THE EFFECTS OF SCALAR MESONS IN A SKYRME MODEL*

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We study the effects of scalar mesons on skyrmion properties by using a mesonic model including a two- and four-quark scalar mesons as well as the pion, rho and omega mesons in a framework of the hidden local symmetry. We show that the scalar mesons reduce the skyrmion mass and the lighter two-quark state scalar meson is, the lighter soliton mass becomes. In addition, we find the lighter soliton for the more two-quark component of the lighter scalar f_{500}. When vector meson mass partially comes from chiral condensate, the smaller chiral invariant vector meson mass is, the larger soliton mass becomes. In addition to the soliton mass, we study the scalar meson effect on the energy and charge radii.

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1. Introduction

For more than half of a century, people have been making lots of efforts to get a Skyrme type model that can reproduce the realistic baryon properties. In the literature, people have already found that the vector resonance ρ and ω meson and also the scalar degree of freedom associated with the spontaneous breaking of scale symmetry play important roles in skyrmion properties [1–6]. Motivated by the significant role of the scalar meson in nuclear physics, we explore the effect of the scalar mesons made of two quarks and four quarks on the skyrmion properties such as the mass and size of the skyrmion in Ref. [7]. Relegating details to Ref. [7], we give a brief summary of the main ideas and the main results of our approach.

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2. The model

For the $N_f = 2$ case, the Lagrangian of the model has the general form

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}\left(\partial_\mu M_{(2)}\partial^\mu M_{(2)}^\dagger\right) + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi \\ &\quad - \left(V_0 - \bar{V}_0\right) - \left(V_{\mathrm{SB}} - \bar{V}_{\mathrm{SB}}\right) + \mathcal{L}_V\,, \end{aligned} \tag{1}$$

where $M_{(2)}$ denotes two-quark state and ϕ denotes four-quark state, V_{SB} denotes spontaneous chiral symmetry breaking term and $\mathcal{L}_V$ denotes the Lagrangian including vector mesons. By requiring that each vertex in the potential terms V_0 and V_{SB} contains less than or equal to four legs, with the quark number less than or equal to eight, we write the potential parts as [8]

$$\begin{aligned}V_0 &= \lambda\,\mathrm{Tr}\left(M_{(2)}M_{(2)}^\dagger M_{(2)}M_{(2)}^\dagger\right) - m_2^2\,\mathrm{Tr}\left(M_{(2)}M_{(2)}^\dagger\right) + \tfrac{1}{2}m_4^2\phi^2 \\ &\quad + \sqrt{2}A\left(\det\left(M_{(2)}\right) + \det\left(M_{(2)}^\dagger\right)\right)\phi\,, \end{aligned} \tag{2}$$

$$V_{\mathrm{SB}} = -\tfrac{1}{2}f_\pi\,\mathrm{Tr}\left(\chi M_{(2)}^\dagger + \chi^\dagger M_{(2)}\right)\,. \tag{3}$$

By using the polar decomposition, we can write $M_{(2)} = \frac{1}{2}\xi_{\mathrm{L}}^\dagger\sigma\xi_{\mathrm{R}}$ with $U = \xi_{\mathrm{L}}^\dagger\xi_{\mathrm{R}} = e^{2i\pi^a\tau^a/f_\pi}$. The potential for σ is arranged in such a way that σ has a vacuum expectation value (VEV), and ϕ also has a VEV though the interaction term to σ expressed by the A-term. Here, we shift the fields σ and ϕ as $\sigma \equiv \langle\sigma\rangle + \tilde{\sigma} = f_\pi + \tilde{\sigma}$, $\phi \equiv \langle\phi\rangle + \tilde{\phi} = \phi_{\mathrm{vac}} + \tilde{\phi}$, where $\tilde{\sigma}$ and $\tilde{\phi}$ are the fluctuation fields.

In this study, we treat physical scalar mesons as the mixing state between two-quark and four-quark states through

$$\begin{pmatrix}\phi_{f500}\\ \phi_{f1370}\end{pmatrix} = \begin{pmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{pmatrix}\begin{pmatrix}\tilde{\sigma}\\ \tilde{\phi}\end{pmatrix}. \tag{4}$$

In this parametrization, $\cos(\theta)$ implies that when $\cos(\theta)\to 0$, the physical state of ϕ_{f500} is almost constructed by the four-quark component and ϕ_{f1370} is almost constructed by the two-quark component.

The vector meson part of the Lagrangian is

$$\mathcal{L}_V = a_{\mathrm{hls}}\left(s_0\sigma^2 + (1-s_0)f_\pi^2\right)\mathrm{Tr}\left(\alpha_{\|\mu}\alpha_{\|}^{\mu}\right) - \frac{1}{2g^2}\,\mathrm{Tr}\left(V_{\mu\nu}V^{\mu\nu}\right) + \mathcal{L}_{\mathrm{anom}}\,, \tag{5}$$

where the parameter s_0 denotes the percentage of the masses of ρ and ω coming from the spontaneous chiral symmetry breaking. Since the potential V_0 has the Z_2 symmetry for σ, we include only the possible lowest order term $s_0\sigma^2$. Note that $ag_{\rho,\omega}^2 f_\pi^2(1-s_0)$ denotes the chiral invariant mass of the vector mesons. The anomaly term of Lagrangian incorporates the $\omega_\mu B^\mu$ term to construct a minimal model in which the ω meson contribution is included.

3. Numerical analyse

3.1. The ansatz

In this section, we make a numerical calculation of the soliton properties with baryon number one in the $U(2)_{\rm hls}$ hidden local symmetry. For the π, ρ and ω, we take the widely used ansatz and boundary conditions given in Refs. [9–11]. Since the scalar meson fields σ and ϕ do not contain space or isospin indices, we choose the following ansatz

$$\sigma = f_\pi \left(1 + \bar{\sigma}(r)\right) , \qquad \phi = \phi_{\rm vac} \left(1 + \bar{\phi}(r)\right) , \tag{6}$$

with $\bar{\sigma}(r)$ and $\bar{\phi}(r)$ being dimensionless quantities. The boundary conditions for scalar meson fields σ and ϕ are, $\bar{\sigma}'(0) = 0$, $\bar{\sigma}(\infty) = 0$, $\bar{\phi}'(0) = 0$, $\bar{\phi}(\infty) = 0$. By numerically solve the equations of motion for $F(r)$, $G(r)$, $W(r)$, $\bar{\sigma}(r)$ and $\bar{\phi}(r)$, one can obtain the physical quantities of skyrmions. Here, we are interested in the skyrmion mass, the mean square charge radii for the baryon number and the soliton mass.

3.2. The scalar mesons contribution

We first consider the case in which the two-quark and four-quark fields decouple from each other by taking $A = 0$. In such a case, there is only a trivial solution for ϕ, *i.e.*, the four-quark field ϕ decouples from our model and, therefore, there are two parameters m_σ and s_0 in the model.

To investigate the effect of the two-quark scalar meson mass m_σ, we set $s_0 = 0$. We show the m_σ dependence of soliton properties in Fig. 1. From this figure, we see that soliton mass $m_{\rm sol}$ increases with the increasing of m_σ. This is because when m_σ becomes heavier, the attractive force made by $\bar{\sigma}$ becomes smaller. In contrast to the soliton mass, we find that with the increasing of m_σ, the charge radii for $\sqrt{\langle r^2 \rangle_E}$ and $\sqrt{\langle r^2 \rangle_B}$ become smaller. This can be understood from the equation of motion of $\bar{\sigma}$ which tells us that the larger m_σ is, the sharper profile for $\bar{\sigma}$ becomes. The shape for σ dominates the shape for energy and charge distributions.

To explore the effect of s_0 in this model, we take a typical value of m_σ, explicitly $m_\sigma = 1370$ MeV. Our results are plotted in Fig. 2. We see for a bigger s_0 that the $m_{\rm sol}$ goes up and baryon charge radius $\sqrt{\langle r^2 \rangle_B}$ also increases. By seeing the equation of motion for the ω, we can define an effective mass $m_{\omega({\rm eff})}$ which is smaller than the vacuum mass; $m_{\omega({\rm eff})} < m_\omega$. Furthermore, the effective mass is lighter for larger s_0 (see Ref. [7] for detail). So that, when s_0 becomes larger, the repulsive force from ω becomes stronger and the effective range of repulsive force becomes broader. Since the baryon number current is proportional to the effect of ω meson, the baryon

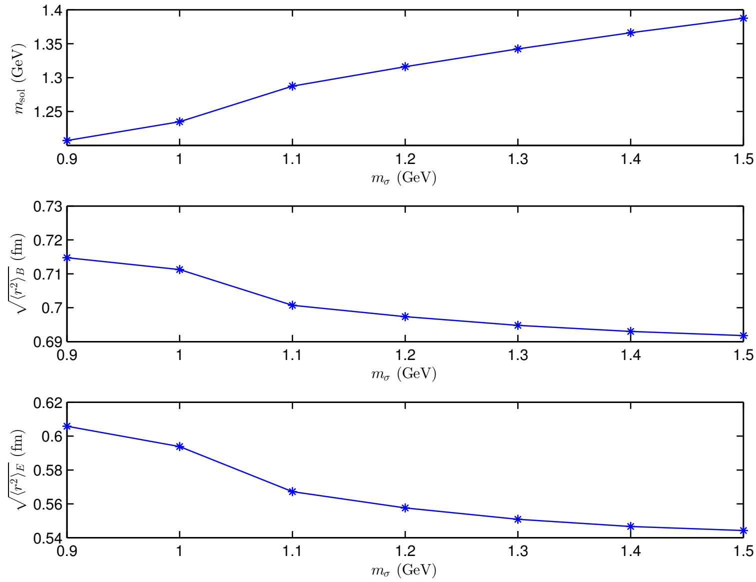

Fig. 1. m_σ dependence of the skyrmion properties.

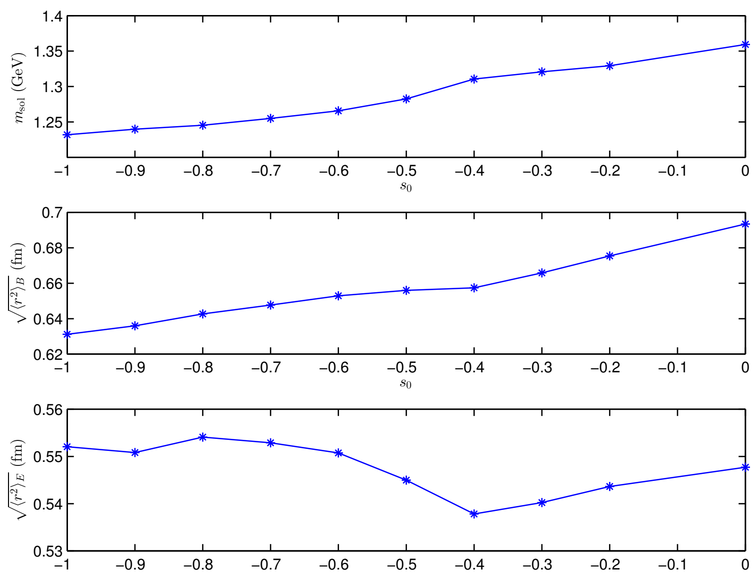

Fig. 2. s_0 dependence of the skyrmion properties.

number charge radius $\sqrt{\langle r^2 \rangle_B}$ becomes larger for larger s_0. The situation for charge radius of energy $\sqrt{\langle r^2 \rangle_E}$ is quite complex. In the present analysis, the numerical error is about 1–3%, so that the third graph in Fig. 2 implies that $\sqrt{\langle r^2 \rangle_E}$ is rather stable against the change of s_0. This implies that the contribution from ω works in an opposite direction to the one from σ, and that they cancel with each other.

We next investigate the effect of the scalar meson structure on skyrmion properties through the mixing angle between the two-quark and four-quark states. Here, we take $s_0 = 0$. Our results are illustrated in Fig. 3. The decreasing and increasing tendencies of Fig. 3 are consistent with the ones of Fig. 1. This is because, in our model, the two-quark state gives a dominant contribution from the scalar mesons. The increasing of mixing strength drops the "effective m_σ" for the two-quark component, therefore the tendencies for soliton mass and charge radii in Fig. 3 is consistent with those in Fig. 1.

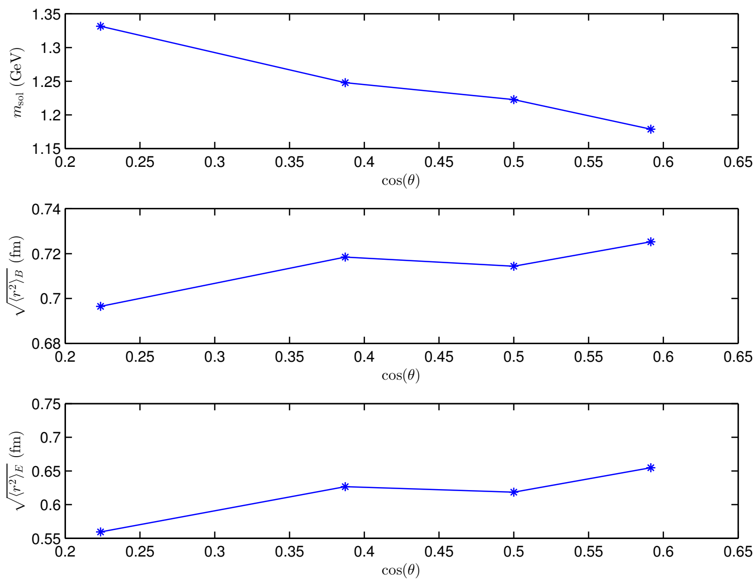

Fig. 3. Dependence of the skyrmion properties on the mixing angle cos(θ).

4. Discussions

In this work, we study the scalar meson effect on the properties of soliton in a Skyrme type model.

The results show the tendency of charge radii and soliton mass dependence on the way how to incorporate scalar meson contribution, which is also pointed out in Ref. [12, 13]. For this study, we show that the increase of m_σ reduces the effective range of attractive force, while the decrease of s_0 reduces the effective range of repulsive force. We also explore the skyrmion properties by considering the mixing structure of the scalar mesons. We find that, with the increases of two-quark component in the lightest scalar meson, soliton mass drops, while $\sqrt{\langle r^2\rangle_B}$ and $\sqrt{\langle r^2\rangle_E}$ become broader.

In this study, the skyrmion mass is not light enough to reproduce the proton mass, it seems the magnitude of attractive force made from scalar meson is not enough, or more specifically speaking, the balance between attractive force and repulsive force for this model do not allow for a lower soliton mass. In our model, the attractive force made by two scalar mesons is dominated by the two-quark component, a more direct contribution of attractive force made from four-quark component might be one way to reach the normal nucleon property. For the present analysis, there is no precursor way to determine the magnitude of s_0, however, this parameters might play a very important role in the dense matter case.

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