013).
[2] I. Caprini, G. Colangelo, H. Leutwyler, *Phys. Rev. Lett.* **96**, 132001 (2006).
[3] R. Garcia-Martin, R. Kaminski, J. Pelaez, J. Ruiz de Elvira, *Phys. Rev. Lett.* **107**, 072001 (2011).
[4] J. Batley *et al.* [NA48/2 Coll.], *Eur. Phys. J.* **C70**, 635 (2010).
[5] B. Adeva *et al.*, *Phys. Lett.* **B704**, 24 (2011).
[6] P. Minkowski, W. Ochs, *Eur. Phys. J.* **C9**, 283 (1999).
[7] P. del Amo Sanchez *et al.* [BaBar Coll.], *Phys. Rev.* **D83**, 072001 (2011).
[8] B. Adeva *et al.* [DIRAC Coll.], *Phys. Lett.* **B735**, 288 (2014).
[9] K. Maltman, *Phys. Lett.* **B462**, 14 (1999).
[10] J.F. Donoghue, J. Gasser, H. Leutwyler, *Nucl. Phys.* **B343**, 341 (1990).
[11] D. Aston *et al.* [LASS Coll.], *Nucl. Phys.* **B296**, 493 (1988).

[12] M. Jamin, J.A. Oller, A. Pich, *Nucl. Phys.* **B622**, 279 (2002).
[13] B. Moussallam, *Eur. Phys. J.* **C53**, 401 (2008).
[14] D. Epifanov *et al.* [Belle Coll.], *Phys. Lett.* **B654**, 65 (2007).
[15] V. Bernard, *J. High Energy Phys.* **1406**, 082 (2014).
[16] S. Weinberg, *Phys. Rev.* **112**, 1375 (1958).
[17] S. Descotes-Genon, B. Moussallam, *Eur. Phys. J.* **C74**, 2946 (2014).
[18] H. Neufeld, H. Rupertsberger, *Z. Phys.* **C68**, 91 (1995).
[19] D. Scora, K. Maltman, *Phys. Rev.* **D51**, 132 (1995).
[20] M. Antonelli *et al.*, *Eur. Phys. J.* **C69**, 399 (2010).
[21] D. Black, A.H. Fariborz, J. Schechter, *Phys. Rev.* **D61**, 074030 (2000).