

DECONFINEMENT IN DENSE (TWO-COLOR) MATTER*

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I review our current understanding of the phase diagram of two-color quark matter with emphasis on the comparison of model and lattice results. Reproducing, even qualitatively, the thermodynamic observables measured on the lattice requires augmenting the standard Polyakov loop Nambu–Jona-Lasinio model with two new elements: explicit chiral symmetry breaking in the contact interaction, and renormalization of the Polyakov loop.

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1. Two-color QCD

If quantum chromodynamics (QCD) at nonzero baryon density did not suffer from the sign problem, there would be no need to write this text. Meanwhile, our understanding of cold and dense nuclear matter remains very rudimentary. Valuable insight could be gained by numerical simulation of QCD-like theories which do not suffer from the sign problem. In this contribution, I will concentrate on the most thoroughly studied one of them: two-color QCD with quarks in the fundamental representation (2cQCD). Eventually, I anticipate an effective model approach that applies to both 2cQCD and QCD and, therefore, allows one to translate the lattice results for 2cQCD directly into the real world. Here, I will however focus only on the first step of this ambitious program, that is, how to use already existing lattice data to improve effective continuum models for 2cQCD itself.

The two-color world differs to a large extent from what we are used to. Firstly, a color singlet can be only made out of an even number of quarks and hence baryons are necessarily bosons. This can be traced back to the fact that the quark representation of the $SU(2)$ color group is pseudoreal. The same fact also implies that 2cQCD with N_f flavors of massless quarks

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possesses an extended $SU(2N_f)$ flavor symmetry [1]. The low-energy spectrum of the theory is determined by the spontaneous breakdown of this symmetry by a chiral condensate in the ground state; the symmetry-breaking pattern is $SU(2N_f) \rightarrow Sp(2N_f)$. For the $N_f = 2$ case, considered in the following, this implies the existence of 5 Nambu–Goldstone (NG) bosons: the usual pion triplet augmented with a diquark–antidiquark pair. When the two quark flavors are endowed with equal, nonzero masses m_0 , the NG modes will acquire a common nonzero mass as well, denoted here as m_π .

The presence of light bosonic states carrying baryon number dramatically changes the topology of the phase diagram. At baryon chemical potential $\mu_B = m_\pi$ (and zero temperature), the diquark mass drops to zero, and at higher μ_B , we expect a Bose–Einstein condensate (BEC) to form. At very high μ_B , on the other hand, the thermodynamics should be dominated by a Fermi sea of weakly interacting quarks, slightly perturbed by Bardeen–Cooper–Schrieffer (BCS) pairing. Since the spontaneous breakdown of baryon number by such pairing is characterized by the same order parameter as diquark BEC, the transition between the two regimes is expected to be a smooth crossover [2]. At low baryon densities, the chiral condensate σ and diquark condensate Δ can be calculated using the model-independent approach of effective field theory, giving a universal prediction for these quantities normalized to the vacuum value of σ

$$\frac{\sigma}{\sigma_0} = \left(\frac{m_\pi}{\mu_B}\right)^2, \quad \frac{\Delta}{\sigma_0} = \sqrt{1 - \left(\frac{m_\pi}{\mu_B}\right)^4} \quad \text{for } \mu_B \geq m_\pi. \quad (1)$$

2. Lattice results and the puzzle

The phase diagram of 2cQCD with two quark flavors has been calculated using both the Nambu–Jona-Lasinio (NJL) model [3] and its Polyakov loop extension [4], and more recently, also in lattice simulations with Wilson fermions [5, 6]. While the results agree on the qualitative level, striking discrepancies appear at a closer look. Firstly, at $\mu_B = 0$, chiral symmetry gets restored via a smooth but sharp crossover at a temperature T_c roughly coinciding with the transition from hadronic to quark degrees of freedom (deconfinement). Based on Eq. (1), one would naively expect the critical temperature T_d for diquark condensation at high μ_B to be of the same order. However, its value is much smaller, roughly $T_d \approx T_c/2$ [6].

Secondly, the expectation value of the color-singlet diquark operator at low (effectively zero) temperature and moderate chemical potential scales as μ_B^2 , resembling the density of states at the Fermi surface of a system of massless relativistic fermions. By the same token, the baryon density and pressure are very close to their Stefan–Boltzmann (SB) limits in the same

thermodynamic regime [5]. All these results suggest an early onset of a BCS regime of weakly-coupled, nearly *massless and gapless* quarks. The anticipated intermediate BEC phase, if present at all, is not resolved.

Such behavior is impossible to understand using any effective model based on the global SU(4) flavor symmetry for a reason easy to understand. Namely, symmetry requires via Eq. (1) that at zero temperature, $\sigma^2 + \Delta^2$ is constant as a function of μ_B . (In effective models of the NJL type, the sum tends to slightly increase with μ_B .) Hence, the quark constituent mass and gap cannot be small simultaneously: at least one of them (or both) must assume a value comparable with that in the vacuum.

3. Improved effective model

In order to be able to explain the above-sketched qualitative features of the lattice results, I propose the following generalized NJL model

$$\begin{aligned} \mathcal{L} = & \bar{\psi} (i\not{D} - m_0) \psi + \kappa D_\mu \bar{\psi} D^\mu \psi + G (\bar{\psi}\psi)^2 \\ & + \lambda G \left[(\bar{\psi} i\gamma_5 \vec{\tau} \psi)^2 + |\bar{\psi}^c \gamma_5 \sigma_2 \tau_2 \psi|^2 \right]. \end{aligned} \quad (2)$$

The κ -term emulates the effects of explicit chiral symmetry breaking in the lattice Wilson action. It leads to the appearance of a new species of heavy fermions with mass scaling as $1/\kappa$ for $\kappa \rightarrow 0$. In the following, I set $\kappa = 0$; see Ref. [7] for an analysis of the consequences of this term.

The Lagrangian (2) contains all operators up to canonical dimension five, consistent with the *exact* Sp(4) symmetry of 2cQCD with two massive quark flavors. The contact four-fermion interaction was chosen in order to allow a straightforward mean-field analysis of the model: the channels corresponding to the NG modes and to the scalar mode that condenses in the vacuum were included. Under the constraints of the full SU(4) symmetry, all three channels would have to come with the same coupling strength. However, explicit breaking of the chiral symmetry splits them, as expressed, by including a new dimensionless coupling: the *chiral twist* λ . Note that the pion and diquark channels are still degenerate, as required by the exact Sp(4) symmetry of the theory.

Let us first inspect the vacuum physics of the model. For illustration of the numerical results, the same parameter set as in Ref. [4] is used, namely

$$G = 7.23 \text{ GeV}^{-2}, \quad \Lambda = 657 \text{ MeV}, \quad m_0 = 5.4 \text{ MeV}. \quad (3)$$

The parameter Λ represents a three-momentum cutoff. The chiral twist λ is treated as a tunable coupling. The above values of the input parameters are fixed among others to reproduce the pion mass in the vacuum, $m_\pi = 140 \text{ MeV}$. Figure 1 shows how the pion mass and decay constant depend on

the chiral twist. The pion mass initially sharply increases with decreasing λ ; in fact, in the strict chiral limit, its asymptotic behavior is $m_\pi \propto \sqrt{1 - \lambda}$ for $\lambda \rightarrow 1$. For $\lambda \approx 0.6$, the pion mass reaches the two-body quark continuum. This defines the range of physically reasonable values of λ as $\lambda \in [0.6, 1]$. From the right panel of Fig. 1, one infers that simultaneously with becoming very weakly bound, the pion ceases to behave as a NG boson at small λ : its coupling to the axial current is strongly reduced. However, roughly in the range $\lambda \in [0.8, 1]$, f_π remains nearly constant, which avoids the necessity to refit the model parameters anew for each value of λ considered.

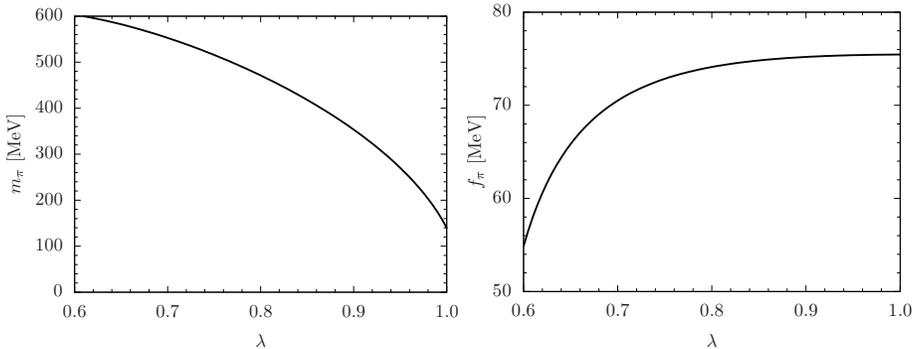


Fig. 1. Pion mass (left panel) and pion decay constant (right panel) in the vacuum as a function of λ . The other parameters of the model are fixed according to Eq. (3).

As the next step, we investigate the effect of the chiral twist on the flavor symmetry at zero temperature and nonzero density. The effective field theory prediction (1) is in a very good agreement with the NJL model at $\lambda = 1$. Reducing λ changes the picture dramatically, see Fig. 2. As one could expect from the Lagrangian (2), the smaller λ is, the more suppressed the diquark condensate becomes. In addition, however, the chiral condensate is suppressed as well. As a consequence, a small enough value of λ can ensure a fast transition to the regime with nearly massless and gapless quarks, as observed on the lattice. The observation that $T_d \approx T_c/2$ together with the fact that, in the BCS theory, the critical temperature for pairing is proportional to the gap at zero temperature, leads to the simple estimate $0.6 \lesssim \lambda \lesssim 0.7$. A similar estimate can be obtained by a direct calculation of the baryon number density and pressure upon the requirement that these saturate their respective SB limits immediately after the onset of diquark condensation.

At nonzero temperature, the nature of thermal excitations becomes important. In order to reproduce the confining property of (2c)QCD, the NJL model is coupled to the Polyakov loop, Φ . However, in contrast to usual treatments in the literature, an additional step is made here to ensure a

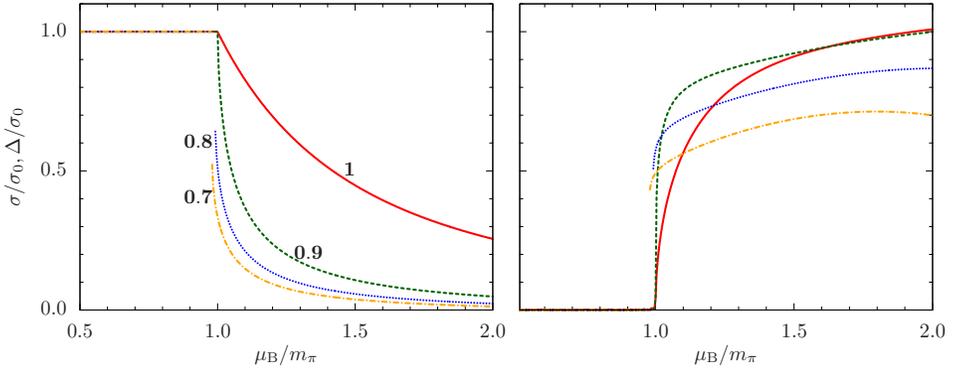


Fig. 2. The dependence of the rescaled chiral (left panel) and diquark (right panel) condensate at zero temperature on μ_B/m_π for several values of λ : 1 (solid/red line), 0.9 (dashed/green line), 0.8 (dotted/blue line) and 0.7 (dash-dotted/orange line). Note that for the sake of comparison, the chemical potential is rescaled by the λ -dependent pion mass.

meaningful comparison of the model predictions with lattice data: renormalization of the Polyakov loop [5]. This temperature-dependent multiplicative renormalization can be interpreted as a consequence of an additive renormalization of the free energy of a static test quark placed in the colored medium. Requiring that at a given reference temperature \bar{T} , the renormalized Polyakov loop takes a fixed value $\bar{\Phi}_R$, the relation between the bare and renormalized Polyakov loops Φ_0 and Φ_R reads

$$\Phi_R(T, \mu_B) = \Phi_0(T, \mu_B) \left[\frac{\bar{\Phi}_R}{\Phi_0(\bar{T}, 0)} \right]^{\bar{T}/T}. \quad (4)$$

The definition of the PNJL model is completed by specifying the thermodynamic potential of the gauge sector, which I take from Ref. [4],

$$\Omega_{\text{gauge}}(\Phi) = -bT \left[24\Phi^2 e^{-a/T} + \log(1 - \Phi^2) \right]. \quad (5)$$

The non-negative quantity Φ is forced to fall between 0 and 1 by the logarithmic term in the potential. The comparison with lattice data is, therefore, carried out by first solving the PNJL model for Φ , and then performing an additional finite renormalization using Eq. (4).

Figure 3 shows the model predictions for the Polyakov loop at zero chemical potential; the parameters a, b were adjusted in order to reproduce the lattice data correctly. In the same graph, I also plot the normalized chiral condensate, which demonstrates that the chiral and deconfinement crossovers overlap for the parameter set used here. Note that thanks to the mean-field approximation, these results for $\mu_B = 0$ are independent of λ .

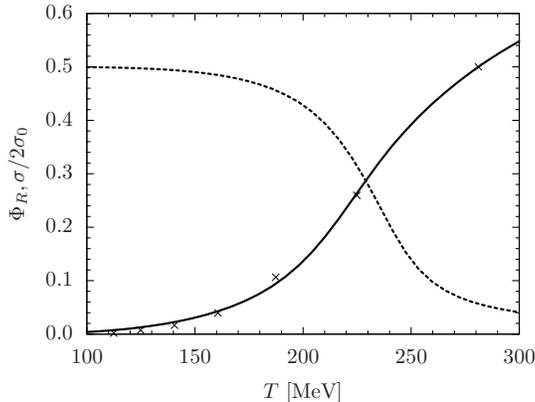


Fig. 3. The expectation value of the rescaled chiral condensate (dashed line) and the Polyakov loop (solid line) at $\mu_B = 0$ as a function of temperature. The data points are taken from Ref. [6].

4. Summary and outlook

The peculiar behavior of the thermodynamic observables in 2cQCD at high baryon density and low temperature can be explained if one assumes strong explicit breaking of chiral symmetry. This is incorporated here by adding a new parameter to the NJL Lagrangian (2): the chiral twist λ . The thus improved model can be used, in conjunction with the lattice data for the Polyakov loop at nonzero chemical potential [5, 6], to gain insight in the back-reaction of the dense medium into the gauge sector [7].

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