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THE $\pi\pi$-SCATTERING AMPLITUDE AND THE σ POLE*

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The multichannel amplitudes used in the analysis of the $\pi\pi$-scattering data and in the description of final-state interaction effects in heavy-meson decays predict too broad and heavy σ meson ($f_0(500)$ resonance) which is in disagreement with results from other, especially recent, analyses. The amplitudes are constructed using a uniformizing variable and proper analytical continuation of the S-matrix elements but the crossing symmetry constraint, important in the S-wave, is not included. We modified, therefore, the amplitudes using the dispersion relations with imposed crossing symmetry (GKPY equations). We also examined the multichannel formalism comparing the single- and two-channel analyses of the $\pi\pi$-scattering data. After the modification and in the single-channel analysis, the σ pole acquired values within the limits accepted in the Particle Data Tables.

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1. Introduction

Analysis of the multichannel $\pi\pi$-scattering data in various partial waves is a suitable tool for studying the spectrum of light-meson resonances. For this purpose, one needs a reliable method for construction of the amplitudes with minimum model assumptions or approximations. Then, a pole structure of the multichannel amplitudes carries true information on existence and properties of resonances decaying into the considered meson channels. These analyses were performed in the S- and P-waves using unitary S-matrix with proper analytical properties on the Riemann surface [1–4]. However, in

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construction of these partial-wave amplitudes, the crossing symmetry constraint (CSC) was not fully included. Since the absence of the CSC affects mainly behavior of the low-energy S-wave amplitude, one may expect that the pole corresponding to the lowest $f_0(500)$ resonance (σ meson) can be shifted. Indeed, the σ pole on the second Riemann sheet, as predicted by the multichannel amplitudes, is typically farther from the origin of the coordinates than the values recommended by the Particle Data Group (PDG); compare, for example, the values $617 - i554$ MeV [2] and $563 - i417$ MeV [5] with the PDG limits, 400–550 $- i$(200–350) MeV [6].

Another suitable method of analysis of the $\pi\pi$ scattering is based on the dispersion relations with imposed crossing symmetry, the Roy-like equations [7, 8]. These analyses are especially convenient for description of the low-energy region in the S-wave $\pi\pi$ scattering giving the position of the σ pole in agreement with the PDG values. The once-subtracted dispersion relations fulfilling the CSC for the S–F-wave $\pi\pi$ amplitudes, the GKPY equations [7], were therefore used to modify some parameters of the S- and P-wave multichannel amplitudes to make the amplitudes consistent also with the CSC [5]. These modified amplitudes describe very well the $\pi\pi$-scattering data in all considered channels from the threshold up to about 1.8 GeV, while their mathematical structure on the Riemann surface is well under control due to the analyticity of the S-matrix.

2. Multichannel amplitudes

The multichannel amplitudes for the S- and P-waves in the $\pi\pi$ scattering were constructed requiring analyticity and unitarity of the S-matrix and applying the uniformization procedure (a conformal mapping of the Riemann surface onto the uniformization w-plane) without any specific assumptions about dynamics of the process. Formulas for the analytic continuation of the S-matrix elements allow to specify an arrangement of poles and zeros on the Riemann surface which is denoted by a cluster. A specific cluster type representing a resonance points to its nature [2, 3]. The uniformization procedure can be applied exactly in the two-channel case [1, 4] but in the three-channel case simplifying approximations have to be done resulting in a bad description of the phase shifts in the threshold region [2]. In the latter case, the channels $K\bar{K}$ and $\eta\eta'$ for S-wave and $\rho 2\pi$ and $\rho\sigma$ for P-wave were considered to couple to the $\pi\pi$ channel. In the uniformizing variable, the left-hand branch point connected with the crossed channels was neglected in Ref. [2] but it was included in [3, 4], partially accounting for the CSC.

The resonance parts of the matrix elements of the S-matrix, expressed via the Le Couteur–Newton relations, are generated by clusters of complex-conjugate poles and zeros on the uniformization w-plane representing considered resonances [2]. For example, the σ meson is represented by a cluster type which possesses zero only in the S_{11} matrix element on the physical sheet. The background part of the S-matrix, which includes less important effects not taken explicitly into account in the formalism, is described by energy-dependent complex phases. Resonance zeros and background parameters were fitted to the inelasticity parameters and phase shifts in all assumed channels selecting also the best scenario (types of clusters) [2].

The data are described satisfactorily as shown in Fig. 1 for the isoscalar S-wave three-channel amplitude. Here, we report on the amplitude used in Ref. [5], "the new refitted $S0$-wave amplitude". The amplitude gets contributions from the $f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ resonances. The pole of the $f_0(500)$ on the second sheet is at $563 \pm 5 - i417 \pm 15$ MeV which is out of the limits set by PDG 2014. In Fig. 1 (a), one can see a poor description of the data below 400 MeV which is due to the approximation in the uniformizing procedure for the three-channel case.

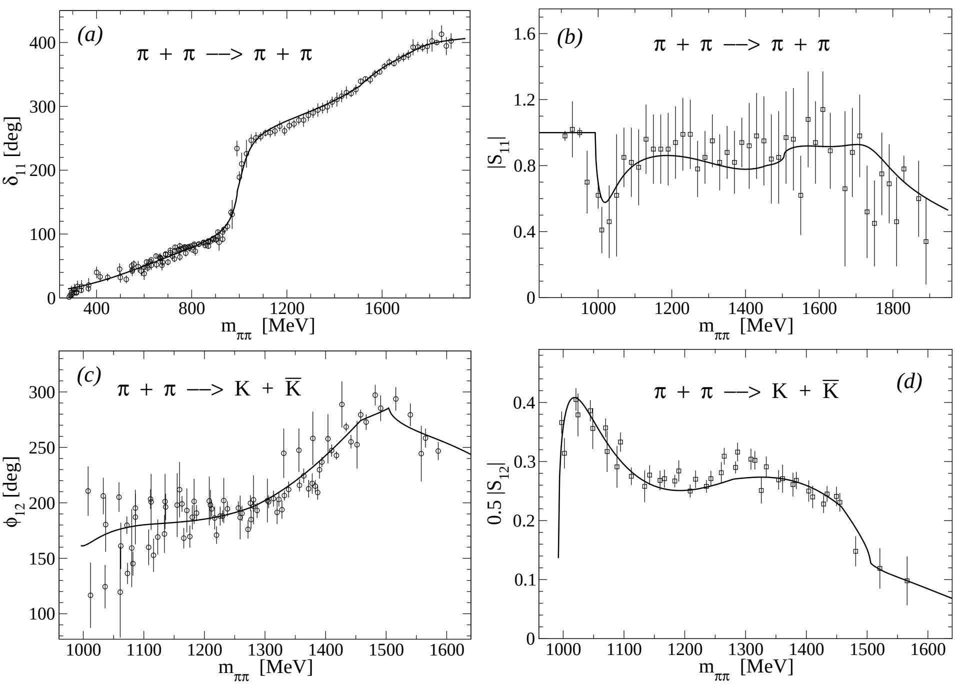

Fig. 1. Phase shifts and modulus of the S-matrix elements in the elastic (a), (b) and inelastic (c), (d) channels for the new $S0$-wave amplitude [5]. Data are taken from [2].

The multichannel amplitudes are suited for description of the final-state interactions in specific heavy-meson decays, *e.g.* $J/\Psi \to \phi\pi\pi$ and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$, in which the pseudoscalar meson pair is produced in the S-wave and the final-state vector meson can be considered as a spectator. Then, one can relate the $\pi\pi$-scattering amplitude to the decay amplitudes [9] and calculate the $\pi\pi$-mass distributions. In Fig. 2, we present results calculated with the amplitudes from the combined analyses of the multichannel $\pi\pi$-scattering data and decays of J/Ψ [9] and $\Upsilon(3\mathrm{S})$ [10]. In the latter, one sees that a destructive interference between the $\pi\pi \to \pi\pi$ and $K\bar{K} \to \pi\pi$ processes naturally describes the two-hump structure in $\Upsilon(3\mathrm{S}) \to \Upsilon(1\mathrm{S})\,\pi\pi$.

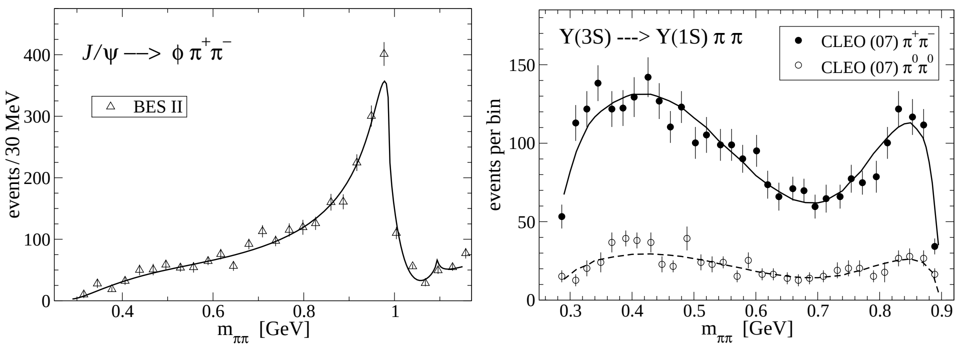

Fig. 2. The $\pi\pi$-invariant mass distributions in the J/Ψ and $\Upsilon(3\mathrm{S})$ decays. The curves and data are taken form Refs. [9] and [10], respectively.

3. Modification of the multichannel amplitudes and the σ pole

As pointed out above, a typical feature of our multichannel amplitudes is a broad and heavy σ meson which disagrees with the widely accepted values in PDG 2012 and later. To better understand reasons for this difference, we have examined the amplitudes using the Roy-like equations [5] and analyzing single- and two-channel data [4].

In the analysis with the once-subtracted dispersion relations GKPY [7], first, we had to correct the bad behavior of the S- and P-wave phase shifts near the threshold using a polynomial expansion in the pion momentum and taking the low-energy parameters from Ref. [7]. After this correction, parameters of the lowest poles, $f_0(500)$, $f_0(980)$, and $f_0(1500)$ in the S-wave and $\rho(770)$ in the P-wave, and the low-energy background parameters were refitted to get the amplitudes consistent with the GKPY equations. Specifically, this means that the difference between the input and output amplitudes in the GKPY equations, $\mathrm{Re}\, f^{\mathrm{out}} - \mathrm{Re}\, f^{\mathrm{in}}$, is minimal. See Ref. [5] for more technical details of the analysis. The modified S- and P-wave amplitudes reveal much better consistency with the GKPY equations indicating that the amplitudes are more consistent with the crossing symmetry constraint than the original ones. Moreover, both amplitudes describe well the data form the threshold up to about 1.8 GeV in all considered channels. Very important result, however, is that in the modified amplitudes the poles are shifted. Whereas only small shifts were observed for the $f_0(980)$, $f_0(1500)$, and $\rho(770)$ poles, a very significant change was seen for the σ pole on the second Riemann sheet: $(563 - i417) \to (459 - i292)$ MeV for the new refitted $S0$-wave amplitude (see Fig. 1). The improved consistency with the GKPY equations, therefore, appears as a vital ingredient in our multichannel formalism having a strong influence on the low-energy behavior of the isoscalar S-wave amplitude.

The σ-meson pole was also studied in the analysis of the S-wave $\pi\pi \to \pi\pi, K\bar{K}$ data using the two-channel formalism [4], which allows a correct description of data in the threshold region (the scattering length). In the uniformizing variable, the left-hand branch point was included taking partially into account effects of the crossing channels. Parameters of the two-channel amplitude were fitted, first only to the elastic $\pi\pi$ data, the single-channel amplitude, and then to both channels which resulted in two variants, two-channel amplitude A and B. Results for the elastic phase shift and inelasticity are shown in Fig. 3. In the single-channel amplitude, the σ pole on the second sheet is located at $448 - i267$ MeV which is in a very good agreement with the PDG limits. The scattering length is $0.222 \pm 0.008\ m_\pi^{-1}$ which agrees very well with $0.220 \pm 0.005\ m_\pi^{-1}$ from the analysis based on ChPT and Roy equations [8]. However, a serious flaw of the single-channel amplitude is that it cannot describe data on inelasticity in the $\pi\pi \to K\bar{K}$ channel for $m_{\pi\pi} > 1100$ MeV as it is seen in Fig. 3.

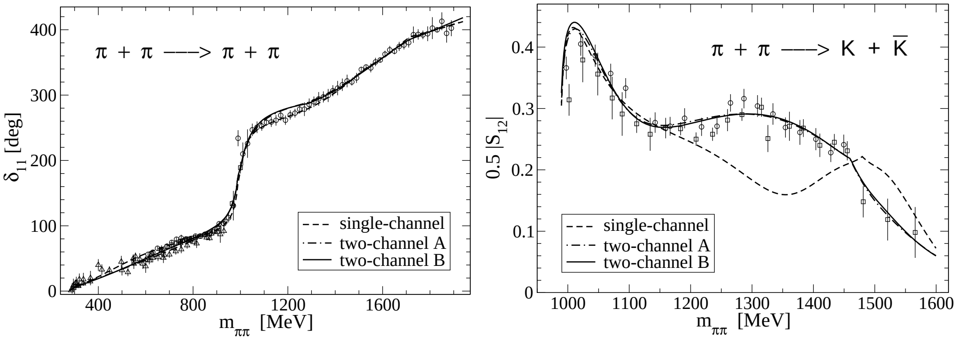

Fig. 3. The elastic phase shift and inelasticity in the coupled channel for the single- and two-channel analyses A and B. The curves and data are taken from Ref. [4].

The σ poles in the two-channel amplitudes A and B are in $517 - i394$ MeV and $551 - i502$ MeV, respectively. These values are rather closer to the three-channel result than to the PDG value. In Fig. 3, one sees different behavior of the elastic phase shift in the region of $400 < m_{\pi\pi} < 800$ MeV for the single- and two-channel solutions which is related with the position of the σ pole. The threshold behavior of the solution A is, however, still in a good agreement with the single-channel solution as the scattering length for the solution A is $0.230 \pm 0.004\ m_\pi^{-1}$.

4. Conclusions

The multichannel amplitudes are well suited to study resonances in the multichannel $\pi\pi$ scattering and also to describe the final-state interaction effects in specific decays of heavy mesons, like $J/\psi, \psi(2S), \Upsilon(nS)$ decaying into V$\pi\pi$, V$K\overline{K}$, or V$\eta\eta$ (V is ϕ, J/ψ, Υ, ...). A peculiar feature of the multichannel analysis is that the σ meson is broader and heavier than predicted by other approaches. We have shown that by imposing the crossing symmetry constraint on the amplitudes via the GKPY equations or restricting the analysis only to the single $\pi\pi$ channel, the σ pole acquires values consistent with the values preferred in the Particle Data Tables.

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