

# QUARK PROPERTIES FROM THE HADRON RESONANCE GAS\* \*\*

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We show how the quark free energy can be determined from a string and the Hadron Resonance Gas model with one heavy quark below the de-confinement phase transition. We discuss the interesting problem of identification of degrees of freedom at increasing temperatures, as well as the relevance of string breaking and avoided crossings.

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## 1. Introduction

What is the maximum temperature where the bulk of QCD thermodynamics can be described in hadronic terms, with no explicit reference to the underlying quarks and gluons? In this contribution, we analyze to what extent can one describe the QCD thermodynamics from the hadronic spectrum. We thus hope to identify the nature of non-hadronic precursors of the crossover to the quark–gluon plasma in terms of low temperature partonic expansions [1, 2] (see [3] for a review). The relation between spectrum and partition function provides the thermodynamic free energy  $F$

$$Z(T) = \sum_n e^{-E_n/T} \equiv e^{-F/T}. \quad (1)$$

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The example of a hydrogen gas in QED illustrates the situation and motivates the discussion regarding the completeness of states which ultimately are interacting protons and electrons. At low temperatures, states are molecular, *i.e.*  $H_2$  states which behave as compact and structureless. When we heat up the system, a rovibrational spectrum of states  $H_2^*$ ,  $H_2^{**}$  emerges and eventually the molecule is dissociated into atoms  $H_2 \leftrightarrow 2H$ . At higher temperatures, atoms are excited  $H^*$ ,  $H^{**}, \dots$  and eventually ionized into a plasma of  $p$  and  $e^-$  in the continuum where the  $p$  and  $e^-$  constituents becomes manifest. Clearly, neutral states (atomic or molecular) are not effectively complete at all temperatures and residual interactions corresponding to background non-resonant scattering become relevant [4].

## 2. QCD *vs.* the Hadron Resonance Gas

In QCD on the lattice,  $F$  is determined by integrating the trace anomaly,

$$\mathcal{A}(T) \equiv \frac{\epsilon - 3P}{T^4} = T \frac{\partial}{\partial T} \left( \frac{P}{T^4} \right), \quad (2)$$

with respect to the temperature and *assuming* a reference value, ideally  $P(T_0) = P_\pi(T_0)$  with  $T_0 \ll m_\pi$ . Once this condition is imposed, we expect that at sufficiently low temperatures all the observables are described in terms of hadronic degrees of freedom; the quark–gluon underlying constituents are disclosed at very high temperatures. This quark–hadron duality *at low temperature* has been confirmed by the most recent and accurate lattice calculations at finite temperature [5, 6] and are confronted with the Hadron Resonance Gas (HRG), a rather simple system of non-interacting point-like and structureless hadrons

$$\mathcal{A}_{\text{HRG}}(T) = \frac{1}{T^4} \int dN(M) \int \frac{d^3p}{(2\pi)^3} \frac{E(p) - \vec{p} \cdot \nabla_p E(p)}{e^{E(p)/T} + \eta}. \quad (3)$$

Here, the sum is over *all* hadronic states including spin–isospin and anti-particle degeneracies,  $\eta = \mp 1$  for mesons and baryons respectively,  $E(p) = \sqrt{p^2 + M^2}$  is the energy and the cumulative number of states (degeneracy included)  $N(M) = \sum_n \theta(M - M_n)$ , where  $M_n$  are the hadron masses. The spectrum can be taken either directly from the PDG [7] or the relativized quark model (RQM) Refs. [8, 9]. Indeed, requiring

$$\chi^2 = \sum_{i=1}^{N_{\text{dat}}} \left[ \frac{\mathcal{A}_{\text{Lat}}(T_i) - \mathcal{A}_{\text{HRG}}(T_i)}{\Delta \mathcal{A}_{\text{Lat}}(T_i)} \right]^2 \sim N_{\text{dat}} \pm \sqrt{2N_{\text{dat}}}, \quad (4)$$

we get  $T \lesssim 175$  MeV for  $N_{\text{dat}} = 10$  lattice points [5, 6] both for PDG and RQM, showing that they can be used to saturate the hadronic spectrum for

light quarks provided  $M \lesssim 2.1$  GeV. This RQM saturation will be assumed also to take place when heavy quarks are involved in which case the PDG lacks many of the needed states. This, of course, suggests to determine *directly*  $N(M)$  from the  $\mathcal{A}$  data, as illustrated in Fig. 1 (left panel).

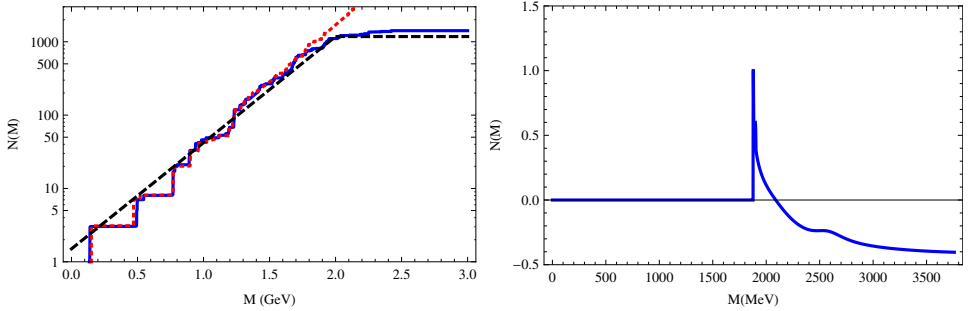


Fig. 1. Left: Total cumulative number  $N(M) = \sum_n \theta(M - M_n)$  for PDG (full), RQM (dotted) and a fit (dashed), Eq. (4) with  $N(M) = A[e^{M/T_H} \theta(M_{\text{th}} - M) + e^{M_{\text{th}}/T_H} \theta(M - M_{\text{th}})]$ . For  $M_{\text{th}} = 2$  GeV gives  $A = 1.5$  and  $T_H = 300$  MeV with  $\chi^2/\nu = 0.94$  and  $T_{\text{max}} = 185$  MeV. Right: The single contribution of the deuteron  ${}^3S_1$ -channel to  $N(M)$  as a function of the invariant  $NN$  mass.

### 3. Counting hadrons

The partition function and its comparison with hadronic states through the trace anomaly  $\mathcal{A}$  displays explicitly the connection between spectrum and thermodynamics yielding the statistically significant identification of  $N_{\text{QCD}}$  with  $N_{\text{HRG}}$  using PDG or RQM where only  $\bar{q}q$  and  $qqq$  states contribute. But, which hadronic states count into the calculation of the thermodynamic properties? As argued long ago [10], counting hadronic states implicitly average over some scale, and so states such as the deuteron (made of  $6q$ ) generate fluctuations in a smaller nuclear scale. The cumulative number in a given channel  $\alpha$  with bound states  $M_{n,\alpha}$  below the threshold  $M_{\text{th}}$  is in general (see *e.g.* [4])

$$N_\alpha(M) = \sum_n \theta(M - M_{n,\alpha}) + \frac{1}{\pi} [\delta_\alpha(M) - \delta_\alpha(M_{\text{th}})] , \quad (5)$$

which becomes  $N(\infty) = n_B + [\delta(\infty) - \delta(M_{\text{th}})]/\pi = 0$  due to Levinson's theorem. This fluctuation is shown in Fig. 1 for the  $NN$  channel where  $M_{\text{th}} = 2M_N$ , and the deuteron mass is  $M_d = 2M_N - B_d$  ( $B_d = 2.2$  MeV). Thus, the deuteron does not count. This is a general feature of weakly bound states, which for the copious new  $X, Y, Z$  states, might affect the thermodynamics if the HRG was blindly identified with the PDG (with *all*  $X, Y, Z$ s).

#### 4. Quark potential, string breaking and avoided crossing

The linearly growing static energy of two  $\bar{Q}$  and  $Q$  sources in the fundamental representation of the  $SU(N_c)$ -group placed at a distance  $r$  is often identified with confinement with a string tension  $\sigma = (0.42 \text{ GeV})^2$ . Including the short distance (perturbative) Coulomb-like behaviour yields the venerable Cornell potential for the ground state

$$V_{Q\bar{Q}}(r) = \sigma r - \frac{4\alpha_s}{3r} + \dots \quad (6)$$

Actually, the mass of a  $\bar{Q}Q$  state becomes unstable at a critical distance  $r_c$  when a light  $\bar{q}q$  pair is created from the vacuum and the heavy–light meson–antimeson  $\bar{B}B \equiv (\bar{Q}q)(Q\bar{q})$  channel opens

$$M_{\bar{Q}Q}(r_c) = V_{\bar{Q}Q}(r_c) + m_{\bar{Q}} + m_Q = M_{\bar{B}} + M_B, \quad (7)$$

where the weak van der Waals-like meson exchange interaction in the  $\bar{B}B$  sector is neglected<sup>1</sup>. We have a diabatic crossing structure which turns into an adiabatic avoided crossing when the transition strength  $V_{\bar{Q}Q \rightarrow \bar{B}B}$  is included. In general, one has excited meson states,  $V_{\bar{Q}q, \bar{q}Q}^{(n,m)}(r) = \Delta_{\bar{q}\bar{Q}}^{(n)} + \Delta_{qQ}^{(m)}$

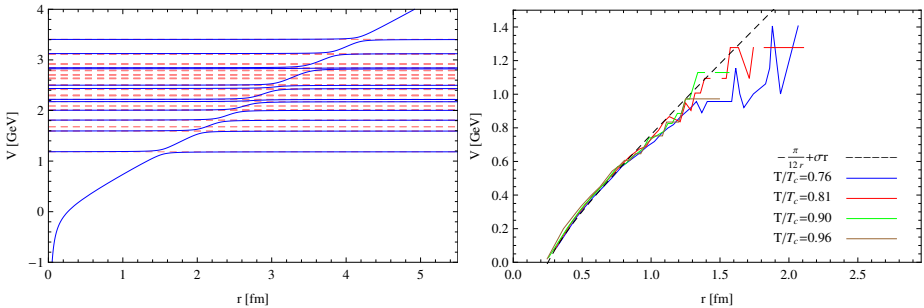


Fig. 2. Left: Potential as a function of distance for the model including heavy–light mesons with a charm quark [8], up to  $\Delta = 1.8 \text{ GeV}$ . We include only a  $g = 50 \text{ MeV}$  mixing between the fundamental and the excited states. Right: Heavy  $\bar{Q}Q$  potential as a function of distance extracted from the lattice data of Ref. [12] for temperatures  $T/T_c = 0.76, 0.81, 0.90$  and  $0.96$  using Eq. (9). The dashed (red) line is the result for the  $\bar{Q}Q$  Cornell potential of Eq. (6) with  $\sigma = (0.4 \text{ GeV})^2$ .

<sup>1</sup> A rough estimate of  $r_c$  proceeds as follows. Due to spontaneous chiral symmetry breaking, the quarks acquire an additive *dynamical* mass  $M_0$  contribution to the total mass  $M_i = M_0 + m_i$ , where  $m_i$  is the *current* mass. For light  $\bar{q}q$ -mesons,  $M_\rho = 2M_0 + 2m_q$  whereas for light–heavy ( $\bar{q}Q$ ) mesons,  $M_B = 2M_0 + m_Q + m_{\bar{q}}$  and thus  $V_{\bar{Q}Q}(r_c) = 4M_0 + 2m_q$  which yields  $r_c \sim 4M_0/\sigma \sim 1.2\text{--}1.4 \text{ fm}$  in fair agreement with the lattice estimate [11].

where  $\Delta_{\bar{q}Q}^{(n)} = M_B^{(n)} - m_Q$  and in Fig. 2 we show the resulting adiabatic spectrum based on the RQM [8] when a mixing strength of 50 MeV is implemented.

## 5. Heavy $\bar{Q}Q$ free energy

Much of our understanding of the QCD dynamics at finite temperature is linked to the Helmholtz free energy defined as the maximum work the system can exchange at fixed temperature between two heavy  $\bar{Q}Q$  at separation  $r$ . In the confined phase, the correlator between Polyakov loops becomes [3]

$$e^{-F_{\text{ave}}(r,T)/T} = \left\langle \text{Tr}_F \Omega(\vec{r}) \text{Tr}_F \Omega(0)^\dagger \right\rangle = \sum_{n,m} e^{-V_{\bar{Q}Q}^{(n,m)}(r)/T}, \quad (8)$$

where, before mixing, one has the crossing among the energy levels up to  $\bar{q}q$  pair creation. Neglecting the avoided crossing, Eq. (8) yields

$$e^{-F_{\text{ave}}(r,T)/T} = e^{-V_{\bar{Q}Q}(r)/T} + \left( \frac{1}{2} \sum_n e^{-\Delta_n/T} \right)^2, \quad (9)$$

where  $\Delta_n = \Delta_{\bar{q}Q}^{(n)} = \Delta_{\bar{Q}Q}^{(n)}$ . We show in Fig. 3 the heavy  $\bar{Q}Q$  free energy obtained with Eq. (9) by using the spectrum of heavy–light mesons with a charm (bottom) quark, and no strangeness, obtained with the Isgur model of Ref. [8] up to  $\Delta = 3.19$  GeV. We have considered  $\sigma = (0.4 \text{ GeV})^2$  and  $\alpha = \pi/16$  (no fit). In Fig. 3, we also show the Polyakov loop computed as

$$L(T) \equiv \lim_{r \rightarrow \infty} e^{-F_{\text{ave}}(r,T)/(2T)} = \frac{1}{2} \sum_n e^{-\Delta_n/T}. \quad (10)$$

The agreement with lattice data is quite good for  $T < 0.8 T_c$ . Thus, the spectrum both for the PDG and the RQM saturates the sum rules at these temperatures<sup>2</sup> whereas the finite temperature does not resolve avoided crossings making them effectively diabatic passages.

<sup>2</sup> By multiplying  $L(T)$  by a factor  $e^{C/T}$  with  $C = -40$  MeV, we get good agreement with the lowest temperature lattice point. This kind of ambiguity comes from renormalization effects. Like in [12], we assume  $F_{\text{ave}}(r,T) \sim V_{\bar{Q}Q}(r)$  for  $rT \ll 1$ . The recent evaluation of Ref. [13], unlike [12] is based on a different renormalization condition and  $N_f = 2 + 1$  for the free energy and the role of avoiding crossing will be analyzed elsewhere.

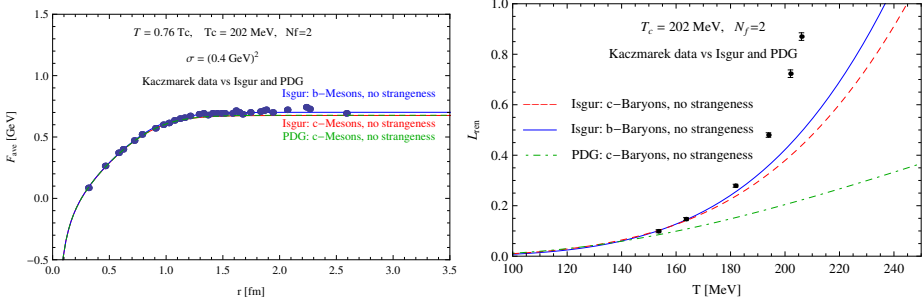


Fig. 3. Left: Colour averaged heavy  $\bar{Q}Q$  free energy as a function of distance. We show as dots the lattice data for  $N_f = 2$ ,  $T = 0.76 T_c$  taken from Ref. [12], and as continuous lines the result by using Eq. (9) with the spectrum of heavy-light mesons with a charm quark (red) and bottom quark (blue), with no strangeness, obtained with the Isgur model of Ref. [8], as well as the spectrum obtained from the PDG (green). Right: Renormalized Polyakov loop as a function of temperature. We show as dots the lattice data for  $T = 0.76 T_c$  taken from Ref. [12], and as continuous lines the result by using Eq. (10).

## 6. Conclusions

While much of the effort has been directed towards a description of the hadron-to-quark-gluon crossover, at present there is no understanding on the mechanism, perhaps because confinement itself is not really well understood. Our analysis of the free energy suggests that lowest temperatures where purely hadronic states account for QCD observables is rather large  $T \sim 170$ – $180$  MeV and close to the critical temperature.

## REFERENCES

- [1] E. Megías, E. Ruiz Arriola, L. Salcedo, *Phys. Rev. Lett.* **109**, 151601 (2012).
- [2] E. Megías, E. Ruiz Arriola, L. Salcedo, *Phys. Rev. D* **89**, 076006 (2014).
- [3] E. Ruiz Arriola, L. Salcedo, E. Megías, *Acta Phys. Pol. B* **45**, 2407 (2014).
- [4] R. Rajaraman, in: *Mesons in Nuclei*, v. III, Amsterdam: North-Holland, 1979.
- [5] S. Borsanyi *et al.*, *Phys. Lett. B* **730**, 99 (2014).
- [6] A. Bazavov *et al.* [HotQCD Collaboration], *Phys. Rev. D* **90**, 094503 (2014).
- [7] K. Nakamura *et al.* [Particle Data Group], *J. Phys. G* **37**, 075021 (2010).
- [8] S. Godfrey, N. Isgur, *Phys. Rev. D* **32**, 189 (1985).
- [9] S. Capstick, N. Isgur, *Phys. Rev. D* **34**, 2809 (1986).
- [10] R.F. Dashen, G.L. Kane, *Phys. Rev. D* **11**, 136 (1975).
- [11] G.S. Bali *et al.* [SESAM Collaboration], *Phys. Rev. D* **71**, 114513 (2005).
- [12] O. Kaczmarek, F. Zantow, *Phys. Rev. D* **71**, 114510 (2005).
- [13] S. Borsanyi *et al.*, *J. High Energy Phys.* **1504**, 138 (2015).