COLLECTIVE HAMILTONIAN AND ITS APPLICATIONS FOR CHIRAL AND WOBBLING MODES*

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The recent progresses of the collective Hamiltonian and its applications for chiral and wobbling modes are reviewed. In particular, the phenomenon of the multiple chiral doublet bands that are given by the collective Hamiltonian is introduced. In the investigation of the wobbling mode, the wobbling frequency as a function of the rotational frequency by the collective Hamiltonian in comparison with the harmonic frozen alignment approximation results for the longitudinal and transverse wobbling is discussed.

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1. Introduction

During the past decades, searching for stable triaxial shapes in nuclei has been a keen interest. The chirality [1] and wobbling motion [2] are the two unique fingerprints associated with stable triaxial shapes, therefore, they have attracted a lot of attention in nuclear physics. The corresponding experimental signals are, respectively, chiral doublet bands [1,3] and wobbling bands [2,4].

Experimentally, more than thirty candidate chiral doublet bands have been reported in the $A \approx 80$, 100, 130 and 190 mass regions; see, *e.g.*, Refs. [5–7]. The wobbling bands have been extensively identified in the triaxial strongly deformed (TSD) region around $N = 94$, such as in ^{161,163,165,167}Lu and ^{167}Ta; see, *e.g.*, Refs. [8,9]. Very recently, the first wobbling bands in the $A \sim 130$ mass region were reported in ^{135}Pr [10].

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Theoretically, the chirality and wobbling motion are mainly investigated by the triaxial particle rotor model (PRM) [11–16], the tilted axis cranking (TAC) model [10, 17, 18], and the random phase approximation (RPA) [19–22]. Their characteristics and applications for chirality and wobbling motion are summarized in Refs. [5, 9, 23, 24].

Recently, based on the tilted axis cranking model, a new approach, collective Hamiltonian, was proposed to describe the chirality and wobbling motion [9, 23]. This approach goes beyond the mean-filed approximation and could microscopically describe not only the yrast sequence but also the highly excited bands. Using this model, the chiral vibration and rotation modes have been successfully described [23]. The simple, longitudinal, and transverse wobblers [8] are systematically studied and the variation trends of wobbling frequency of these three types of wobblers are confirmed [9]. Very recently, this model was also extended to study the wobbling bands in ^{135}Pr [25].

In this proceeding, the recent progresses of the collective Hamiltonian and its applications for chiral and wobbling modes will be briefly reviewed.

2. Theoretical framework

Collective Hamiltonian is an effective tool for investigating various collective processes which involve small velocities [2]. The well-known Bohr Hamiltonian is aimed to describe the collective rotational and vibrational degrees of freedom with the five collective intrinsic variables β, γ, and Euler angles $\boldsymbol{\Omega}$ [2]. To describe the chiral and wobbling modes, the orientation of the nucleus in the rotating mean field is considered as the collective degree of freedom. The detail theoretical framework of the collective Hamiltonian is presented in Refs. [9,23]. In the present framework of the collective Hamiltonian, the azimuth angle of nucleus orientation φ is considered as the collective variable, the quantized form of the collective Hamiltonian is written as the sum of kinetic and potential terms

$$\hat{H}_{\mathrm{coll}} = \hat{T}_{\mathrm{kin}}(\varphi) + V(\varphi) = -\frac{\hbar^2}{2\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi}\frac{1}{\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi} + V(\varphi)\,, \tag{1}$$

in which the collective potential $V(\varphi)$ can be obtained by the TAC solutions, and the mass parameter can be either obtained by the cranking formula [23] or harmonic frozen alignment (HFA) approximation formula [9]. Solving this collective Hamiltonian by the diagonalization, the collective levels and the corresponding wave functions can be obtained.

3. Results and discussion

In the investigation of the chiral modes by the collective Hamiltonian [23], a system with one $h_{11/2}$ proton particle and one $h_{11/2}$ neutron hole coupled to a triaxial rigid rotor with $\gamma = -30°$ is considered. The moment of inertia is chosen as $\mathcal{J}_0 = 40\ \hbar^2/\mathrm{MeV}$. The collective Hamiltonian is constructed by Eq. (1) with the collective potential and the mass parameter obtained from TAC calculations. The diagonalization of the collective Hamiltonian yields the energy levels and wave functions for each cranking frequency.

In Fig. 1, the six lowest energy levels, labeled 1–6, obtained from the collective Hamiltonian are shown together with the potential energy $V(\varphi)$. It is seen that with the increasing frequency, the three pairs of energy levels, *i.e.*, levels 1 and 2, levels 3 and 4 as well as levels 5 and 6, become close to each other. For example, the energy difference between levels 1 and 2 decreases from 1.19 MeV at $\hbar\omega = 0.25$ MeV to 0.01 MeV at $\hbar\omega = 0.50$ MeV. This is because the potential barrier becomes higher and wider with the increase of cranking frequency, the tunneling penetration probability is more and more suppressed. The appearance of multiple degenerate energy levels is the signal of the existence of multiple chiral doublet bands (MχD) with identical configuration [26–31]. This indicates that the phenomenon of the MχD is naturally obtained in the collective Hamiltonian, which will be an interesting topic in the future study. The energy spectra of the chiral doublet bands 1 and 2 obtained by the collective Hamiltonian are in comparison with the results calculated by the PRM. A good agreement between this two models is achieved. For details, see Ref. [23].

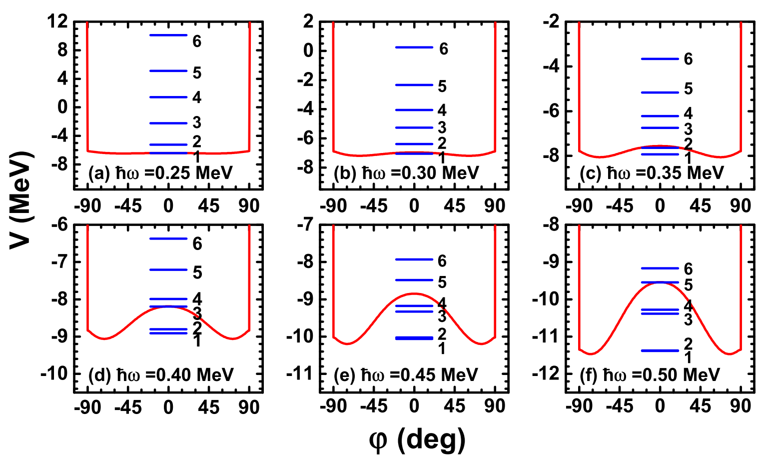

Fig. 1. (Color on-line) The six lowest energy levels, labeled 1–6, obtained from the collective Hamiltonian. The potential energy $V(\varphi)$ is included as well. Taken from Ref. [23].

In the investigation of the wobbling modes by the collective Hamiltonian [23], the simple, longitudinal, and transverse wobbling motions are studied. A triaxial rotor is considered as a simple wobbler [2]. Its angular momentum geometry was investigated very recently [16]. For a system of a high-j particle coupled to a triaxial rotor, the longitudinal (transverse) wobbler is achieved by arranging the orientation of quasiparticle angular momentum vector to be (perpendicular to) the axis with largest moments of inertia [8]. The collective Hamiltonian is also constructed by Eq. (1) but with the collective potential obtained from TAC model and the mass parameter obtained from HFA formula. The diagonalization of the collective Hamiltonian yields the wobbling energy and the corresponding wave functions.

In Fig. 2, the extracted wobbling frequencies of the longitudinal (left panel) and transverse (right panel) wobbling from the collective Hamiltonian are shown in comparison with those of the HFA approximation. It is found that both the collective Hamiltonian and HFA predict the similar trend of the wobbling frequency, *i.e.*, the wobbling frequency increases with the rotational frequency in longitudinal wobbling motions, while decreases in transverse wobbling motion. These variation trends can be understood from the stiffness of the collective potential in the collective Hamiltonian [9]. The HFA results are larger than the collective Hamiltonian ones in the longitudinal wobbling. This is due to the stiffness of the collective potential calculated by HFA that is larger than that in the collective Hamiltonian [9]. For the transverse wobbling, the HFA gives a more rapid decreasing trend than the collective Hamiltonian since the quantum fluctuations are not taken into account in the HFA beyond the region of transverse wobbling motion. The obtained wobbling energy spectra are compared with the results of the PRM. The results of the collective Hamiltonian are in a good agreement with those exact solutions by PRM. For details, see Ref. [9].

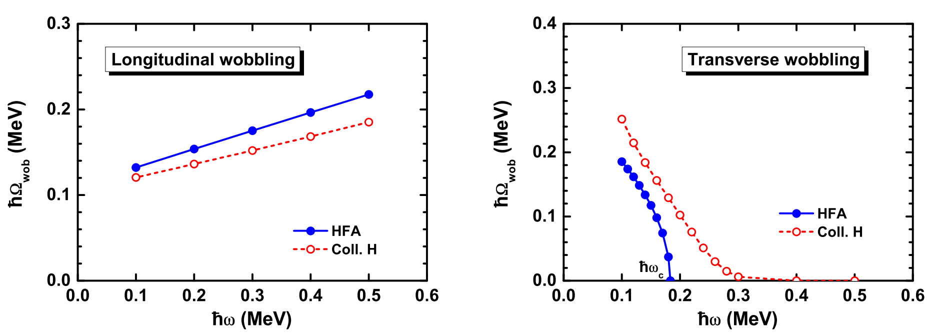

Fig. 2. (Color on-line) The wobbling frequencies $\hbar\Omega_{\mathrm{wob}}$ of the longitudinal (left panel) and transverse (right panel) wobbling as functions of the rotational frequency in comparison with those of the HFA approximation. Taken from Ref. [9].

The success of the collective Hamiltonian guarantees its applications for the newly observed wobbling bands in ^{135}Pr [10]. With the collective potential calculated by TAC model and the mass parameter by HFA formula, the collective Hamiltonian reproduces the experimental wobbling energy spectra and wobbling frequency very well. The variation trend of the wobbling frequency can be understood from the collective potential. The ability of the collective Hamiltonian in the description of realistic wobbling nuclei is thus confirmed. For details, see Ref. [25].

4. Summary

The recent progresses of the collective Hamiltonian based on the tilted axis cranking model and its applications for both chiral and wobbling modes are reviewed. For the chiral modes, the focused is concentrated at the introduction of the appearance of the nearly degenerate collective energy levels obtained by the collective Hamiltonian. For the wobbling modes, the variation trend of the wobbling frequency as a function of the rotational frequency obtained by the collective Hamiltonian in comparison with the HFA results is introduced for the longitudinal and transverse wobbling.

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