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REFINED LATTICE/MODEL INVESTIGATION OF $ud\bar{b}\bar{b}$ TETRAQUARK CANDIDATES WITH HEAVY SPIN EFFECTS TAKEN INTO ACCOUNT*

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We investigate four-quark systems consisting of two heavy anti-bottom quarks and two light up/down quarks. We propose to solve a coupled Schrödinger equation for the anti-bottom–anti-bottom separation using potentials computed via lattice QCD in the limit of static anti-bottom quarks. This coupled Schrödinger equation allows to incorporate effects due to the heavy anti-bottom spins. First exploratory numerical tests are discussed.

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1. Introduction

In recent papers [1–5], we have studied heavy tetraquark candidates combining lattice QCD and quark model techniques proceeding in two steps. First, we have computed potentials of two static antiquarks $\bar{Q}\bar{Q}$ in the presence of two quarks of finite mass qq ($q \in \{u, d\}$ throughout this work) using lattice QCD [1,2] (such potentials have also been computed by other groups, *cf.* *e.g.* [6–12]). The static approximation is expected to be a rather good approximation for $\bar{Q}\bar{Q} = \bar{b}\bar{b}$ and allows for a comparably easy computation of the potentials. For larger $\bar{Q}\bar{Q}$ separations, some of these potentials can be interpreted as potentials of two B and/or B^* mesons, which are degenerate in the static limit. In a second step, we have inserted these potentials into the Schrödinger equation for the relative coordinate of the two B/B^* mesons. We have then checked, whether they are sufficiently attractive to

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host bound states, which would indicate stable $qq\bar{b}\bar{b}$ tetraquarks. For a specific potential (isospin $I = 0$, light quark spin $j = 0$), we have found a bound state with a confidence level $\approx 2\sigma$ and binding energy ≈ 90 MeV, while there seems to be no bound state in any of the other channels [3, 5].

In the static limit effects due to the spin of $\bar{b}$ quarks are neglected, *e.g.* there is no mass difference of B and B^* for infinitely heavy $\bar{b}$ quarks. These effects, however, could be of the same order as the ≈ 90 MeV binding energy of the tetraquark predicted in [3, 5], as can *e.g.* be estimated from the mass difference $m_{B^*} - m_B \approx 50$ MeV. The goal of this work is to take the heavy $\bar{b}$ spins into account, in particular, to estimate their effect on the binding energy of the above mentioned $(I = 0, j = 0)$ $qq\bar{b}\bar{b}$ tetraquark.

2. Incorporating heavy $\bar{b}\bar{b}$ spin effects

2.1. Relating $qq\bar{Q}\bar{Q}$ potential and $B^{(*)}B^{(*)}$ creation operators

Due to static quark symmetries, it is essential to couple the light spin indices and the static spin indices separately, when defining $qq\bar{Q}\bar{Q}$ potential creation operators. To interpret the meson–meson structure generated by such operators, one needs to express them in terms of static-light bilinears. We do this by using the Fierz identity[1]

$$\begin{aligned} & L_{AB} S_{CD} \left(\bar{Q}_C\left(\vec{r}_1\right) q_A^{(1)}\left(\vec{r}_1\right)\right)\left(\bar{Q}_D\left(\vec{r}_2\right) q_B^{(2)}\left(\vec{r}_2\right)\right) \\ & = \frac{1}{16} \operatorname{Tr}\left(\varGamma_b S^T \varGamma_a^T L\right)\left(\bar{Q}\left(\vec{r}_1\right) \varGamma^a q^{(1)}\left(\vec{r}_1\right)\right)\left(\bar{Q}\left(\vec{r}_2\right) \varGamma^b q^{(2)}\left(\vec{r}_2\right)\right) , \quad (1) \end{aligned}$$

where A, B, C, D denote spin indices, $\varGamma^a \in \{\gamma_5, \gamma_0\gamma_5, \mathbb{1}, \gamma_0, \gamma_j, \gamma_0\gamma_j, \gamma_j\gamma_5, \gamma_0\gamma_j\gamma_5\}$ and $\varGamma_a$ is the inverse of $\varGamma^a$. The left-hand side of this equation has the structure of a $qq\bar{Q}\bar{Q}$ potential creation operator (*cf.* *e.g.* [5], Eq. (6)), while the right-hand side allows to read off, which linear combination of B meson pairs it excites.

In the following, we are interested in those matrices L and S generating B and/or B^* mesons ($\bar{Q}(\mathbb{1}+\gamma_0)\gamma_5 q$ and $\bar{Q}(\mathbb{1}+\gamma_0)\gamma_j q$, respectively). After some linear algebra, one finds that there are 16 such combinations, $L, \mathcal{C}S^T\mathcal{C} \in \{\mathcal{C}(\mathbb{1}+\gamma_0)\gamma_5, \mathcal{C}(\mathbb{1}+\gamma_0)\gamma_j\}$ ($\mathcal{C}$ denotes the charge conjugation matrix). The $qq\bar{Q}\bar{Q}$ potentials, which have been computed in the static limit, depend only on the light spin coupling L, but not on the heavy spin coupling S. There are two different potentials: (1) $V_5(r)$ (corresponding to $L = \mathcal{C}(\mathbb{1}+\gamma_0)\gamma_5$), attractive for isospin $I = 0$, repulsive for isospin $I = 1$, and (2) $V_j(r)$ (corresponding to $L = \mathcal{C}(\mathbb{1}+\gamma_0)\gamma_j$), repulsive for isospin $I = 0$, attractive for isospin $I = 1$, where $r = |\vec{r}_1 - \vec{r}_2|$.

[1] Similar techniques have recently been applied to relate meson–meson and diquark–antidiquark creation operators [13].

Note that it is not possible to choose S and L in a way that exclusively B mesons appear on the right-hand side of Eq. (1). One always finds linear combinations of B and B^* mesons, *e.g.* for $L = \mathcal{C}S^T\mathcal{C} = \mathcal{C}(\mathbb{1} + \gamma_0)\gamma_5$ the right-hand side of Eq. (1) is proportional to $B(\vec{r}_1)B(\vec{r}_2) + B^*_x(\vec{r}_1)B^*_x(\vec{r}_2) + B^*_y(\vec{r}_1)B^*_y(\vec{r}_2) + B^*_z(\vec{r}_1)B^*_z(\vec{r}_2)$ (the indices x, y, z denote the spin orientation of B^*). Taking this mixing of B and B^* mesons into account, which differ in mass by ≈ 50 MeV, is the goal of this work, as already mentioned in the introduction.

2.2. The coupled channel Schrödinger equation

We study a coupled channel Schrödinger equation

$$H\Psi(\vec{r}_1, \vec{r}_2) = E\Psi(\vec{r}_1, \vec{r}_2) \ , \tag{2}$$

where the Hamiltonian H acts on a 16-component wave function Ψ. The components of Ψ correspond to the 16 possibilities to combine $(B(\vec{r}_1), B^*_x(\vec{r}_1), B^*_y(\vec{r}_1), B^*_z(\vec{r}_1))$ and $(B(\vec{r}_2), B^*_x(\vec{r}_2), B^*_y(\vec{r}_2), B^*_z(\vec{r}_2))$, *i.e.* the first component corresponds to $B(\vec{r}_1)B(\vec{r}_2)$, the second to $B(\vec{r}_1)B^*_x(\vec{r}_2)$, *etc.*

The Hamiltonian can be split in a free and an interacting part, $H = H_0 + H_{\text{int}}$. The free part is given by

$$H_0 = M \otimes \mathbb{1} + \mathbb{1} \otimes M + \frac{\vec{p}_1^{\,2}}{2} (M \otimes \mathbb{1})^{-1} + \frac{\vec{p}_2^{\,2}}{2} (\mathbb{1} \otimes M)^{-1} \tag{3}$$

with $M = \text{diag}(m_B, m_{B^*}, m_{B^*}, m_{B^*})$. The interacting part can be written according to

$$H_{\text{int}} = T^{-1} V(r) \,, \tag{4}$$

where

$$V(r) = \text{diag}\Big(\underbrace{V_5(r), \ldots V_5(r)}_{4\times}, \underbrace{V_j(r), \ldots V_j(r)}_{12\times}\Big) \tag{5}$$

and T is a 16×16 matrix relating the 16 choices for L, S (*cf.* Section 2.1 and Eq. (1)) to the 16 components of Ψ (the entries of T can be computed using the Fierz identity (1)).

3. Numerical solution of the coupled channel Schrödinger equation

Rotational symmetry allows to bring the coupled channel Schrödinger equation (2) to block diagonal form, *i.e.* to split it into independent simpler equations corresponding to definite total spin J and isospin I [2]:

[2] J and I are related, because quarks are fermions and have to obey the Pauli principle (*cf.* [5] for a detailed discussion of quantum numbers of $qq\bar{b}\bar{b}$ tetraquarks).

— a single 2×2 coupled channel equation:
$J = 0$, $I = 1$, meson pairs $B(\vec{r}_1)B(\vec{r}_2)$ and $B^*(\vec{r}_1)B^*(\vec{r}_2)$;

— three identical 1×1, *i.e.* uncoupled equations:
$J = 1$, $I = 1$, meson pairs $B(\vec{r}_1)B^*(\vec{r}_2)$ and $B^*(\vec{r}_1)B(\vec{r}_2)$;

— three identical 2×2 coupled channel equations:
$J=1$, $I=0$, meson pairs $B(\vec{r}_1)B^*(\vec{r}_2)$, $B^*(\vec{r}_1)B(\vec{r}_2)$ and $B^*(\vec{r}_1)B^*(\vec{r}_2)$;

— five identical 1×1, *i.e.* uncoupled equations:
$J = 2$, $I = 1$, meson pairs $B^*(\vec{r}_1)B^*(\vec{r}_2)$.

For the remainder of this section, we focus on the $J = 0$ coupled channel equation, where

$$H_{0,J=0} = \begin{pmatrix} 2m_B & 0 \\ 0 & 2m_{B^*} \end{pmatrix} + \left(\frac{\vec{p}_1^{\,2}}{2} + \frac{\vec{p}_2^{\,2}}{2} \right) \begin{pmatrix} 1/m_B & 0 \\ 0 & 1/m_{B^*} \end{pmatrix} , \tag{6}$$

$$H_{\mathrm{int},J=0} = \begin{pmatrix} (1/4)(V_5(r) + 3V_j(r)) & (\sqrt{3}/4)(V_5(r) - V_j(r)) \\ (\sqrt{3}/4)(V_5(r) - V_j(r)) & (1/4)(3V_5(r) + V_j(r)) \end{pmatrix} . \tag{7}$$

Introducing center of mass and relative coordinates, the partial differential equation in $\vec{r}_1$ and $\vec{r}_2$ can analytically be reduced to an ordinary differential equation for r,

$$\left(\begin{pmatrix} 2m_B - \frac{1}{m_B} \frac{d^2}{dr^2} & 0 \\ 0 & 2m_{B^*} - \frac{1}{m_{B^*}} \frac{d^2}{dr^2} \end{pmatrix} + H_{\mathrm{int},J=0} \right) \chi(r) = E\chi(r) \, , \tag{8}$$

where the first component of χ represents a $B(\vec{r}_1)B(\vec{r}_2)$ pair and the second component a $B^*(\vec{r}_1)B^*(\vec{r}_2)$ pair. Following standard textbooks on quantum mechanics, one can show that the radial wave function of an s wave bound state is subject to the boundary conditions

$$\chi(r) \sim \begin{pmatrix} Ar \\ Br \end{pmatrix} \quad \text{as} \quad r \to 0 \, , \qquad \lim_{r \to \infty} \chi(r) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{9}$$

with $A, B \in \mathbb{R}$.

First exploratory numerical tests have been performed with $I = 0$ potentials[3], *i.e.* with an attractive V_5 and a weakly repulsive V_j. We integrate Eq. (8) using the Runge–Kutta–Fehlberg method starting with the linear asymptotic behavior (9) at tiny $r = \varepsilon > 0$ to $r = r_{\max}$ with sufficiently

[3] Even though this $(J = 0, I = 0)$ channel is excluded by the Pauli principle, it is conceptually interesting to compare numerical results with existing results from [3,5], where heavy spin effects have not been taken into account.

large $r_{\rm max} \gtrsim 10$ fm. This integration is iterated many times as part of a standard shooting procedure to find parameters A/B and E such that also $\chi_1(r_{\rm max}) = \chi_1(r_{\rm max}) = 0$ is fulfilled.

In Fig. 1, we show results obtained with an unphysically strong attractive potential V_5 (roughly a factor 1.5 stronger than the lattice QCD result for V_5). The intersection of the red line (squares) and the green line (circles) at $E \approx 10.4$ GeV corresponds to $\chi_1(r_{\rm max}) = \chi_1(r_{\rm max}) = 0$, *i.e.* represents an energy eigenstate. Since $E < 2m_B \approx 10.6$ MeV ($2m_B$ is the upper boundary of the plot), this eigenstate is a bound four quark state.

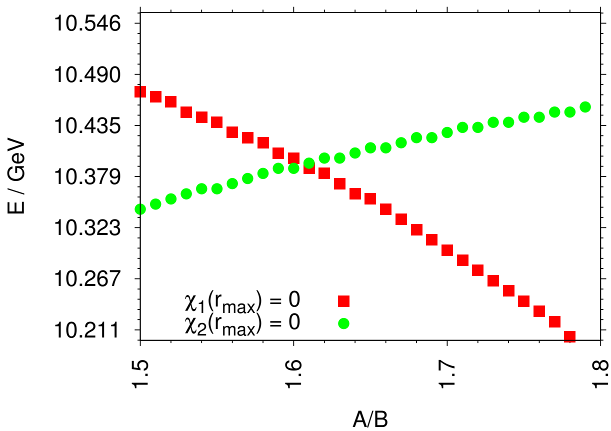

Fig. 1. Isolines $\chi_1(r_{\rm max}) = 0$ (red squares) and $\chi_2(r_{\rm max}) = 0$ (green circles) in the A/B–E plane (for unphysically strong attractive potential V_5).

When repeating these calculations with crude fits to the lattice QCD results for V_5 and V_j, the situation is less clear, *i.e.* $E \approx 2m_B$. A more careful analysis and treatment of statistical and systematic errors similar to what has been done in [5] is needed, to confirm or rule out a bound state. In any case, one can conclude that the heavy $\bar{b}\bar{b}$ spins counteract four-quark binding.

4. Outlook

Most interesting will, of course, be an investigation of the physical channels listed at the beginning of Section 3, in particular the $(J = 1, I = 0)$ channel, which has a stronger attractive potential than the $I = 1$ channels. We are currently in the process of studying corresponding Schrödinger equations for all these channels.

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