NEUTRINO MASS AND UNIQUE FORBIDDEN BETA DECAYS*

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A possibility to use the first, second, third and fourth unique forbidden β decays for the determination of the absolute mass of neutrinos is addressed. For selected nuclear systems with small Q value, the energy distribution of emitted electrons is presented. Calculations are based on the exact Dirac wave functions of the electron with finite nuclear size and the electron screening taken into account. It is shown that the Kurie plot near the endpoint is within a good linear accuracy in the limit of massless neutrinos like the Kurie plot of the superallowed β decay of tritium.

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1. Introduction

The discovery of neutrino oscillations, and hence, non-zero neutrino masses and mixing implies physics beyond the Standard Model. The neutrino oscillation data, accumulated over many years allowed to determine the parameters which drive the solar, atmospheric and reactor neutrino oscillations (2 mass squared differences and three angles) with a high precision.

Determining the absolute scale of neutrino masses, the type of neutrino mass spectrum, which can be either with normal or inverted ordering, the nature of massive neutrinos, and getting information about the Dirac and Majorana CP-violation phases in the neutrino mixing matrix, remain the most pressing and challenging problems of the future research in the field of neutrino physics.

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Neutrino oscillation experiments cannot tell us about the overall scale of neutrino masses. Terrestrial experiments such as tritium beta decay gives a constraint on the absolute neutrino mass scale [1–3]. The first measurement was performed by Hanna and Pontecorvo in 1949 [4]. Currently, from the Mainz and Troitsk experiments for the upper bound on the effective neutrino mass m_β, we have $m_\beta < 2.2$ eV [2]. The KATRIN experiment [5], which will start taking data soon, aims at reaching a sensitivity of 0.2 eV. Another promising way to determine absolute neutrino mass scale is to exploit the first unique forbidden β decay of ^{187}Re due to its low transition energy of 2.47 keV [6].

The aim of this contribution is to determine the Kurie plot near the endpoint in the case of the first, second, third and fourth unique forbidden β decays with low Q values, which might be used to measure the absolute neutrino mass scale.

2. Unique forbidden beta decays

In Table I, we present the unique forbidden β decays with $Q < m_e$ and half-life larger than 10 years. In what follows, we shall describe the theoretical spectral shape of emitted electrons.

TABLE I

The first, second, and fourth unique forbidden β decays with $T_{1/2} > 10$ yrs and $Q < m_e$ from the nuclear database. The Q values are from [7]. Both transitions to ground state with spin and parity J^π and first excited state J_1^π of final nucleus are considered.

Parent $(J_i^{\pi_i})$	Daughter $\left(J_f^{\pi_f}\right)$	ΔJ^π	Q value [keV]	$T_{1/2}$ [yrs]
^{48}Ca(0^+)	^{48}Sc(5_1^+)	5^+	151.1	—
^{60}Fe(0^+)	^{40}Co(5^+)	5^+	237	—
^{79}Se$(7/2^+)$	^{79}Br$(3/2^-)$	2^-	151	—
^{93}Zr$(5/2^+)$	^{93}Nb$(1/2_1^-)$	2^-	60	1.61×10^6
^{99}Tc$(9/2^+)$	^{99}Ru$(3/2_1^+)$	3^+	204	2.111×10^5
^{107}Pd$(5/2^+)$	^{107}Ag$(1/2^-)$	2^-	34.1	6.5×10^6
^{115}In$(9/2^+)$	^{115}Sn$(3/2_1^+)$	3^+	0.189	4.41×10^{14}
^{129}I$(7/2^+)$	^{129}Xe$(1/2^+)$	3^+	194	—
^{135}Cs$(7/2^+)$	^{135}Ba$(1/2_1^+)$	3^+	47.76	—
^{135}Cs$(7/2^+)$	^{135}Ba$(11/2_1^-)$	2^-	0.5	—
^{138}La(5^+)	^{138}Ce(2_1^+)	3^+	255.3	1.02×10^{11}
^{182}Hf(0^+)	^{182}Ta(2^-)	2^-	104.6	8.9×10^6
^{187}Re$(5/2^+)$	^{187}Os$(1/2^-)$	2^-	2.469	4.33×10^{10}

The differential decay rates of the first ($l = 2$), second ($l = 3$), third ($l = 4$), and fourth ($l = 5$) unique forbidden β transitions can be written in a compact form as follows:

$$\frac{d\Gamma}{dE_e} = \frac{2}{\pi^2} G_{\mathrm{F}}^2 V_{ud}^2 \sum_j p_e E_e p_\nu E_\nu |U_{ej}|^2 \Theta(E_0 - E_e - m_j) \frac{g_A^2}{2J_i + 1} \times \sum_{I=1}^{2l} c_I \left| \left\langle J_f^{\pi_f} \middle\| \sum_n \tau_n^+ T_I(E_e, r_n) \{\sigma_1(n) \otimes Y_{l-1}(n)\}_l \middle\| J_i^{\pi_i} \right\rangle \right|^2 , \quad (1)$$

where

$$\begin{aligned} T_{2k-1}(E_e, r_n) &= g_{-k}(E_e, r_n) j_{l-k}(p_\nu r_n) , \\ T_{2k}(E_e, r_n) &= f_{+k}(E_e, r_n) j_{l-k}(p_\nu r_n) . \end{aligned} \quad (2)$$

Here, l and c_I are given in Table II and $k = 1, \ldots, l$. G_{F} stands for the Fermi constant. V_{ud} is the element of the Cabibbo–Kobayashi–Maskawa matrix which is assumed to be real, *i.e.* no CP violation in the quark sector is assumed here. p_e, E_e, and E_0 are the momentum, energy, and maximal endpoint energy (in the case of zero neutrino mass) of the electron, respectively. Neutrino energy and momentum are $E_\nu = E_0 - E_e$ and $p_\nu = \sqrt{(E_0 - E_e)^2 - m_j^2}$. U_{ej} and m_j are the element of neutrino mixing matrix and the neutrino mass, respectively. $\Theta(x)$ is a theta (step) function. g_A denotes an axial-vector coupling constant and r_n is the position of the n^{th} nucleon. The radial electron wave functions $g_{-k}(E_e, r_n)$ and $f_k(E_e, r_n)$ satisfy the radial Dirac equations with the potential of Coulomb field of daughter nucleus distorted by the screening potential of the electrons of the

TABLE II

Coefficients c_l and l entering the decay rate of the first, second, third, and fourth unique forbidden β transitions given in Eq. (1). l corresponds to the change of angular momenta between the initial and final nuclei.

Unique forbidden β decay	$l = \Delta J$	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
first	$l = 2$	1	1	1	1						
second	$l = 3$	1	1	$\frac{6}{5}$	$\frac{6}{5}$	1	1				
third	$l = 4$	1	1	$\frac{9}{7}$	$\frac{9}{7}$	$\frac{9}{7}$	$\frac{9}{7}$	1	1		
fourth	$l = 5$	1	1	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{10}{7}$	$\frac{10}{7}$	$\frac{4}{3}$	$\frac{4}{3}$	1	1

daughter atom. The electron radial wave functions are evaluated by means of the subroutine package RADIAL [8]. In the case of neutrino, radial wave functions can be expressed with the help of the spherical Bessel functions $j_{l-k}(p_\nu r_n)$.

It is known that mostly nucleons close to the nuclear surface undergo β-decay transition. This fact allows to simplify the calculation of differential decay rate in Eq. (1) by evaluating the electron wave function at $r_n = R$, where R is the nuclear radius ($R = 1.2A^{1/3}$ fm). By keeping the first term in the Taylor expansion of spherical Bessel functions, we get

$$\frac{d\Gamma}{dE_e} = \frac{2}{\pi^2} G_{\rm F}^2 V_{ud}^2 \sum_j p_e E_e p_\nu E_\nu |U_{ej}|^2 \theta(E_0 - E_e - m_j) \mathcal{B}_A \mathcal{G}_A(E_e, E_\nu, R)\,, \quad (3)$$

where strength functions $\mathcal{B}_A$ and the functions $\mathcal{G}_A(E_e, E_\nu, R)$ for the first ($A = 2^-$), second ($A = 3^+$), and fourth ($A = 5^+$) unique forbidden β transitions are given by

$$\begin{aligned}
\mathcal{B}_{2^-} &= \frac{g_A^2}{2J_i+1} \left| \left\langle J_f^{\pi_f} \right|\right| \sum_n \frac{r_n}{R} \tau_n^+ \{\sigma_1(n) \otimes Y_1(n)\}_2 \left|\left| J_i^{\pi_i} \right\rangle \right|^2 , \\
\mathcal{B}_{3^+} &= \frac{g_A^2}{2J_i+1} \left| \left\langle J_f^{\pi_f} \right|\right| \sum_n \frac{r_n^2}{R^2} \tau_n^+ \{\sigma_1(n) \otimes Y_2(n)\}_3 \left|\left| J_i^{\pi_i} \right\rangle \right|^2 , \\
\mathcal{B}_{5^+} &= \frac{g_A^2}{2J_i+1} \left| \left\langle J_f^{\pi_f} \right. \right. \| \sum_n \frac{r_n^4}{R^4} \tau_n^+ \{\sigma_1(n) \otimes Y_4(n)\}_5 \left|\left| J_i^{\pi_i} \right\rangle \right|^2 , \qquad (4)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{G}_{2^-}(E_e, E_\nu, R) &= \tfrac{1}{9} \left(F_{p_{3/2}}(E_e, r)(p_e R)^2 + F_{s_{1/2}}(E_e, r)(p_\nu R)^2 \right) , \\
\mathcal{G}_{3^+}(E_e, E_\nu, R) &= \tfrac{1}{45} \left(\tfrac{1}{5} F_{d_{5/2}}(E_e, r)(p_e R)^4 + \tfrac{2}{3} F_{p_{3/2}}(E_e, r)(p_e p_\nu)^2 R^4 \right. \\
&\quad \left. + \tfrac{1}{5} F_{s_{1/2}}(E_e, r)(p_\nu R)^4 \right) , \\
\mathcal{G}_{5^+}(E_e, E_\nu, R) &= \tfrac{1}{945^2} \left(F_{g_{9/2}}(E_e, r)(p_e R)^8 + 12 F_{f_{7/2}}(E_e, r) p_e^6 p_\nu^2 R^8 \right. \\
&\quad + \tfrac{126}{5} F_{d_{5/2}}(E_e, r)(p_e p_\nu)^4 R^8 + 12 F_{p_{3/2}}(E_e, r) p_e^2 p_\nu^6 R^8 \\
&\quad \left. + F_{s_{1/2}}(E_e, r)(p_\nu R)^8 \right) . \qquad (5)
\end{aligned}$$

Here, the Fermi function associated with the emission of the electron in the $s_{1/2}$, $p_{3/2}$, $d_{5/2}$, $f_{7/2}$, and $g_{9/2}$-states is defined as

$$
\begin{aligned}
F_{s_{1/2}}(E_e,R) &= g_{-1}^2(E_e,R)+f_{+1}^2(E_e,R)\,,\\
F_{p_{3/2}}(E_e,R) &= \frac{g_{-2}^2(E_e,R)+f_{+2}^2(E_e,R)}{(p_eR)^2/3^2}\,,\\
F_{d_{5/2}}(E_e,R) &= \frac{g_{-3}^2(E_e,R)+f_{+3}^2(E_e,R)}{(p_eR)^4/15^2}\,,\\
F_{f_{7/2}}(E_e,R) &= \frac{g_{-4}^2(E_e,R)+f_{+4}^2(E_e,R)}{(p_eR)^6/105^2}\,,\\
F_{g_{9/2}}(E_e,R) &= \frac{g_{-5}^2(E_e,R)+f_{+5}^2(E_e,R)}{(p_eR)^8/945^2}\,.
\end{aligned}
\tag{6}
$$

The current upper limit on neutrino mass from tritium β decay holds in the degenerate neutrino mass region, *i.e.* $m_1 \simeq m_2 \simeq m_3 \simeq m_\beta = \sum_{j=1}^3 |U_{ej}|^2 m_j$. Therefore, we substitute the effective neutrino mass m_β for all the neutrino masses m_i $(i=1,2,3)$.

The Kurie functions for the first, second, and fourth unique forbidden β decay are given by

$$
\begin{aligned}
K_{2^-}(E_e,m_\beta) &= \sqrt{\frac{d\Gamma/dE_e}{p_eE_e\left(F_{p_{3/2}}(E_e,r)(p_eR)^2/3^2\right)}}\\
&= G_{\mathrm F}V_{ud}g_A\sqrt{\frac{2}{\pi^2}\mathcal{B}_{2^-}}(E_0-E_e)\sqrt[4]{1-\frac{m_\beta^2}{(E_0-E_e)^2}}\sqrt{S_{2^-}(E_e)}\,,\\
K_{3^+}(E_e,m_\beta) &= \sqrt{\frac{d\Gamma/dE_e}{p_eE_e\left(F_{d_{5/2}}(E_e,r)(p_eR)^4/15^2\right)}}\\
&= G_{\mathrm F}V_{ud}g_A\sqrt{\frac{2}{\pi^2}\mathcal{B}_{3^+}}(E_0-E_e)\sqrt[4]{1-\frac{m_\beta^2}{(E_0-E_e)^2}}\sqrt{S_{3^+}(E_e)}\,,\\
K_{5^+}(E_e,m_\beta) &= \sqrt{\frac{d\Gamma/dE_e}{p_eE_e\left(F_{g_{9/2}}(E_e,r)(p_eR)^8/945^2\right)}}\\
&= G_{\mathrm F}V_{ud}g_A\sqrt{\frac{2}{\pi^2}\mathcal{B}_{5^+}}(E_0-E_e)\sqrt[4]{1-\frac{m_\beta^2}{(E_0-E_e)^2}}\sqrt{S_{5^+}(E_e)}\,.
\end{aligned}
\tag{7}
$$

The corresponding shape factor $S_A(E_e)$ takes the form

$$\begin{aligned} S_{2^-}(E_e) &= \left(1 + \frac{F_{s_{1/2}}(E_e,r)p_\nu^2}{F_{p_{3/2}}(E_e,r)p_e^2}\right), \\ S_{3^+}(E_e) &= \left(1 + \frac{2/3F_{p_{3/2}}(E_e,r)(p_e p_\nu)^2}{1/5F_{d_{5/2}}(E_e,r)p_e^4} + \frac{F_{s_{1/2}}(E_e,r)p_\nu^4}{F_{d_{5/2}}(E_e,r)p_e^4}\right), \\ S_{5^+}(E_e) &= \left(1 + \frac{12F_{f_{7/2}}(E_e,r)p_e^6 p_\nu^2}{F_{g_{9/2}}(E_e,r)p_e^8} + \frac{126/5F_{d_{5/2}}(E_e,r)(p_e p_\nu)^4}{F_{g_{9/2}}(E_e,r)p_e^8}\right. \\ &\quad \left. + \frac{12F_{p_{3/2}}(E_e,r)p_e^2 p_\nu^6}{F_{g_{9/2}}(E_e,r)p_e^8} + \frac{F_{s_{1/2}}(E_e,r)p_\nu^8}{F_{g_{9/2}}(E_e,r)p_e^8}\right). \end{aligned} \tag{8}$$

In Fig. 1, the shape factors $S_A(E_e)$ for nuclear transition presented in Table I are plotted as a function of the electron energy in the region up to 10 keV before the endpoint. We notice a very small deviation from the unity, which is below 1%. In the case of β decay of ^{115}In, ^{135}Cs, and ^{187}Re with Q value below 3 keV (not plotted in Fig. 1) this correction factor is even significantly smaller.

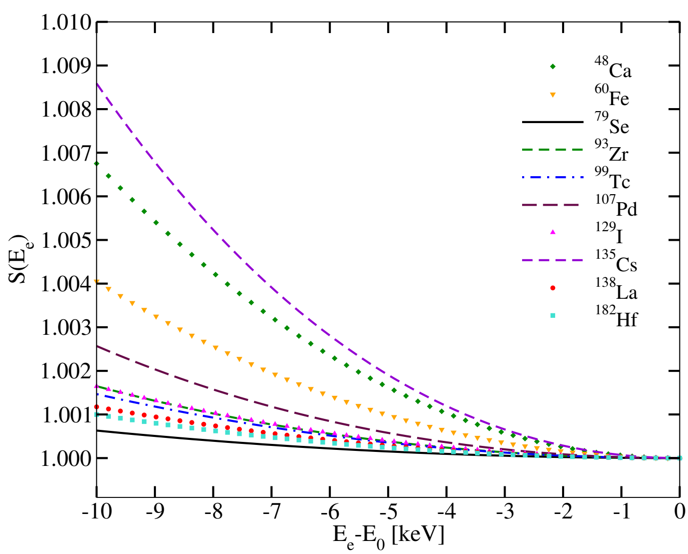

Fig. 1. Shape factor $S_A(E_e)$ *versus* the electron energy E_e close to the endpoint E_0 for selected unique forbidden β decays.

If we adopt an approximation $S_A(E_e) \sim 1$ close to the endpoint, for the Kurie function K_A for the first, second, and fourth unique forbidden β decays ($A = 2^-$, 3^+, and 5^+), we obtain

$$K_A(E_e, m_\beta) \cong G_{\mathrm{F}} V_{ud} g_A \sqrt{\frac{2}{\pi^2} \mathcal{B}_A} (E_0 - E_e) \sqrt[4]{1 - \frac{m_\beta^2}{(E_0 - E_e)^2}} \,. \tag{9}$$

We note that K_A is linear near the endpoint for $m_\beta = 0$.

3. Conclusions

Till now, the kinematical measurement of the neutrino mass has been performed in the laboratory by taking the advantage of β decay of tritium and ^{187}Re. We argue that this goal can be addressed also with the unique forbidden β decays, if one could reach sufficient statistics in a real experiment. It is shown that if the Q value of such a transition is below the mass of the electron, the Kurie function for the unique forbidden β decays close to the endpoint coincides up to a factor to the Kurie function of superallowed β decay of tritium having the same dependence on the effective neutrino mass m_β.

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