

DIFFERENT KINDS OF LIGHT MESONS AT LARGE N_c^*

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In this paper, we review the leading $1/N_c$ behavior of different configurations of light mesons. Recently, we studied not just the usual configurations with a fixed number of constituents, like $q\bar{q}$, tetraquark, molecules, gluonia and $\bar{q}qg$, but also the so-called “polyquark” which is the natural generalization to large N_c of the diquark–antidiquark configuration, whose number of constituents grows with N_c . With the exception of this polyquark, which has an $O(1)$ width in the large- N_c limit, all other configurations have a vanishing width at large N_c .

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1. Introduction

We report here on our recent work on the large- N_c behavior of different configurations of light mesons [1] in terms of quark and gluon degrees of freedom. The nature of light scalar mesons has been the matter of a

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longstanding debate, since they do not easily fit into a $\bar{q}q$ scheme. Actually, many alternatives have been proposed, such as tetraquarks [2] in different configurations [3], molecules [4], glueballs [5] or most likely a mixture of different configurations in which the “meson-cloud”-component or the meson-rescattering or unitarization play an essential role to explain their features [6, 7]. Here, the words “configuration” or “composition” should only be interpreted in the intuitive meaning of “valence”, in the same sense that we say a proton is made of three quarks or in the usual Fock-expansion sense

$$|M\rangle = \sum \int \left(\alpha_{q\bar{q}} |q\bar{q}\rangle + \alpha_{gg} |gg\rangle + \alpha_{qq\bar{q}\bar{q}} |qq\bar{q}\bar{q}\rangle \dots \right), \quad (1)$$

when it is well approximated by one or just a few states. The glueball, of course, can only be present in isoscalar mesons. However, the setback of this full quantum-mechanical answer is that it is frame and gauge dependent. Unfortunately, the standard QCD perturbative expansion breaks down at low energies and is not useful to calculate meson properties. In particular, none of those states in the Fock expansion are observable as well-defined asymptotic states.

With these caveats in mind, the $1/N_c$ expansion [8, 9] becomes particularly interesting, since it provides an alternative expansion parameter valid at all energies. In addition, in the $N_c \rightarrow \infty$ limit, at least ordinary $\bar{q}q$ mesons are known to become stable, since their mass behaves as $M \sim O(1)$ whereas their width is $O(1/N_c)$. Moreover, the $1/N_c$ behavior of the low energy constants (LECs) of the QCD low energy effective Lagrangian, called Chiral Perturbation Theory (ChPT) [10], is known [10, 11]. This is relevant because one of the most reliable descriptions of light scalars is from the unitarization of the ChPT meson–meson scattering amplitudes [7] in which generically these LECs play the role of subtraction constants in a dispersive formalism. Light scalars are then generated together with light vectors, but when the LECs are rescaled according to their N_c behavior, the vectors follow nicely the expected $\bar{q}q$ $1/N_c$ behavior, whereas the light scalars do not, at least for moderate N_c . Actually, for moderate N_c their width grows with N_c and their associated pole moves away from the physical real axis. At the time, these features were considered “... *the only reliable identifications of observed effects that may be examples of a different class of hadrons*” [12]. The non-ordinary behavior at moderate N_c has been confirmed later within different approaches, although in the large- N_c limit different behaviors can be found [13]. For the particular case of the $f_0(500)$ or sigma meson, two large- N_c behaviors are found: either it keeps moving away from the physical part of the real axis and its effect becomes less and less visible, or it turns back but at a considerably higher mass, suggesting some mixing with another subdominant component that behaves similar to $\bar{q}q$. The latter scenario leads to a natural fulfillment of semi-local duality sum rules [14].

This was our motivation to calculate within QCD the behavior of non-ordinary configurations and their possible mixing, *i.e.* their couplings to one another. It is important to remark that the $1/N_c$ leading behavior can only separate classes of equivalence of states whose mass and decays behave in the same way under N_c . Thus, instead of the usual Fock expansion, we are actually considering

$$|M\rangle = \sum \int \left(\alpha_{q\bar{q}} |q\bar{q} - \text{like}\rangle_M + \alpha_{gg} |gg - \text{like}\rangle_M + \alpha_{qq\bar{q}\bar{q}} |qq\bar{q}\bar{q} - \text{like}\rangle_M \dots \right), \quad (2)$$

where the $|\dots - \text{like}\rangle_M$ states above are the projection of the M meson component within the linear subspace defined by the states within each equivalence class. For the sake of brevity, we customarily drop the “ $- \text{like}$ ” suffix. We also refer to “light” mesons because we will not allow for baryon–antibaryon decays.

The calculations are rather involved to be explained in detail in these proceedings and we refer the reader to the original paper, but we have gathered the results in Table I and Table II.

TABLE I

Leading $1/N_c$ behavior the mass and width for various meson configurations.

	$q\bar{q}$	gg	$q\bar{q}g$	$\pi\pi$	$T_0(q\bar{q}q\bar{q})$	$(N_c - 1)q\bar{q}$
M	$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(N_c)$
Γ_{Tot}	$O(1/N_c)$	$O(1/N_c^2)$	$O(1/N_c)$	$O(1)$	$O(1/N_c)$	$O(1)$

TABLE II

Couplings between configurations with fixed constituent number to leading order in the large N_c expansion. Note that the diagonal counts, of course, as the propagator (mass) and is of the order of 1.

	$q\bar{q}$	$\pi\pi$	gg	$T_0(qq\bar{q}\bar{q})$
$q\bar{q}$	$O(1)$	$O\left(\frac{1}{\sqrt{N_c}}\right)$	$O\left(\frac{1}{\sqrt{N_c}}\right)$	$O(1)$
$\pi\pi$		$O(1)$	$O\left(\frac{1}{N_c}\right)$	$O\left(\frac{1}{\sqrt{N_c}}\right)$
gg			$O(1)$	$O\left(\frac{1}{\sqrt{N_c}}\right)$
$T_0(qq\bar{q}\bar{q})$				$O(1)$

The configurations we have studied are the ordinary $\bar{q}q$ mesons, the gluonium gg , the qqg exotic, a meson–meson state generically denoted by $\pi\pi$, a conventional tetraquark $T_0(q\bar{q}q\bar{q})$ and the so-called “polyquark” made of

$(N_c - 1)$ $q\bar{q}$ pairs. The latter is the natural extension of a tetraquark formed by an antisymmetrized diquark $\bar{D}^i \equiv \epsilon^{ijk} q^j q^k$ and an antisymmetrized antidiquark $D^i \equiv \epsilon^{ijk} \bar{q}^j \bar{q}^k$, which behave as an antiquark and a quark, respectively. Thus $\bar{D}D$ has the quantum numbers of a meson. However, for $N_c > 3$ the antisymmetrized structure contains $2(N_c - 1)$ constituents, namely

$$\epsilon^{a j_1 \dots j_{N_c-1}} \epsilon^{a i_1 \dots i_{N_c-1}} q^{i_1} \dots q^{i_{N_c-1}} \bar{q}^{j_1} \dots \bar{q}^{j_{N_c-1}}. \quad (3)$$

The behavior of the mass and width of ordinary-mesons and gluonium were, of course, known for long [8,9]. However, the classic tetraquark configuration, with the number of constituents fixed to four, has been the subject of recent debate. Actually, Coleman, in his Erice lectures [15], maintained that tetraquarks did not exist (presumably implying that they were broad) in the large N_c limit, since the two-point function of the $\bar{q}q\bar{q}q$ current is dominated by the creation and annihilation of two-meson states. However, Weinberg has recently pointed out [16] that such an argument applies to leading order disconnected diagrams, whereas tetraquark poles should appear in the connected part, which excludes the leading order two-meson propagation. Were they to exist in the large N_c limit, then they would be narrow, *i.e.* their width would scale as $1/N_c$ at least. Later on, Knecht and Peris [17] have classified various tetraquarks according to their flavor content and shown that some specific configurations can be suppressed even further. Note that this does not imply that tetraquarks must exist at large N_c but only how they should behave if they existed. In this respect, in [18] it was argued that in the case of exotic channels and under the conventional assumptions used in large N_c analysis, either tetraquarks do not exist in the $N_c \rightarrow \infty$ limit or their widths should scale as $1/N_c^2$ or more.

Therefore, all meson configurations with a fixed number of constituents have vanishing widths in the $N_c \rightarrow \infty$ limit. This makes rather unnatural any possible explanation of the very broad light scalar states like the σ and κ , also called $f_0(500)$ and $K^*(800)$, respectively. However, in [12,19] it was noticed that the diquark–antidiquark meson could be extended to larger N_c in two different ways. The first leaves the number of constituents fixed, that is, $qq\bar{q}\bar{q}$ for all N_c , which corresponds to a tetraquark or molecule. As we already commented, the second scales both the number of quarks and antiquarks as $(N_c - 1)$, which what we have called polyquark.

The polyquark at large N_c was discussed qualitatively by Witten [9], who argued that it must exist and that its width to a fixed number of mesons should vanish at large N_c , although he suggested that the decay to one meson and an $(N_c - 2)\bar{q}q$ polyquark should be of the order of one. However, Jaffe [20] argued that these states, while weakly coupled to channels in which it annihilates into mesons is, in fact, parametrically broad, having a width for decaying into nucleon–antinucleon plus mesons of the order of $N_c^{1/2}$ or more.

In [1], we explicitly calculated that, for a light polyquark, its total width is $O(1)$ because it is dominated by the sequential emission of a meson, to form an $(N_c - 2)\bar{q}q$ state, that then emits another meson, *etc.*, but we have shown that the n^{th} -step in this chain process is $O(\sqrt{1 + 1/n}) \sim O(N_c^0)$. Actually, the first step, which yields the behavior of the total width is $O(\sqrt{2}N_c^0)$. We have also calculated other non-sequential decays into other final states, like $N_c - 1$ mesons or two pions, as well as the couplings to the other meson configurations, which are all exponentially suppressed.

In summary, we have recently reviewed, calculating explicitly, the leading order $1/N_c$ behavior of the most common configurations of meson states in terms of quarks and gluons. We have studied their masses, total decay widths and coupling between different configurations. All the structures that keep the number of constituents fixed for all N_c are found to become stable at large N_c . However, we have explicitly calculated that the natural extension of the popular diquark–antidiquark configuration into $(N_c - 1)$ antisymmetrized quarks and $(N_c - 1)$ antisymmetrized antiquarks, does have an $O(1)$ total width.

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