HYDRODYNAMICS OF QCD*

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I briefly review the use of hydrodynamics to model heavy-ion collisions at ultrarelativistic energies, and what such modelling has taught us about the properties of QCD matter.

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1. Fluid dynamics

The goal of ultrarelativistic heavy-ion collisions is to form strongly interacting matter — matter in a sense that the thermodynamical concepts like temperature and pressure apply. Thus, it is natural to try to use fluid dynamics to describe the expansion stage of the collision. In a case of no conserved charges, the equations of motion are the conservation laws for energy and momentum

$$\partial_\mu T^{\mu\nu} = 0\,, \qquad \text{where} \qquad T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}\,,$$

and ϵ is energy density in the rest frame of the fluid, P equilibrium pressure, Π bulk pressure, u^μ is the fluid 4-velocity, $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ the metric tensor, and $\pi^{\mu\nu}$ the shear-stress tensor. These four equations contain eleven unknowns. To close the set of equations, we need an equation of state (EoS) connecting equilibrium pressure to energy density, $P = P(\epsilon)$, and constitutive equations for bulk pressure and shear stress. A relativistic generalisation of the Navier–Stokes equations, where the dissipative quantities are directly proportional to the gradients of flow velocity, leads to non-causal behaviour. Therefore, heavy-ion collisions are modelled using

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the so-called Israel–Stewart type, a.k.a. transient, fluid dynamics where $\pi^{\mu\nu}$ and Π are dynamical variables relaxing to their Navier–Stokes values on characteristic relaxation times τ_π and τ_Π.

Once the equation of state and constitutive equations are chosen, the expansion dynamics is uniquely defined, but the actual solution depends on the boundary conditions: The initial distribution of matter and the criterion for the end of evolution. Hydrodynamics does not provide either of these, but they have to be supplied by other models. The end of evolution is usually taken to be a hypersurface of constant temperature or energy density, where the fluid is converted to particles (particlization). In pure hydrodynamical models, all interactions are assumed to cease at this point and particle distributions freeze out. In so-called hybrid models, particles formed at the end of fluid dynamical evolution are fed into a hadron cascade describing the late dilute hadronic stage.

2. Azimuthal anisotropies of final particle distribution

The particle production in the primary collisions is azimuthally isotropic, but the distribution of observed particles in $A+A$ collisions is not. The anisotropy can be easily explained in terms of rescatterings of the produced particles: In a non-central collision, the collision zone has an elongated shape. If a particle is heading to a direction where the collision zone is long, it has a larger probability to scatter and change its direction than a particle heading to a direction where the collision zone is short. Thus more particles end up in direction where the edge of the collision zone is close. Or, in a hydrodynamical language, the pressure gradient between the center of the system and the vacuum is larger in the "short" direction, the flow velocity is thus larger in that direction, and more particles are emitted in that direction.

This anisotropy is quantified in terms of Fourier expansion of the azimuthal distribution. The coefficients of this expansion v_n, and the associated event angles ψ_n, are defined as

$$v_n = \langle \cos[n(\phi - \psi_n)] \rangle \,, \qquad \text{and} \qquad \psi_n = \frac{1}{n} \arctan \frac{\langle p_\mathrm{T} \sin(n\phi) \rangle}{\langle p_\mathrm{T} \cos(n\phi) \rangle} \,.$$

Of these coefficients v_1 is called directed, v_2 elliptic, and v_3 triangular flow. Elliptic flow of charged hadrons as a function of centrality was one of the first measurements at the RHIC [1].

The measured elliptic flow was seen to be quite large and to increase with decreasing centrality, as expected if it has the described geometric origin. Thus there must be rescatterings among the particles formed in the collision, and an $A+A$ collision is not just a sum of independent pp collisions. The elliptic flow is also very close to the hydrodynamically calculated one [2], which is a very strong indication of hydrodynamical behaviour of the matter.

3. Equation of State has many degrees of freedom

The Equation of State (EoS) of strongly interacting matter is an explicit input to hydrodynamical models. Thus one might expect hydrodynamical modelling of heavy-ion collisions to tell us a lot about the Equation of State, but unfortunately that is not the case. The collective motion of the system is directly affected by the pressure gradients in the system, and thus by the EoS, but the effects of the EoS on the final particle $p_{\rm T}$ distributions can, to very large extent, be compensated by changes in the initial state of the evolution and the final decoupling temperature. This makes constraining the properties of the EoS very difficult. However, what we do know is that the number of degrees of freedom has to be large.

It was already seen when modelling S + Au collisions at the CERN SPS at $E_{\rm lab} = 200$ AGeV energy, that if we use ideal pion gas EoS, we cannot simultaneously reproduce the pion rapidity and transverse momentum distributions. If the rapidity distribution is reproduced, the $p_{\rm T}$ distribution is too flat, and if the freeze-out temperature is chosen to reproduce the $p_{\rm T}$ distribution, rapidity distribution is too narrow [3]. On the other hand, if we use an EoS containing several hadrons and resonances and/or transition to a partonic phase, the distributions can be fitted.

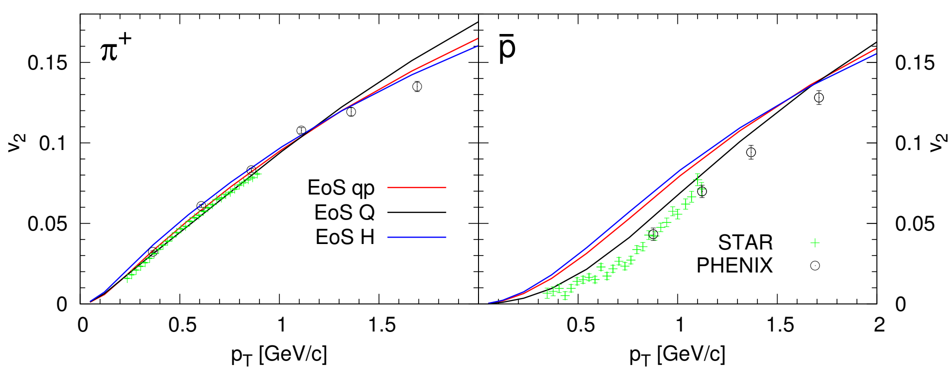

Fig. 1. $p_{\rm T}$ differential elliptic flow $v_2(p_{\rm T})$ of pions and antiprotons in minimum bias Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV calculated using three different EoSs [4] and compared with the data by the STAR and PHENIX collaborations [5]. The labels stand for a lattice QCD inspired quasiparticle model (qp), EoS with a first order phase transition (Q), and pure hadron resonance gas with no phase transition (H). Figure taken from Ref. [6].

One might want to use the elliptic flow to constrain the EoS after the initial state and freeze-out temperature are fixed to reproduce the p_T distributions. Unfortunately, the elliptic flow is only very weakly sensitive to the details of the EoS [4]: The only flow observable affected by the EoS seems to be the p_T-differential anisotropy of heavy particles, *e.g.*, protons. As shown in Fig. 1, the $v_2(p_T)$ of pions is unchanged within the experimental errors no matter whether one uses an EoS with (EoS A) or without phase transition (EoS H), or an EoS with a first order phase transition (EoS A) or a smooth crossover (EoS qp). On the other hand, the proton $v_2(p_T)$ is sensitive to the EoS, but surprisingly the EoS with the first order phase transition is closest to the data. Consequently, distinguishing between different parametrizations of the lattice QCD EoSs is very difficult, see Ref. [7].

4. η/s has very low minimum

Once it became clear that the ideal fluid dynamics can describe the particle spectra and their anisotropies fairly well, it was reasonable to assume that the matter formed in the collision has very low shear viscosity coefficient to entropy density ratio η/s. But how low in particular? It has been shown that the shear viscosity strongly reduces v_2 [8]. Thus, in principle, extracting the η/s ratio from the data is easy: One needs to calculate the p_T-averaged v_2 of charged hadrons using various values of η/s and choose the value of η/s which reproduces the data. Unfortunately, this approach is hampered by our ignorance of the initial state of the evolution. The values of v_2 calculated using non-zero value of η/s fit the data best [9], but the preferred value depends on how the initial state of hydrodynamic evolution is chosen: Whether one uses the so-called MC-Glauber [10] or MC-KLN [11] model causes a factor two difference in the preferred value ($\eta/s = 0.08$–0.16).

The calculations have been improved since Ref. [9] by a better treatment of the hadronic phase (see, *e.g.*, Ref. [12]), but the same uncertainty remains. This uncertainty can be reduced by studying the higher flow coefficients ($v_n, n > 2$). Because of the fluctuations of the positions of nucleons in the nuclei, the initial collision region has an irregular shape which fluctuates event-by-event, see Fig. 2, and thus all the coefficients v_n are finite [13]. As illustrated in Fig. 3, the larger the n, the more sensitive the coefficient v_n is to viscosity [14]. This provides a possibility to distinguish between different initialisations, and preliminary results for the p_T-dependence of v_2 and v_3 seem to favour the MC-Glauber initialisation [15].

On the other hand, in event-by-event studies, it is not sufficient to reproduce only the average values of v_n, but the fluctuations of the flow coefficients should be reproduced as well. Neither MC-Glauber nor MC-KLN model seems to be able to reproduce the measured fluctuations [16], whereas the recent calculations using the so-called IP-Glasma [17] and EKRT [18] initialisations reproduce both the fluctuations and the average values of v_2, v_3 and v_4 [18, 19], making these approaches very promising.

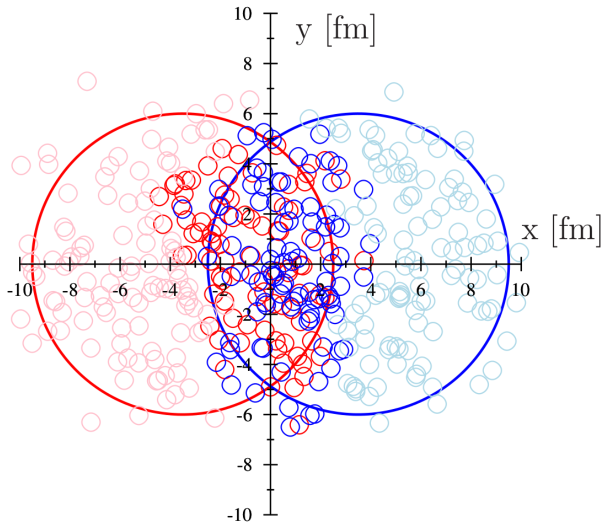

Fig. 2. An example of the positions of interacting nuclei in MC-Glauber model. Figure taken from Ref. [20] and reprinted with permission.

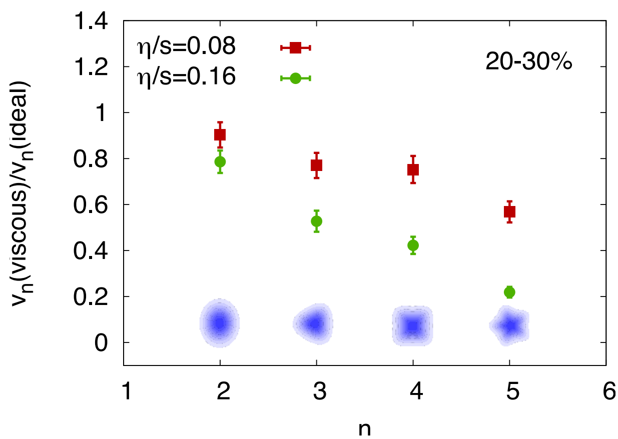

Fig. 3. Ratio of the anisotropy coefficients of charged hadrons in viscous calculation to the coefficients in ideal fluid calculation [14]. Figure taken from Ref. [6], courtesy to Bjoern Schenke.

However, in the calculations discussed above, the η/s-ratio is assumed to be constant. We know no fluid where the η/s-ratio would be temperature independent, and there are theoretical reasons to expect it to depend on temperature with a minimum around $T_{\rm c}$ [21]. Thus the temperature-independent η/s is only an effective viscosity, and its connection to the physical, temperature-dependent shear-viscosity coefficient is unclear. What complicates the determination of the physical shear-viscosity coefficient, is that the sensitivity of the anisotropies to dissipation varies during the evolution of the system. As studied in Ref. [22] at RHIC ($\sqrt{s_{NN}} = 200$ GeV), v_2 is insensitive to the value of η/s above $T_{\rm c}$, but very sensitive to its minimum value around $T_{\rm c}$, and to its value in the hadronic phase below $T_{\rm c}$. At the present LHC energy ($\sqrt{s_{NN}} = 2.76$ TeV), the shear viscosity in the plasma phase does affect the final v_2, but not more than the shear viscosity in the hadronic phase. Thus disentangling the effect of viscosity during different stages of the evolution is challenging. Because of all these complications, we can only say that the minimum value of the η/s ratio of strongly interacting matter is small, and in the vicinity of the postulated minimum of $\eta/s = 1/4\pi$, but how small, is too early to say.

5. Further reading

My talk was based on my recent review [23] and H. Niemi's Quark Matter proceeding [24]. A short write-up of my talk can also be found in Ref. [6].

A reader interested in the theory of hydrodynamics in ultrarelativistic heavy-ion collisions can find a good introduction in Ref. [25]. More general reviews about hydrodynamics and flow can be found in Refs. [26] and [27].

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