CENTER VORTEX VERSUS ABELIAN MODELS OF THE QCD VACUUM*

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We present evidence that the center vortex model of confinement is more consistent with lattice results than other currently available models.

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1. Introduction

Quantum chromodynamics (QCD) at low energies is dominated by the non-perturbative phenomena of quark confinement and spontaneous chiral symmetry breaking (χSB). Center vortices are promising candidates for explaining confinement. They form closed magnetic flux tubes, whose flux is quantized, taking only values in the center of the gauge group. These properties are the key ingredients in the vortex model of confinement, which is theoretically appealing and was also confirmed by a multitude of numerical calculations, in the lattice Yang–Mills theory, see [1] and references therein, and within infrared effective models of random center vortex lines in continuous 3D space-time [2] and world-surfaces in discrete 4D lattices [3, 4].

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Lattice QCD simulations indicate further that vortices are responsible for the spontaneous breaking of chiral symmetry (χSB), dynamical mass generation and the axial $\mathrm{U_A}(1)$ anomaly [5–16], and thus successfully explain the non-perturbative phenomena which characterize the infrared sector of strong interaction physics.

In these proceedings, we present recent results on so-called "double-winding" Wilson loops, a gauge-invariant observable suitable to test center vortex and Abelian models of confinement by comparison with full the SU(2) gauge theory. In view of the ongoing interest in monopole/caloron confinement mechanisms [17–21], it is reasonable to examine those conjectured mechanisms critically.

2. Abelian fields and Abelian models

Magnetic monopole confinement mechanisms, in either the monopole plasma [22,23] or (closely related) dual superconductor incarnations [24,25], provide a durable image of the mechanism underlying quark confinement in non-Abelian gauge theories. The more recent notion that long-range field fluctuations in QCD are dominated by caloron gas ensembles [26, 27], fits nicely into the framework of the earlier monopole plasma conjectures. The mechanisms we are discussing have this point in common: there is some choice of gauge in which the large scale quantum fluctuations responsible for disordering Wilson loops are essentially Abelian, and are found primarily in the gauge fields associated with the Cartan subalgebra of the gauge group. For the SU(2) gauge group, which is sufficient for our purposes, let this Abelian field be the A^3_μ color component. The question we are concerned with is: what do typical configurations drawn from the Abelian field distribution look like? Do they resemble what is predicted by monopole plasma, caloron gas, and dual superconductor models? To be clear, we do not challenge the notion that, in some gauge, most of the confining fluctuations are Abelian in character. The purpose is to subject a qualitative feature of those predicted distributions to a numerical test.

3. Double-winding Wilson loops

Let C_1 and C_2 be two coplanar loops, with C_1 lying entirely in the minimal area of C_2, which share a point $\vec{x}$ in common. Consider a Wilson loop in SU(2) gauge theory which winds once around C_1 and once, winding with the same orientation, around C_2. It will also be useful to consider Wilson loop contours in which C_1 lies mainly in a plane displaced in a transverse direction from the plane of C_2 by a distance δz comparable to a correlation length in the gauge theory. We will refer to both of these cases as "double-winding" Wilson loops. In both cases, we imagine that the extension of loops C_1, C_2 is much larger than a correlation length, so in the latter example the displacement of loop C_1 from the plane of C_2 is small compared to the size of the loops. Let A_1, A_2 be the minimal areas of loops C_1, C_2 respectively. What predictions can be made about the expectation value $W(C)$ of a double-winding Wilson loop, as a function of areas A_i?

In the Abelian models summarized in the previous section, the answer for the displaced loops simply $W(C) = \exp[-\sigma(A_1 + A_2) - \mu P]$, where P is a perimeter term, equal to the sum of the lengths of C_1 and C_2. Assuming that the large scale fluctuations are Abelian in character, we can make the "Abelian dominance" approximation

$$\begin{aligned} W(C) &= \frac{1}{2}\left\langle \mathrm{Tr}\, P \exp\left[i \oint_C dx^\mu A^a_\mu \frac{\sigma^a}{2}\right]\right\rangle \approx \left\langle \exp\left[i\frac{1}{2}\oint_C dx^\mu A^3_\mu\right]\right\rangle \\ &= \left\langle \exp\left[i\frac{1}{2}\oint_{C_1} dx^\mu A^3_\mu\right] \exp\left[i\frac{1}{2}\oint_{C_2} dx^\mu A^3_\mu\right]\right\rangle . \end{aligned} \tag{1}$$

If loops C_1 and C_2 are sufficiently far apart, then the expectation value of the product is approximately the product of the expectation values, *i.e.*,

$$\begin{aligned} W(C) &\approx 2\left\langle \exp\left[i\frac{1}{2}\oint_{C_1} dx^\mu A^3_\mu\right]\right\rangle \left\langle \exp\left[i\frac{1}{2}\oint_{C_2} dx^\mu A^3_\mu\right]\right\rangle \\ &\approx \exp[-\sigma(A_1 + A_2)] , \end{aligned} \tag{2}$$

which we refer to as a "sum-of-areas falloff". Now, as $\delta z \to 0$, we would argue that this limit does not really change the sum-of-areas behavior. Analytical arguments can be found in [28]. The question is whether this sum-of-areas behavior corresponds to the actual behavior of double-winding Wilson loops.

In the center vortex picture of confinement, and also in the strong coupling lattice gauge theory, the behavior of the double-winding loops, whether coplanar or slightly shifted, is $W(C) = \alpha \exp[-\sigma|A_2 - A_1|]$. The same difference-of-areas law is obtained in SU(3) pure gauge theory, in the vortex picture and from strong-coupling expansions, for a Wilson loop which winds twice around loop C_1 and once around the coplanar loop C_2. For simplicity, however, we will restrict our discussion to SU(2), where $W(C)$ picks up a center element (-1) each time any of the loops C_1 or C_2 is pierced by a vortex. So the vortex crossing can only produce an effect if it pierces the minimal area of C_2 but not the minimal area of C_1, resulting in a "difference-of-areas" falloff. A slight shift of loop C_1 by δz in the transverse direction does not make any difference to the argument, providing the scales of A_1 and A_2 are so large compared to δz that a vortex piercing the smaller area A_1 is guaranteed to also pierce the larger area A_2.

4. Sum or difference of areas?

We consider the contour shown in Fig. 1 (a), where $\delta L = 1, L = 7$, L_2 and therefore the size of C_2 are fixed and we vary L_1, *i.e.*, the size of the inner loop C_1. If we increase L_1, the sum-of-areas (and the perimeter) increases, however difference-of-areas decreases. Thus, if $W(C_1 \times C_2)$ increases with L_1, the dominant behavior must be difference-of-areas, and sum-of-areas is completely ruled out. This is clearly the case for center-projected loops in maximal center gauge (Fig. 1 (b)) and, more importantly, for gauge-invariant loops as shown in Fig. 1 (d). The difference-of-areas law is not evident for Abelian-projected loops in maximal Abelian gauge (Fig. 1 (c)). In Fig. 2 (a), we plot results for fixed perimeter P, *vs.* the difference in area $A_2 - A_1$ of the contour shown in Fig. 1 (a) with $\delta L = 0$. Note that the points seem to cluster around universal lines, regardless of perimeter. The results clearly show that difference-of-areas is the dominant effect, and the sum-of-areas behavior is definitely ruled out.

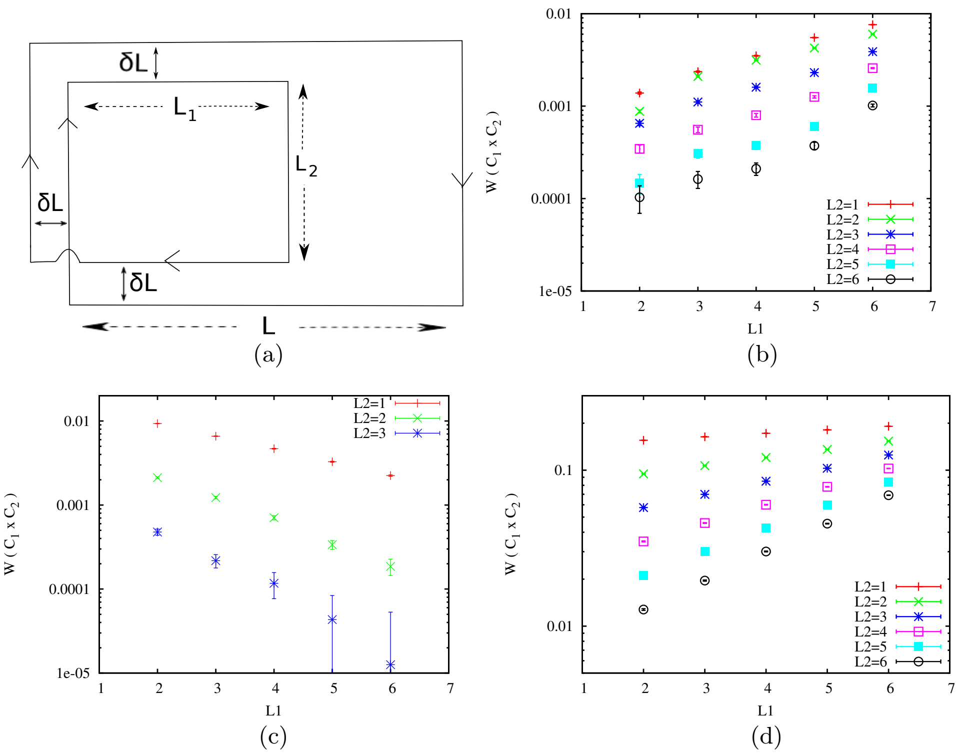
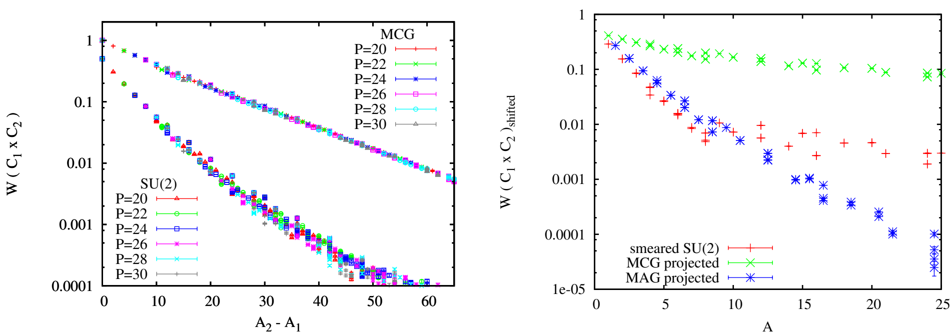


Fig. 1. (a) The coplanar double-winding loop 16^4 lattices with $L = 7$, L_2 fixed and $\delta L = 1$. Both unprojected SU(2) loops on smeared links (b), and center-projected loops in maximal center gauge (d) show a difference-of-areas, MAG projected loops (c) show sum-of-areas behavior.

We also look at shifted double-winding loops with $C_1 = C_2 = C$, so that the difference in areas is zero. For a transverse shift $\delta z = 0$, the situation is trivial. We can make use of an SU(2) group identity $\mathrm{Tr}[U(C)U(C)] = -1 + \mathrm{Tr}_A\, U(C)$, where the trace on the right-hand side is in the adjoint representation. Since, apart from very small loops, $\langle \mathrm{Tr}_A\, U(C) \rangle \ll 1$, we have, almost independent of loop size, $W(C) \approx -\frac{1}{2}$, which is obviously consistent with difference-in-area behavior, just like for center-projected loops, where the result is $W(C) = 1$ exactly. For loops $C_1 = C_2$ shifted by $\delta z = 1a$, where a is the lattice spacing, there is still almost no effect for center projected loops, and for smeared SU(2) loops $W(C)$ levels off for large areas A, see Fig. 2 (b). Abelian-projected loops seem to follow a sum-of-areas falloff in this range, although they may level out eventually.



(a) (b)

Fig. 2. Double-winding loops (a) for the contours of Fig. 1 (a) with $\delta L = 0$ for fixed perimeter P, smeared SU(2) and center projected (MCG) results; (b) for $C_1 = C_2$ (difference in area is zero) shifted in a transverse direction by one lattice spacing. $W(C)$ for the unprojected SU(2) loops levels off at $A_1 = A_2 \approx 8$.

5. Conclusions

We draw the obvious conclusion that if confinement can be attributed, in some gauge, to the quantum fluctuations of gauge fields in the Cartan subalgebra of the gauge group, then the spatial distribution of the corresponding Abelian field strength cannot follow any of the models discussed in Section 2. The difference-of-areas law could be obtained in these models once the neglected double charged matter, namely W-bosons (or any other double-charged objects in the model), are properly taken into account. But then, the typical distribution of Abelian fields in the vacuum must be arranged so as to be consistent with the difference-of-areas behavior. For example, in a monopole picture, the field distribution at a fixed time would very likely resemble a chain of monopoles and anti-monopoles rather than a monopole Coulomb gas, with the magnetic flux collimated, from monopole to anti-monopole, along the line of the chain. In other words, rather than being a monopole plasma, this is a vacuum consisting of center vortices, and the difference-in-area law follows. This is exactly what happens in compact QED with a double-charged Higgs and numerical evidence for this picture was also provided in the context of Abelian projection in maximal Abelian gauge [29, 30]. In general, vortices have a non-trivial color structure, which in Abelian projection leads to monopole lines on vortex surfaces and is a key ingredient for chiral symmetry breaking of center vortices [14, 15].

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