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THE ROLE OF THE KINEMATICAL CONSTRAINT AND NON-LINEAR EFFECTS IN THE CCFM EQUATION*

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We report on recent study [M. Deak, K. Kutak, *J. High Energy Phys.* **1505**, 068 (2015)] of the role of the kinematical constraint in the CCFM equation and its non-linear extension. We compare numerical results obtained by solving the CCFM equation and argue that kinematical constraint represents an important correction.

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1. Introduction

In the regime of high energy scattering, where the center-of-mass energy is larger than any other available scale, perturbative approach to processes with high momentum transfer allows factorization of the cross section into a hard matrix element with initial off-shell gluons and an unintegrated gluon density [1,2]. The unintegrated gluon density is a function of the longitudinal momentum fraction x and transverse momentum k_{T} of a gluon. After taking into account formally subleading corrections coming from coherence of gluon emissions, one is lead to the CCFM equation which introduces gluon density dependent on hard scale related to probe. In principle, it is an equation that should be the ideal framework for application to final states at high energies and covering DGLAP and BFKL domain in gluon channel. It has been implemented in the Monte Carlo event generator [3]. However, so far, good agreement with high precision data has been successfully achieved only in rather inclusive processes like F_2 and Drell–Yan [4]. It is known that on the theory side, the CCFM physics is still to be completed. For instance: the impact of the kinematical effects introducing energy conservation in the CCFM evolution has been not investigated in all detail. As it turns

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out from our study, it is necessary to revisit the inclusion of the so-called kinematical constraint [5] into CCFM equation. Only recently, the CCFM has been promoted to non-linear equation [6–8] allowing, therefore, for the possibility to investigate interplay of coherence effects and saturation [1]. In particular, the important question is what is the role of the angular ordering and kinematical effect in the evolution at the non-linear level. The optimal form of initial conditions is also not known.

We present here a continuation of the work [9] done in [10, 11].

1.1. The CCFM equation and the kinematical constraint

The CCFM equation for gluon density reads

$$\begin{aligned}\mathcal{A}\left(x, k_{\mathrm{T}}, p\right) \;=\; & \mathcal{A}_0(x, k_{\mathrm{T}}, p) + \bar{\alpha}_{\mathrm{S}} \int \frac{d^2\bar{\boldsymbol{q}}}{\pi \bar{\boldsymbol{q}}^2} \int\limits_x^{1-\frac{Q_0}{\bar{q}}} dz\, \theta(p - z\bar{q})\, \mathcal{P}(z, k_{\mathrm{T}}, \bar{q}) \\ & \times \varDelta_{\mathrm{S}}(p, z\bar{q})\, \mathcal{A}\left(x/z, {k_{\mathrm{T}}}', \bar{q}\right) , \qquad (1)\end{aligned}$$

where $k' = |\boldsymbol{k} + (1-z)\bar{\boldsymbol{q}}|$ and the modulus of two dimensional vectors transversal to the collision plane are denoted $|\boldsymbol{k}| \equiv k_{\mathrm{T}}$, $|\boldsymbol{q}| \equiv q_{\mathrm{T}}$ and x is gluon's longitudinal momentum fraction and $\bar{\alpha}_{\mathrm{S}} = N_c \alpha_{\mathrm{S}}/\pi$. Also the rescaled momentum is introduced as $\bar{q} = |\bar{\boldsymbol{q}}| = q_{\mathrm{T}}/(1-z)$ where $\mathcal{P}\left(z, k_{\mathrm{T}}, \bar{q}\right)$ is the gluon splitting function. The function $\varDelta_{\mathrm{S}}(p, z\bar{q})$ is the Sudakov form-factor. The non-Sudakov form-factor $\varDelta_{\mathrm{NS}}(z, k_{\mathrm{T}}, \bar{q})$ regularizes $1/z$ singularity.

1.2. The kinematical constraint

The integration over $\bar{\boldsymbol{q}}$ in equation (1), although being constrained from below by the soft cut-off Q_0, is not constrained by an upper limit thus violating the energy-momentum conservation. Moreover, in the low-x formalism, one requires that in the denominator of the off-shell gluon propagator one keeps terms that obey $|k^2| = k_{\mathrm{T}}^2$. In order to be consistent, the non-Sudakov form-factor should be accompanied by a kinematical constraint limiting the above integration over $\bar{q}$. In an approximated form, it reads $k_{\mathrm{T}}^2 > z\,\bar{q}^2$ and at $z \ll 1$ guaranties that $|k^2| \simeq k_{\mathrm{T}}^2$. In [5], it has been extended to a region including also the case when $z \sim 1$. Careful derivation leads to the following form of the kinematical constraint $k_{\mathrm{T}}^2 > \frac{z\, {q_{\mathrm{T}}}^2}{1-z} = z\,(1-z)\,\bar{q}^2$. The lower bound on $z > x$ results in the upper bound on $q_{\mathrm{T}}^2 < k_{\mathrm{T}}^2/x \simeq \hat{s}$ providing local condition for energy-momentum conservation. We include the kinematical constraint in the CCFM equation. The non-Sudakov form-factor after inclusion of the full form of the kinematical constraint assumes the form

$$\begin{aligned}\Delta_{\mathrm{NS}}(z,k_{\mathrm{T}},\bar{q}) &= \exp\left\{-\bar{\alpha}_{\mathrm{S}}\int\limits_{z}^{1}\frac{dz'}{z'}\,\Theta\left(\frac{(1-z')k_{\mathrm{T}}^2}{(1-z)^2\bar{q}^2}-z'\right)\right.\\ &\qquad \left.\times\int\frac{dq'^2}{q'^2}\,\Theta\left(k_{\mathrm{T}}^2-q'^2\right)\,\Theta\left(q'-z'\bar{q}\right)\right\}. \qquad (2)\end{aligned}$$

Please note the presence of the function $\theta(\frac{k_{\mathrm{T}}^2}{(1-z)\bar{q}^2}-z)$. The authors of [5] solve the CCFM at small z limit, therefore, the function $\theta(\frac{k_{\mathrm{T}}^2}{(1-z)\bar{q}^2}-z)$ is neglected and in most of the phenomenological and theoretical applications of the CCFM this term is neglected [4, 12–18]. The following form of non-Sudakov form-factor is usually being used:

$$\begin{aligned}\Delta_{\mathrm{NS}}\left(z,k_{\mathrm{T}},\bar{q}\right) &= \exp\left(-\bar{\alpha}_{\mathrm{S}}\int\limits_{z}^{z_0}\frac{dz'}{z'}\int\frac{dq'^2}{q'^2}\Theta\left(k^2-q'^2\right)\Theta\left(q'-z'\bar{q}\right)\right)\\ &= \exp\left(-\bar{\alpha}_{\mathrm{S}}\log\left(\frac{z_0}{z}\right)\log\left(\frac{k^2}{z_0 z\bar{q}^2}\right)\right), \qquad (3)\end{aligned}$$

where $z_0=1$, if $(k_{\mathrm{T}}/\bar{q})\geq 1$; $z_0=k_{\mathrm{T}}/\bar{q}$, if $z<(k_{\mathrm{T}}/\bar{q})<1$; $z_0=z$, if $(k_{\mathrm{T}}/\bar{q})=z$. The discussed above θ-function is not taken into account.

1.3. Saturation effects and kinematical constraint combined

To account for gluon recombination at large gluon densities, the CCFM equation has been promoted to non-linear equation by including a quadratic term [6–8] and it reads:

$$\begin{aligned}&\mathcal{A}(x,k_{\mathrm{T}},p)=\mathcal{A}_0(x,k_{\mathrm{T}},p)+\bar{\alpha}_{\mathrm{S}}\int\frac{d^2\bar{\boldsymbol{q}}}{\pi\bar{\boldsymbol{q}}^2}\int\limits_{x}^{1-\frac{Q_0}{\bar{q}}}dz\,\theta\left(\frac{k_{\mathrm{T}}^2}{(1-z)\bar{q}^2}-z\right)\theta(p-z\bar{q})\\ &\times\mathcal{P}(z,k_{\mathrm{T}},\bar{q})\,\Delta_{\mathrm{S}}(p,z\bar{q})\left(\mathcal{A}\left(\frac{x}{z},k_{\mathrm{T}}',\bar{q}\right)-\delta\left(\bar{q}^2-\bar{k}_{\mathrm{T}}^2\right)\bar{q}^2\,\mathcal{A}^2\left(\frac{x}{z},\bar{q},\bar{q}\right)\right), \qquad (4)\end{aligned}$$

where $\bar{k}_{\mathrm{T}}=k_{\mathrm{T}}/(1-z)$ and we included the kinematical constraint of the form (2) in the kernel. Simpler versions of the equation above have been already analyzed in [11], and it has been observed that the equation leads to phenomenon called saturation at the saturation scale [11, 16] and the saturation strongly suppresses the gluon density at low x and low k_{T}. The natural question arises how these results are modified when some of the approximations are not taken and how they are modified if the kinematical effect is imposed in the full form.

2. Numerical results

2.1. Linear equations

We use an initial condition which includes resummed virtual and unresolved contributions, according to [11] and [17], in the form $\mathcal{A}_0\,(x, k_{\mathrm{T}}, p) = A\,\Delta_{\mathrm{R}}(x, k_{\mathrm{T}})\,\Delta_{\mathrm{S}}(p, Q_0)/k_{\mathrm{T}}$, with $A = 1/2$ and $\Delta_{\mathrm{R}}(z, k_{\mathrm{T}})$ is the Regge form-factor.

The observation we make from the plots (like plots in Fig. 1) is that the solutions of equations we study differ significantly. The solutions exhibit also similar features. Solutions of both versions of the kernel with kinematical constraint exhibit a local maximum as functions of k_{T} and p. The positions of local maxima in the plots of p dependence are correlated with the value of k_{T}, with a shift to higher k_{T} for the solution of the equation with kinematical constrain θ-function included. The peak can be explained by the fact that the contribution of the integral on the right-hand side of the equation peaks at around $k_{\mathrm{T}} \sim p$. The peak, therefore, is a result of presence of $\theta\,(p - z\bar{q})$ — angular ordering condition. Similar peaks are present also in the plots of k_{T} dependence and resemble Sudakov suppression of k_{T} scales of the order of p [19]. However, in the case without the θ-function, it seems that the position of the peak does not depend on the value of p. The peak observed in solution of the equation without kinematical constraint θ-function is 'hidden' under the result of the evolution. We can conclude that the peak in the k_{T} dependence is an interplay result of inclusion of the explicit kinematical contraint via θ-function factor and the Sudakov effect.

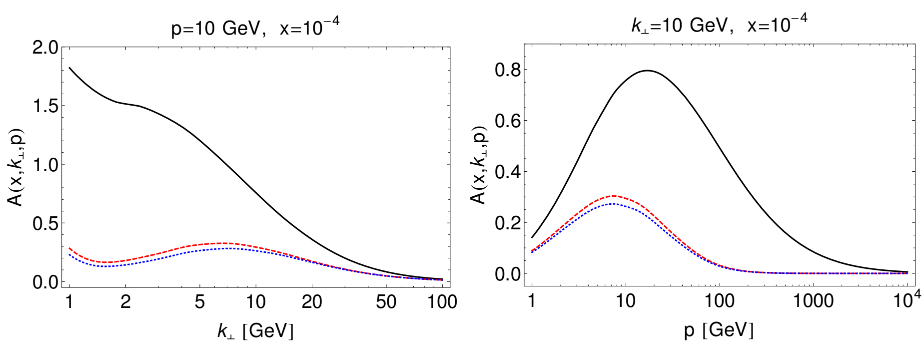

Fig. 1. Relative ratio of CCFM and KGBJS solutions. Distributions with definite p for varying value of k_{T}.

2.2. Non-linear equation

We set the parameter characterizing the strength of the non-linear term R the value $R = \sqrt{1/\pi}$ GeV in equation (4). By comparing the CCFM and KGBJS equations, we see that the kinematical constraint suppresses the growth of the gluon so much that the non-linear effects enter only at very low x. Observations made in previous paragraphs are confirmed in 2-dimensional plots (Fig. 2), where we plot absolute relative difference of two amplitudes, solutions of the CCFM and KGBJS equations, defined by the quantity

$$\beta\left(x, k_{\mathrm{T}}, p\right)=\frac{\left|\mathcal{A}_{\mathrm{CCFM}}\left(x, k_{\mathrm{T}}, p\right)-\mathcal{A}_{\mathrm{KGBJS}}\left(x, k_{\mathrm{T}}, p\right)\right|}{\mathcal{A}_{\mathrm{CCFM}}\left(x, k_{\mathrm{T}}, p\right)}. \tag{5}$$

The function $\beta\left(x, k_{\mathrm{T}}, p\right)$, introduced before in [11], can be used to measure the strength of the non-linear effects and to define a saturation scale using the conditions:

$$\beta\left(x, Q_{\mathrm{s}}\left(x, p\right), p\right)=\mathrm{const.}, \qquad \beta\left(x, k_{\mathrm{T}}, P_{\mathrm{s}}\right)=\mathrm{const.} \tag{6}$$

The second condition in (6) defines p-related saturation scale.

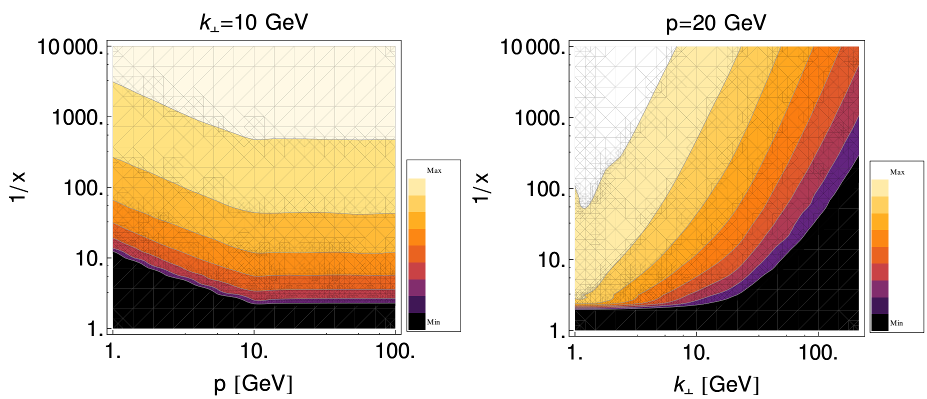

Fig. 2. Relative ratio of CCFM and KGBJS solutions. Distributions with definite p for varying value of k_{T}.

The conditions above, can be seen as equipotential lines in the plots in Fig. 2, where different equipotential lines correspond to different constants on the right-hand side of the equation above. The change in the slope of the $\beta(x, Q_{\mathrm{s}}(x,p), p)$ at around $k_{\mathrm{T}}=p$ reported in [11], can be seen clearly in Fig. 2, and can be understood in the context of the peak at $p \sim k_{\mathrm{T}}$ (Fig. 1).

By comparing the plots in Fig. 2 to analogous plots in [11], we see that the main features are very similar. We, therefore, conclude that the *low-x* approximation of the KGBJS and CCFM equations taken in [11] does not, at least, modify the relative difference between linear and non-linear equation.

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