

IS SPACE-TIME EUCLIDEAN “INSIDE” HADRONS?*

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Solution of Schwinger–Dyson and Bethe–Salpeter equations for excited mesons in Minkowski space in the ladder-rainbow approximation is presented. The invalidity of Wick rotation, which historically raised the question, does not prevent the existence of solution in the physical Minkowski space. No analytical continuation in complex Euclidean spacetime was needed in the presented model.

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1. Introduction

The physical spectrum of QCD theory is made out of hadrons with possibly small admixture of glueballs, tetraquarks and exotics. The most of models based on Dyson–Schwinger equations (DSEs) and lattice calculations unavoidably are defined in Euclidean space from the very beginning. Without doubt, both approaches implement confinement of quarks and gluons, describe dynamical symmetry breaking and, up to date, they provide good description of ground and low excited mesons [1–3], weak mesonic decays and electromagnetic form factors [4–8]. The results based on modern truncation of DSEs have been recently derived for glueballs [9], tetraquarks [10, 11] and for nucleon and its excitations as well [12–14]. Recall here, most of models listed here are based on the UV-improved Stainsby–Cahill kernel [15], the model which has not removable singularity at the timelike ultraviolet momenta.

2. Ladder-rainbow DSE/BSE model for pions

In this contribution, I will present two types of the solutions of DSE/BSE, both are based on the ladder-rainbow approximation. The first is described in paper [16], while the details of the second can be found in [17].

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As mentioned above, the Minkowski metric is employed. The scalar product of two four-vectors is defined as $p \cdot q = g_{\mu\nu} p^\mu q^\nu$ with metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, thus square of momentum p^2 is negative for space-like configuration.

In order to get the meson spectrum from BSE, one first solves the gap equation (another name for quark DSE), whose solution is required to complete BSE kernel, and then solve the BSE for pion and all its radial excitations. The propagator satisfies the DSE, which reads in momentum space

$$\begin{aligned} S^{-1}(p) &= Z_2 \not{p} - Z_4 m_q(\mu) - Z_1 \Sigma(p), \\ \Sigma(p) &= i \int \frac{d^4 k}{(2\pi)^4} G_{\mu\nu}(p-k) \frac{\lambda^a}{2} \gamma_\mu S(k) \Gamma_\nu^a(p, q), \end{aligned} \quad (1)$$

where $m(\mu)$ is the renormalized quark mass at the scale μ , $\Gamma_\nu^a(p, q)$ is the quark–gluon vertex satisfying its own SDE and $G_{\mu\nu}$ is gluon propagator.

The BSE vertex function $\Gamma_{\pi_n}^j(p, P)$ as well as the BSE wave function $\chi(k, P) = S(k_+) \Gamma(k, P) S(k_-)$ is a solution of bound state BSE

$$\Gamma_{\pi_n}^j(p, P) = i \int \frac{d^4 k}{(2\pi)^4} K(k, p, P) \chi(k, P)_{\pi_n}^j, \quad (2)$$

where $K(k, p, P)$ is the renormalized quark–antiquark interaction kernel and the total momentum satisfies $P^2 = m_{\pi_n}^2$. Strictly speaking, the homogeneous BSE (2) represents narrow mass approximation at the resonance mass for $P^2 = m_{\pi_n}^2$, where m_{π_n} is the mass of the pion (for $n = 1$) or of the arbitrary excitation $\pi(1300)$, $\pi(1800)$, ... Ignoring the resonance width has two technical advantages, the first is that one does not need to solve more complicated inhomogeneous set of coupled BSEs. The second is that it allows to consider heavy resonant states as physical asymptotic states including thus many of those which will be never observed, since the corresponding wide resonant peak is completely hidden in the background of given experimental channel. The latter property can be exploited when testing the old-fashionable idea of Regge trajectories.

The rainbow-ladder approximation, which preserves pseudo-Goldstone boson character of ground state pion, enables us to write down the effective charge α

$$\frac{g^2}{4\pi} \Gamma^\mu(k, p) G_{\mu\nu}(k-p) \rightarrow \gamma^\mu G_{\mu\nu}^{[\text{free}]}(k-p) \alpha(k-p), \quad (3)$$

defined as a simplified product of the quark–gluon vertex and the gluon propagator. The kernel of BSE and DSE must be identical. In the first model [16], the kernel is

$$K_1(x) = \frac{\alpha(x)}{\pi x} = (8\pi^3) C a_1 \frac{e^{-\frac{\sqrt{-x}}{a_1}} \Theta(-x) + e^{i\frac{\sqrt{x}}{a_1}} \Theta(x)}{(x - a_1^2)^2 + a_1^4}, \quad (4)$$

where $x = (p - k)^2$, C is an effective coupling strength and the scale $a \simeq 1$ GeV has been set from the pion mass $M_\pi = 140$ MeV.

In the second model, the exponential fall in (4) has been suppressed by power of $x^{-1/2}$ only

$$K_2(x) = -\frac{d}{dx} \left[\exp(xf) \ln \left(\frac{(x - a_2^2) + a_2^4}{2a_2^4} \right) \right] \quad (5)$$

for spacelike $x = p^2$, where the function f is some log damping (for details, see [17]). Further, the correct UV part

$$K_{uv}(x) = \frac{13}{(33 - 2N_f) 2\pi} \frac{d}{dx} \ln \ln \left(e^2 + \left[\frac{x}{4a_2^2} \right]^2 \right) \quad (6)$$

has been added as well. The kernel is consistent with the asymptotic freedom approximated by one-loop perturbative tail in the deep spacelike ultraviolet. However, the change in K_2 affects the all Minkowski region of the solution nontrivially and the dynamical mass M becomes oscillating in this model. Remind here the conventional parameterization of the solution

$$S^{-1}(p) = \not{p} A(p^2) - B(p^2) = 1/Z(p^2) [\not{p} - M(p^2)] \quad (7)$$

for completeness.

The dynamical mass function is a complex function, providing there is no real pole in the quark propagator. The solutions for the models are shown in Fig. 1. To get the numerical solution of the QCD gap equation in Minkowski space is a nontrivial task, and unhappily, the success of convergence does depend on a numerical details which are not completely clear to the author. As a random attempt usually fails, the author keeps the sample of working codes available and public at [24].

The most striking difference between the models is not the oscillating behavior of quark propagator, but the scale, which is $a_2 = 195$ GeV for the second model. However, it is quite interesting that the both models provide very similar spectra, including few unphysical states (or rather say experimentally unobserved states) as well. Furthermore, as a consequence of refined numerical method in the second model, spectrum of very heavy states is available. Actually, the Regge trajectory for radial excitations is observed in the second model, while the author cannot state similar for the first model due to the lost of numerical convergence.

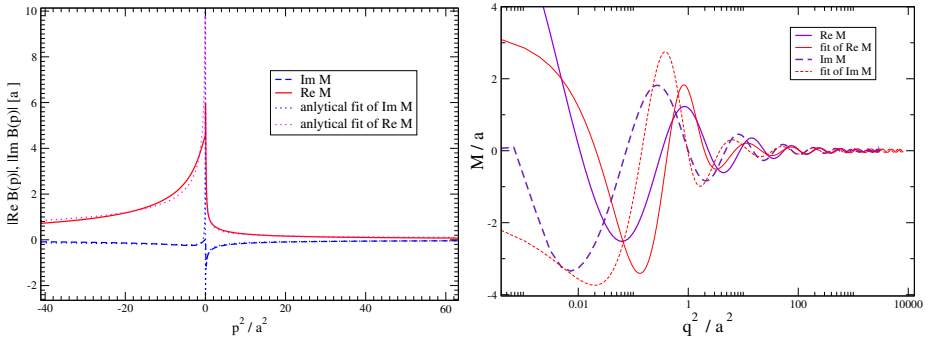


Fig. 1. Dynamical mass for the first (left) and for the second model (right).

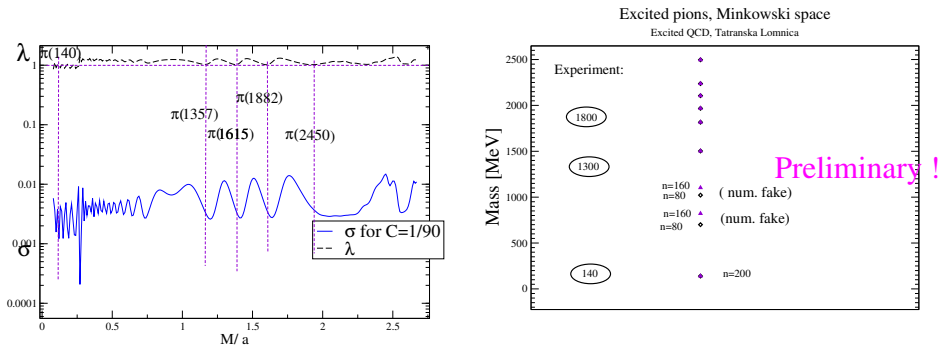


Fig. 2. Spectra for the first (left) and the second model (right).

To conclude, two models based on the ladder-rainbow truncation of DSEs system were presented, both exhibit pionic spectra reasonably. The models differ by the kernels, roughly say, they have different strength at intermediate momenta. Having no other hints at these days, both scenaria can be

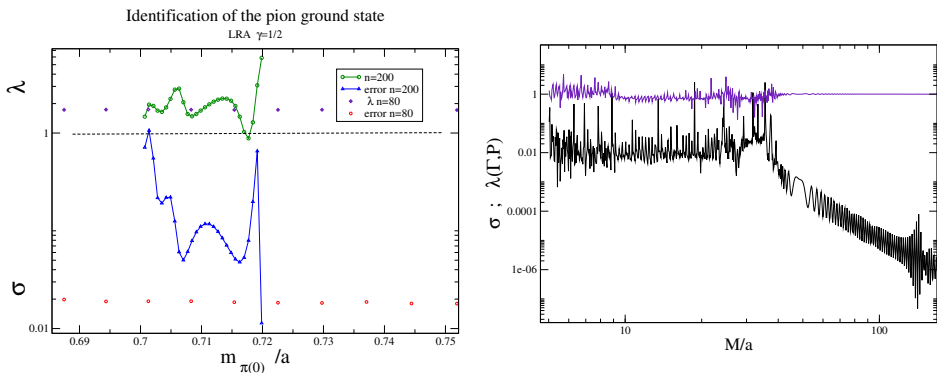


Fig. 3. Graphs of numerical search shown at vicinity of pion mass (left) and global view (right), both for the second model.

considered as equal. Further effort is needed to distinguish, which will be more reliable at the end. Unhappily, due to many unknowns, the calculation of DSEs for gluon propagators, neither the equations for higher points QCD vertices are not recently working numerically due to the lost of stability associated with Minkowski metric (according to the author’s pure experience).

3. Observable and unobservable

This short section is the answer due to Prof. Thomas Cohen, who raised many interesting philosophical questions during the conference.

Recall here that the production of QCD processes is the example where lattice and Euclidean DSE calculations provide almost no results since the physics there has predominantly timelike character. Most striking is the example of the pion electromagnetic form factor. This single variable quantity is measured very indirectly in the spacelike domain and must be reconstructed there in a tricky way from the experimental data. On the other hand, its form is accurately measured in the timelike domain in two pions production for many decades (actually, vector meson dominance is a very old idea). Theorists, at least the ones who prefer to work with QCD degrees of freedom, do exactly opposite, they can calculate pion form factors at spacelike domain for decades. It is a matter of the fact that the tools do not allow to perform analytical continuation of form factor building blocks (QCD Green’s functions) and form factor itself easily [4–8].

Here, Green’s function is complex in the timelike domain as well as in the spacelike, the latter property explicitly exhibits intrinsic inequivalence of the quantum field theory with Euclidean and Minkowski metric as definite ones. While the author generally believes that Euclidean approximation must be a good one, the difference from usage of different metrics (within similar approximations in DSEs system) can be expected. Whether there is some nontrivial consequence for observable is a nontrivial task for a future.

In nonconfining theory, the observable made from unobservable Green’s functions shares and remembers their analytical properties. Thus “correct and usual” analyticity is ensured by the unitarity of S-matrix and *vice versa*, and as a consequence, for instance, one gets form factors which are complex only above the particle production thresholds. In QCD, the observable are composed from Green’s functions in a nonperturbative way. The consequences are less than trivial, and for instance, there is no tree level graph contributing to any hadron transition matrix element. All quark and gluon lines are presented in the loops only, further, they are sandwiched by appropriate BSE amplitudes — the hadrons that appear for asymptotically infinite time. This is a consequence of confinement. The charge conservation and chiral symmetry, together with the proper normalization of BSE states en-

sure the reality and correct values of the form factors, say for instance $F(0) = 1, 0$ for charged or neutral mesons. However, there is no theorem, which would forbid additional momentum-dependent phase of $F(Q)$ elsewhere. Actually, one can expect a presence of such redundant phases in Minkowski space as remnant of confinement. They however should cancel against each other in observable cross sections, $\sigma \simeq F(Q)F^*(Q)$.

Of course, another related question is what can, in principle, follow from the oscillating character of propagators. Obviously, such propagators can interfere in the amplitudes in complicated fashion and one such possible, indirect but observable hint is discussed in the paper [17].

4. Summary

The excited states of pion have been calculated within DSEs formalism. In addition to almost massless 140 MeV pion, we have found four or five excited states below 2.5 GeV. In the case of the second model, we were able to show that the spectrum continues and shows up approximative Regge linear trajectory for higher states. The main purpose of the presented paper is to present the first Minkowski space nonperturbative calculation of light (in a sense of light flavour) mesons. This was possible due to the special property of Green's functions describing confined quarks and gluons. In the case of the second model, the numerical stability was enforced by analytical integration over the angular momenta. Even so, the approximations of DSEs are limited by numerical convergence in Minkowski space, the presented model is the example, where working directly in Minkowski momentum space gives accurate and reliable results. It does not require (a numerically sometimes impossible) analytical continuation of the data coming from the auxiliary Euclidean space.

The main of the limitations of the method presented here is that it relies upon the method of iterations. It is not a secret of the author that many attempts to build an infrared kernel, which would define an alternative ladder-rainbow DSEBSEs models otherwise, do not allow implementation of the correct UV part known from perturbative QCD. In many cases, the iteration process simply collapse before a wanted strength of the interaction is achieved. The principal reason why some models lead to convergent solution while the others do not, remains unknown to the author. While I expect that many solutions in approximations already known and well studied in the Euclidean space will be simply not available in momentum Minkowski space due to the collapsing numerics, the ones giving fruits, should be extremely useful for our understanding. Future work, using an integral transformations, which will allow to perform momentum integration analytically, should clarify this unanswered technical question.

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