

SOME CONSIDERATIONS ABOUT PHOTONS*

LARRY McLERRAN

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA
and

Physics Department, Central China Normal University, Wuhan, China

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I discuss photon production from the Glasma. In particular, I outline the consequences of a power law tail in the distributions of quarks and gluons for the photon production rates and for the times scales of evolution within the Glasma.

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1. A simple model for the Glasma

In what follows, I will construct a simplified model of the Glasma that illustrates some simple features of the Glasma, and may be useful for phenomenological applications [1]. I will assume that distributions are approximately isotropic and, again, the considerations presented here might be generalized to the anisotropic case.

Let us begin with the definition of the gluon distribution function

$$\frac{1}{\tau\pi R^2} \frac{dN}{d^3p} = f(p), \quad (1)$$

where R is the transverse size of the system, and τ is the proper time. For a non-expanding system, the proper time is just the time, but for a longitudinally expanding system, $\tau = \sqrt{t^2 - z^2}$. We take as initial conditions

$$f(p) \sim \frac{1}{\alpha_S}, \quad p \leq Q_{\text{sat}} \quad (2)$$

and

$$f(p) \rightarrow 0, \quad p \geq Q_{\text{sat}}. \quad (3)$$

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At some point, the distribution function must go to zero and will have a value of the order of 1, so we see that the UV scale is defined from

$$f(\Lambda_{\text{UV}}) \sim 1. \quad (4)$$

Generically, the transport equation for a highly occupied Bose gas, with $f \gg 1$ is of the form

$$\frac{df}{dt} \sim \alpha_S^2 f^3. \quad (5)$$

Implicit in this relationship are integrations on the right-hand side of the equation with weight associated with the scattering kernel. The factor of α_S^2 is the coupling strength. In scattering, there are two particles in the initial and two particles in the final state, so we would naively expect that the scattering term in the transport equations to be of the order of f^4 , but this leading term cancels in the forward and backward going processes leaving a term of the order of f^3 .

Let us assume that the distribution function is classical for $E \ll \Lambda_{\text{UV}}$, then

$$f \sim \frac{1}{\alpha_S} \frac{\Lambda_{\text{IR}}}{E}. \quad (6)$$

More generally, we can write

$$f \sim \frac{1}{\alpha_S} \frac{\Lambda_{\text{IR}}}{\Lambda_{\text{UV}}} f(E/\Lambda_{\text{UV}}). \quad (7)$$

Now, plugging this into the transport equation and integrating over momentum gives an equation

$$\frac{d}{dt} \Lambda_{\text{IR}} \Lambda_{\text{UV}}^2 \sim \Lambda_{\text{IR}}^3 \Lambda_{\text{UV}}. \quad (8)$$

Taking

$$1/t \sim \frac{1}{\Lambda_{\text{IR}} \Lambda_{\text{UV}}^2} \frac{d}{dt} \Lambda_{\text{IR}} \Lambda_{\text{UV}}^2, \quad (9)$$

we can identify the scattering time as

$$t_{\text{scat}} \sim \frac{\Lambda_{\text{UV}}}{\Lambda_{\text{IR}}^2}. \quad (10)$$

Note that the coupling constant has entirely disappeared from this equation. One can show that this form of the time dependence persists when one includes higher order corrections associated with the inelastic particle production.

If there is a Bose condensate present then, there is a term in the transport equation associated with scattering from a condensate. In this case, the dependence upon the infrared and ultraviolet scales for the scattering time is different, but can also be explicitly obtained.

The relationship between the dynamical scale and the scattering time, $t \sim t_{\text{scat}}$ gives one equation determining the evolution of the scales. The other equation is energy conservation. The energy density is

$$\epsilon \sim \frac{1}{\alpha_S} \Lambda_{\text{IR}} \Lambda_{\text{UV}}^3. \quad (11)$$

The solution to these equations in a fixed box or an expanding box gives power law dependences in time for the infrared and ultraviolet scale.

It is useful to consider a simple model for the Glasma that is explicit and has the properties described above. Let us take the gluon distribution function to be an over-occupied Bose–Einstein distribution [1]

$$f(p) = \frac{\gamma(t)}{e^{E/\Lambda(t)} - 1}. \quad (12)$$

In this form, we see that Λ is an effective temperature, and that $\Lambda = \Lambda_{\text{UV}}$. The factor γ is the over-occupation factor for the Bose–Einstein distribution. For a thermally equilibrated distribution, $\gamma = 1$. For the Glasma, we take

$$\gamma = \frac{1}{\alpha_S} \frac{\Lambda_{\text{IR}}}{\Lambda_{\text{UV}}}. \quad (13)$$

At some time in the evolution,

$$\gamma(t) = 1. \quad (14)$$

At this time, the system is thermal, and t_{th} is determined from

$$T = \Lambda_{\text{UV}}(t_{\text{th}}). \quad (15)$$

Beyond this time, $\gamma(t) = 1$, but the temperature may evolve.

The entropy density of these over-occupied distributions is

$$s = \int d^3p \{ (1+f) \ln(1+f) - f \ln(f) \} \sim \Lambda_{\text{UV}}^3 \ln \left\{ \frac{\Lambda_{\text{IR}}}{\alpha_S \Lambda_{\text{UV}}} \right\}. \quad (16)$$

On the other hand, the number density of gluons is

$$\rho \sim \frac{1}{\alpha_S} \Lambda_{\text{IR}} \Lambda_{\text{UV}}^2. \quad (17)$$

The entropy per particle becomes

$$s/n \sim \alpha_S \Lambda_{UV}/\Lambda_{IR}. \quad (18)$$

This means that early on when the system is highly coherent, the entropy per particle is small. By the time of thermalization, the entropy per particle has become of the order of 1.

We can also estimate the quark to gluon number density. We take for the quark distribution function

$$f_{\text{quark}} = \frac{1}{e^{E/\Lambda(t)} + 1}. \quad (19)$$

The quarks cannot be over-occupied because they are fermions. We assume the UV scale is the same for quarks and gluons. The total number of quarks is of the order of

$$q \sim \Lambda_{UV}^3. \quad (20)$$

This means that the ratio of quarks to gluons is

$$q/g \sim \alpha_S \Lambda_{UV}/\Lambda_{IR} \quad (21)$$

and like the entropy to gluon ratio, it begins small but at thermalization has achieved a ratio of the order of one. This underabundance of quarks at early times has no relationship to the rate of quark production. It simply reflects the overabundance of gluons, and that Fermi statistics forbid the over-occupation of fermions.

2. Saturation, the Glasma, and photons

If both the Glasma and the Thermalized Quark–Gluon Plasma obey approximate hydrodynamic behaviour, it will be difficult to disentangle which is the source of bulk properties of matter produced in heavy ion collisions. As suggested by Shuryak many years ago, the internal dynamics of an evolving QGP might be best addressed by looking at penetrating probes such as photons and dileptons. These particles can probe the internal dynamics of the QGP and, in principle, resolve the difference between a Glasma and a Thermalized QGP. It is not easy however, as most experimental observables have significant contributions from other sources, such as the matter produced at late times as a hadron gas, and from the fragmentation of produced jets into photons.

Nevertheless, we can first try to see if saturation dynamics has anything to do with photon production. We can first see whether or not the available photon data has geometric scaling. This should be a generic feature of emission from the Color Glass Condensate and early time emission from

the Glasma. In these cases, the only scale in the problem is the saturation momentum. We, therefore, expect that the distribution of photons will be of the form

$$\frac{1}{\pi R^2} \frac{d^2 N}{dy d^2 p_T} = F \left(\frac{Q_{\text{sat}}}{p_T} \right). \quad (22)$$

The saturation momentum for nucleus–nucleus collisions is determined by

$$Q_{\text{sat}}^2 = N_{\text{part}}^{1/3} \left(\frac{E}{p_T} \right)^\delta. \quad (23)$$

Here, N_{part} is the number of nucleon participants and $\delta \sim .22\text{--}.28$ is determined by both fits to deep inelastic scattering data and high energy pp interactions.

When these ideas are applied to the RHIC and ALICE data on photon production, they convert results from different beams, energies and centralities, which vary over 4 orders of magnitude, into a single scaling curve. The agreement with geometric scaling is very remarkable.

The underlying mechanism behind this remarkable scaling behaviour might be jet production and fragmentation into photons. Such a fragmentation process should be approximately scale invariant, and would preserve the geometric scaling of the initial conditions in the Color Glass Condensate.

We can also try to describe photon production using the Glasma. Schenke and I used the known lowest order formula for photon production, with the distribution functions replaced by the over-occupied distribution functions above. The result is that one can obtain a good description of the spectrum of produced photons in the 1–4 GeV transverse momentum range. To do this requires a factor of 5–10 increase in the rates relative to the computed rates. Similar results with related mechanisms are found in the semi-QGP analysis. The Thermalized QGP computations with realistic hydrodynamic simulation are off by a factor of 2–5, so this is a common problem for both computations.

The remarkable result of the photon measurements at the RHIC and LHC is the observation that photons flow almost like hadrons. This is difficult to achieve in Thermalized QGP computations of photon production. This is because the photons are produced early before much flow develops. It might be that such photons are produced late in the collision, but then it would be difficult to explain the geometric scaling seen in the data. At very late times, there are scales of the order of Λ_{QCD} which become important. The Glasma is producing significant entropy per gluon during its expansion, and therefore cools more slowly than does a Thermalized QGP. This allows more time for flow to develop. It is possible to get acceptable flow from the Glasma emission, at the expense as mentioned above, of reducing rates of photon emission which are already somewhat low.

3. A power law tail for the quark and gluon distributions

We will study the effect of power law non-thermal tails, implemented by replacing the modified Bose–Einstein and Fermi–Dirac distributions of the Glasma by the corresponding Tsallis distribution [2]. This will have a significant effect on photon yields and typical emission times: As opposed to most other mechanisms that have been suggested to resolve the thermal photon puzzle, photon emission times are delayed and, at the same time, production rates are increased. This could resolve the puzzle by generating both increased photon yields and elliptic flow.

To compute the photon yield from an expanding ideal gas in 1+1 dimensions, we integrate this rate over time using the time dependence of the temperature, which for an ideal gas of relativistic quarks and gluons is given by

$$t/t_0 = T_0^3/T^3. \quad (24)$$

The four volume is

$$\int d^4x = A_T \int t dt, \quad (25)$$

where A_T is the transverse area of the interaction region. Since

$$t dt \propto \frac{dT}{T} \frac{1}{T^6}, \quad (26)$$

upon inserting the rate at a given T into the integral over the expanding system, we get

$$E_\gamma \frac{dN_{\text{th}}}{d^3p} \propto \int \frac{dT}{T} \exp[-E_\gamma/T - 4 \ln(T/T_0)]. \quad (27)$$

By determining the stationary point of the exponent, we obtain the typical emission energy for photons to be

$$E_\gamma \sim 4T \quad (28)$$

up to logarithmic corrections. (The best way to do the stationary phase distribution is in logarithmic coordinates, $\chi = \ln(T/T_0)$, where $dT/T = d\chi$.)

This emission occurs when we are in the tail of the exponential distribution.

Let us now suppose that the quark and gluon distributions were not purely Bose–Einstein or Fermi–Dirac distributions. Since emission occurs at somewhat hard momenta, we might expect the distributions would be well approximated by an exponential with a power law tail. An example which has this property is the Tsallis distribution,

$$f(E) = [1 + E/(aT)]^{-a}, \quad (29)$$

where E is the quark or gluon energy, and a is a free parameter that determines the power law at large E . For $E \ll aT$, this distribution is well approximated by an exponential, $f \sim e^{-E/T}$, but for large energy, it goes as $f \sim (E/aT)^{-a}$. For a thermal distribution undergoing 1+1 dimensional Bjorken expansion, the integral

$$\rho = \int d^3p f \sim 1/t, \quad (30)$$

as it should for a non-interacting gas. For proton+proton collisions, the measured distribution of produced charged particles is approximately a Tsallis distribution with $a \approx 6$.

Note that the distribution (29) tends to 1 as $E \rightarrow 0$. At high p_T on the other hand, this distribution scales as $(T/E)^a$. The dependence of the multiplicity upon the number of participants at any time t is

$$\frac{dN}{d^3p} = A^{2/3} f, \quad (31)$$

where A is the number of participating nucleons. So for low energies, where $f \rightarrow 1$, the multiplicity distribution scales as the number of participants.

However, at high energies, it scales as $A^{2/3}T^a$. In saturation models, the initial temperature scales like $T \sim A^{1/6}$ so that for $a \sim 6$, we get a very rapid $A^{5/3}$ growth in the multiplicity.

Note also that the low momentum part of the distribution does not evolve very rapidly in time, while the high momentum piece falls as $t^{-a/3}$ so that for $a = 6$ it fall as $f \sim 1/t^2$ at fixed E . This is, however, less rapid than $e^{-(t/t_0)^{1/3}}$ for the Boltzman distribution. This means, there can be more radiation at later time.

These considerations suggest that introducing power law tails for quark and gluon distributions can enhance the photon radiation rate and allow the radiation to appear at later times. At later times, more flow will have been built up and the produced photon spectra will reflect that. Hence, this mechanism has the potential to solve the photon flow problem discussed in the introduction.

To understand how the tails might affect the observed distributions of photons, let us consider the radiation from an exponential distribution and compare it to that of a Tsallis distribution. For the following estimate, we simply replace the photon distribution by a Tsallis distribution. We will improve on that by replacing quark and gluon distributions by Tsallis distributions and recomputing the photon rate in the following section.

Let us take the formula for thermal radiation as a pure exponential, ignoring logarithms and constant factors and integrate it over time

$$h = \int \frac{dT}{T} \frac{T_0^4}{T^4} e^{-E_\gamma/T} = \Gamma(4) \frac{T_0^4}{E_\gamma^4}. \quad (32)$$

This assumes that the temperature of emission $T \sim E_\gamma/4$ is within the range of integration over temperatures. Now, take a Tsallis distribution

$$g = \int \frac{dT}{T} \frac{T_0^4}{T^4} (1 + E_\gamma/aT)^{-a}. \quad (33)$$

The stationary phase point of the integral is at

$$E_\gamma/T = \frac{4a}{a-4}. \quad (34)$$

For $a = 6$, this is 12, corresponding to a large change in the temperature of emission, which would make for a huge shift in the emission time, which goes as the cube of the temperature. Clearly, such a big shift would move the emission outside of the range of integration over temperature where the QGP assumption is motivated, which will reduce this effect. Notice also that the value of the integrand at the stationary phase point is $256/9$ which is much greater than the corresponding numerical factor for a Boltzman distribution.

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