THE ANOMALOUS MAGNETIC MOMENT OF THE MUON AND LATTICE QCD*

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(Received June 16, 2015)

The anomalous magnetic moment of the muon, defined as the fractional difference of its gyromagnetic ratio from the naive value of 2, has been measured with an impressive accuracy of 0.54 parts per million in experiment (BNL E821), thus providing one of the most stringent tests of the Standard Model. Intriguingly, the experimentally measured anomaly disagrees by around 3.6 standard deviations with the calculated value from the Standard Model. The current theoretical uncertainty is dominated by that from the calculation of the QCD contribution — lowest order "hadronic vacuum polarization (HVP)" and the "hadronic light-by-light (HLBL)" diagrams. Improvements in the experimental uncertainty by a factor of 4 in the upcoming experiment at Fermilab (E989) are expected and improvements in the theoretical determination would make the discrepancy (if it remains) really compelling in trying to ascertain the possibility of new physics beyond the Standard Model. I will review the current status of the lattice calculation of the HVP and HLBL contributions with a particular emphasis on the recent progress in the HVP using our (HPQCD) new lattice QCD method [B. Chakraborty *et al.*, *Phys. Rev. D* **89**, 114501 (2014)].

DOI:10.5506/APhysPolBSupp.8.349
PACS numbers: 11.15.Ha, 12.38.Gc, 13.40.Em, 14.60.Ef

1. Introduction

The anomalous magnetic moment of the muon, a_μ, defined as the fractional difference of its gyromagnetic ratio from the naive value of 2, ($a_\mu = (g-2)/2$) is a result of the interactions of muon with a cloud of virtual particles. It has been measured with an impressive accuracy of 0.54 parts per million (ppm) [1] in experiment (BNL E821), thus providing the most stringent test of the SM in its flavor singlet sector. Theoretically, a_μ has been calculated with an even better precision of 0.42 ppm, but surprisingly,

* Presented at "Excited QCD 2015", Tatranská Lomnica, Slovakia, March 8–14, 2015.

shows a tantalizing discrepancy of about 3.6σ [2–5] with the experimental result which could be an exciting indication of the existence of new virtual particles. Improvements of a factor of 4 in the experimental uncertainty are expected and improvements in the theoretical determination would make the discrepancy (if it remains) really compelling.

In the SM, a_μ gets contribution from Quantum Electrodynamics (QED) for virtual leptons, from Electroweak (EW) theory for virtual gauge bosons and from Quantum Chromodynamics (QCD) for virtual hadrons. The QED contribution (largest) and the EW contribution (tiny) have been calculated with high precision and have a little impact on the overall theoretical uncertainty in a_μ as listed in Table I. It is dominated by the QCD contributions, mainly from the lowest order hadronic vacuum polarization (HVP) contribution (0.36 ppm), $a_\mu^{\mathrm{HVP,LO}}$ (Fig. 1) and hadronic light-by-light (HLBL) contributions, a_μ^{HLBL} (0.22 ppm) (Fig. 2). $a_\mu^{\mathrm{HVP,LO}}$ has been obtained from

TABLE I

This table shows the separated contributions to the overall theoretical uncertainty in a_μ coming from QED, EW theory and QCD.

Contribution	Result $(\times 10^{-10})$	Error
QED (leptons)	11658471.8	0.00 ppm
HVP(lo) [3, 5]	692.3	0.36 ppm
HVP(ho)	−9.8	0.01 ppm
HLbL [6]	10.5	0.22 ppm
EW	15.4	0.02 ppm
Total SM	11659180.2	0.42 ppm

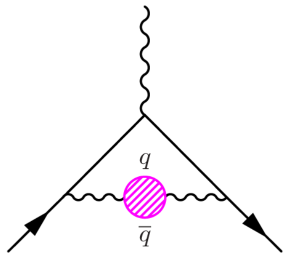

Fig. 1. The hadronic vacuum polarization contribution to the muon anomalous magnetic moment is represented as a shaded blob inserted into the photon propagator (represented by a wavy line) that corrects the point-like photon–muon coupling at the top of the diagram.

dispersion relation with experimental inputs from e^+e^- to hadrons scattering with an uncertainty of 0.7% [3, 5], but needs to be calculated to better than 0.5% uncertainty from first-principle lattice QCD calculations without any experimental inputs to achieve the overall theoretical precision to be comparable to the experiment.

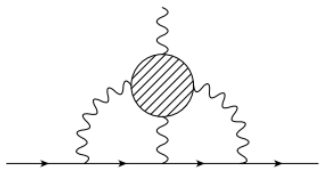

Fig. 2. Representative HLBL contribution to the muon anomalous magnetic moment is denoted by a shaded blob inserted into the photon propagators (represented by a wavy lines) where the blob includes all possible hadronic states.

2. Calculation of HVP from lattice QCD

On lattice, $a_\mu^{\mathrm{HVP,LO}}$ associated with a given quark flavour, f, is obtained by inserting the quark vacuum polarization $\hat{\Pi}$ into the photon propagator [7, 8]

$$a_{\mu,\mathrm{HVP}}^{(f)} = \frac{\alpha}{\pi} \int_0^\infty dq^2 f\left(q^2\right) \left(4\pi\alpha Q_f^2\right) \hat{\Pi}_f\left(q^2\right) , \tag{1}$$

where $\alpha \equiv \alpha_{\mathrm{QED}}$ and Q_f is the electric charge of quark f in units of e. Here, we need the renormalized vacuum polarization function, $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$ and $f(q^2)$ is a known analytic function of four-momentum squared. The integrand in Eq. (1) is strongly peaked around small $q^2 \sim m_\mu^2/4 \sim 0.003$ GeV^2. Extrapolating from higher values of q^2 leads to model uncertainties.

To get around this issue, we (HPQCD) have developed a new method [9] in which the HVP contribution was expressed as a Taylor series of small number of derivatives of the vacuum polarization function $\hat{\Pi}$ evaluated at $q^2 = 0$. The derivatives at $q^2 = 0$ of $\hat{\Pi}$ are readily and accurately given by $t^n\times$ vector meson correlators (time moments) for $n = 4, 6, 8, 10$

$$\begin{aligned} G_{2n} &\equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \left\langle j^i(\vec{x},t) j^i(0) \right\rangle \\ &\equiv (-1)^n \left.\frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}\left(q^2\right)\right|_{q^2=0} . \end{aligned}$$

Using Padé approximants instead of Taylor approximation allows us to deal with high momenta and the q^2 integral has been performed numerically.

2.1. Lattice simulation

Highly Improved Staggered Quark (HISQ) formalism [10] has been used for the valence quarks on MILC HISQ gauge configurations [11, 12] with light (up/down), strange and charm quarks in the sea (2+1+1) using three different lattice spacings a $\approx$ 0.15 fm (very coarse), 0.12 fm (coarse) and 0.09 fm (fine), determined [13] using the Wilson flow parameter w_0 [14]. At each lattice spacing, we have used three different values for u/d quark mass: one fifth the s quark mass, one tenth the s quark mass and the physical value ($m_s/27.5$). At $m_l = m_s/10$ on a particular lattice spacing ~ 0.12 fm, we have three different volumes corresponding to a lattice length in units of the π meson mass of $M_\pi L = 3.2$, 4.3 and 5.4 to test for finite volume effects. We tune the valence s quark mass accurately [15] using the mass of the η_s meson (688.5(2.2) MeV) [13]. However, to test the tuning effects, we de-tuned the valence s quark mass by 5% on a particular set. We have used high statistics, about 1000 configurations on each of the sets and about 16 time sources on each of the configurations. The s quarks are combined into a correlator with a local vector current at either end to form the vector meson ϕ. The local current is not the conserved vector current for the HISQ quark action and has to be renormalized. We have obtained the local vector current renormalization constant ($Z_{V,\bar{s}s}$) completely non-perturbatively with 0.1% uncertainty on the finest $m_l = m_s/5$ lattices [15]. Similarly, combining the light quark propagators using a local vector current, the vector meson ρ has been formed and the correlators have been renormalized using the same method as in [15].

2.2. Our (HPQCD) results for HVP

Using our method, we have achieved [9] a remarkably small uncertainty of 1.1% for the strange quark connected pieces of HVP (Fig. 3): $a^s_{\mu,\mathrm{HVP}} = 53.41(59) \times 10^{-10}$. Finite volume effects seemed to be negligibly small. But the valence HISQ strange quark mass tuning effect was significant. Our result for $a^s_{\mu,\mathrm{HVP}}$ in the continuum limit agrees with the lattice results by the ETM [16] and RBC/UKQCD collaborations [17].

The charm quark contribution (connected) to $a_\mu^{\mathrm{HVP,LO}}$ has also been calculated from the previously obtained moments [18, 19] and found to be $14.42(39) \times 10^{-10}$. Table II gives a comparison of our results for $a^s_{\mu,\mathrm{HVP}}$ and $a^c_{\mu,\mathrm{HVP}}$ with ETMC and RBC/UKQCD lattice calculations and the existing most accurate other calculations. The calculation of the light quark contribution to $a_\mu^{\mathrm{HVP,LO}}$ is particularly challenging because of poor signal-to-noise ratio. However, this issue has been overcome by calculating the time moments from the reconstructed correlators using the best fit parameters

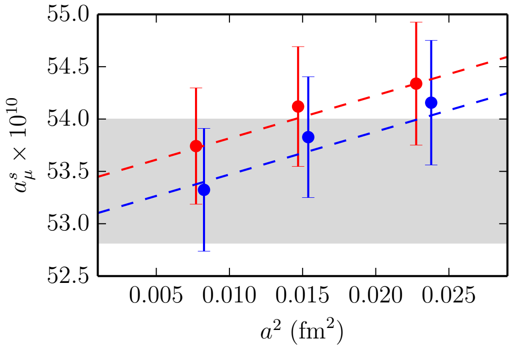

Fig. 3. $a^s_{\mu,\mathrm{HVP}}$ for different lattice spacings and different light quark masses shown; the grey band gives the final results after chiral-continuum fit and finite volume correction.

(instead of using the original correlators). Moreover, Gaussian smearing of the quark fields at both the meson creation and annihilation points on lattice has been used to obtain a better precision from the matrix fit and around 1–2% uncertainty looks achievable for $a^{\mathrm{light}}_{\mu,\mathrm{HVP}}$. This is by far the most precise result from first principle QCD calculation presented on physical point 2+1+1 lattices and considering the finite volume effects.

TABLE II

Comparison of our results for $a^s_{\mu,\mathrm{HVP}}$ and $a^c_{\mu,\mathrm{HVP}}$ [9] with ETMC [16] and RBC/UKQCD [17] lattice calculations and the results using the dispersion relation and the experimental results on $e^+e^- \to$ hadrons or τ decay.

$a^{s/c}_\mu$	Dispersion + experiment	Our results	ETMC (preliminary)	RBC/UKQCD (preliminary)
a^s_μ	$55.3(8) \times 10^{-10}$ [3,9]	$53.41(59) \times 10^{-10}$	$53(3) \times 10^{-10}$	$52.4(21) \times 10^{-10}$
a^c_μ	$14.4(1) \times 10^{-10}$ [20]	$14.42(39) \times 10^{-10}$	$14.1(6) \times 10^{-10}$	—

Nevertheless, the disconnected contribution, though expected to be very small, may be a significant source of systematic uncertainty and has to be included in the calculation of HVP. Getting a signal for the disconnected diagrams in this context is very challenging because of the large noise in the correlators. We (HPQCD) have used one-link spatial current and all-to-all propagator method with random noise vectors (stochastic sources), combined with the methods of time moments to calculate a conservative upper bound (4–5% of the connected piece at present [21]) for the systematic uncertainty from neglecting the disconnected contribution.

3. Status of HLBL calculations on lattice

To reach an overall theoretical goal where the theoretical results of a_μ will supplant the experiment, the HLBL contributions have to be calculated with $\sim$ 15% uncertainty. These contributions are impossible to measure from experiments and have been determined with 25%–40% uncertainty [6] using models leading to different QCD model assumptions. They are difficult enough to extract from lattice QCD because it requires the calculations of the four-point functions. The first lattice QCD calculations introduced by RIKEN BNL group use QCD + QED on lattice, therefore reducing the four-point functions in terms of the differences of the three-point functions. This provides promising preliminary results even if calculated for unphysical quark and muon masses for the connected HLBL contributions [22].

4. Conclusion

To conclude, we (HPQCD) have achieved 1.1% uncertainty for $a^s_{\mu,\mathrm{HVP}}$ [9] along with 1–2% uncertainty for $a^{\mathrm{up/down}}_{\mu,\mathrm{HVP}}$ and an accurate result for $a^c_{\mu,\mathrm{HVP}}$. A conservative upperbound for not including the disconnected contributions to $a_\mu^{\mathrm{HVP,LO}}$ currently exists as 4–5% [21] and we are trying to achieve a more strict limit. In future, HPQCD will collaborate with MILC to use an ensemble size of 10,000 (on 0.15 fm, 0.12 fm lattices) being made by MILC with an aim of 3-fold improvement in the uncertainty.

$a_\mu^{\mathrm{HLBL,conn}}$ calculated by RIKEN BNL group with unphysical quark and muon mass looks promising [22]. This calculation needs to be done with physical quark and muon masses on multiple lattice spacings and multiple volumes. Finite volume correction and other systematic improvements are needed including the calculation of HLbL disconnected contribution.

We are grateful to the MILC Collaboration for the use of their gauge configurations. Our calculations were done on the Darwin Supercomputer as part of STFC's DiRAC facility jointly funded by STFC, BIS and the Universities of Cambridge and Glasgow. This work [9] was funded by STFC, the Royal Society, the Wolfson Foundation and the National Science Foundation.

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