PAIRING PROPERTIES OF THE N2LO SKYRME PSEUDO-POTENTIAL*

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We present the first study of the pairing properties of the Skyrme pseudo-potential including higher order gradient terms in cold symmetric nuclear matter.

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1. Introduction

The nuclear energy density functional (NEDF) theory [1] based on non-relativistic Skyrme functionals [2] has been successful in describing properties of the atomic nucleus such as ground state binding energies and radii [3], and excited states as giant resonances [4] or low-lying excitations [5].

To describe the pairing correlations of atomic nuclei [6] in the particle–particle (*pp*) channel, most of the Skyrme functionals adopt a very simple functional form [7] which is not related to the one used in the particle-hole (*ph*) channel. Using a different functional form for the *ph* and *pp* channels poses serious issues in configuration mixing calculations [8], in particular giving rise to self-interaction problems which are very difficult to handle [9].

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A way to overcome such a problem is to use the same effective interaction in both channels. Although this is the case of other effective interactions [10, 11], it is quite uncommon for the Skyrme interaction. As an example, we refer to the work of Refs. [12,13] for more details.

In this article, we present a preliminary study of the pairing properties of the extended Skyrme interaction [14–16], including higher order derivative terms in symmetric nuclear matter (SNM). Although SNM can be considered as an ideal model, it gives an insight and some very useful informations concerning some basic behaviour of the effective interaction.

The article is organised as follows: in Sec. 2, we present the Skyrme interaction with higher order gradients and we discuss some properties of the residual pairing interaction. In Sec. 3, we present the basic BCS equations used in this work and we discuss our results. Finally, we give our conclusions in Sec. 4.

2. The Skyrme pseudo-potential

The most general form of the Skyrme functional up to 6^{th} order in the gradient expansion has been derived by [17]. In the present article, we prefer to relate this functional to an effective interaction, thus reducing the number of free coupling constants, as shown in [15]. The corresponding N3LO Skyrme interaction reads [14,16]

$$\begin{aligned} v_{\text{Sk}} \;=\; & t_0^{(0)}\left(1+x_0^{(0)}P_\sigma\right)+\tfrac{1}{2}t^{(2)}{}_1\left(1+x_1^{(2)}P_\sigma\right)\left[\boldsymbol{k'}^2+\boldsymbol{k}^2\right] \\ & +t_2^{(2)}\left(1+x_2^{(2)}P_\sigma\right)\boldsymbol{k'}\cdot\boldsymbol{k}+\tfrac{1}{6}t_3^{(0)}\left(1+x_3^{(0)}P_\sigma\right)n^\alpha(\boldsymbol{R}) \\ & +\tfrac{1}{4}t_1^{(4)}\left(1+x_1^{(4)}P_\sigma\right)\left[\left(\boldsymbol{k'}^2+\boldsymbol{k}^2\right)^2+4\left(\boldsymbol{k'}\cdot\boldsymbol{k}\right)^2\right] \\ & +t_2^{(4)}\left(1+x_2^{(4)}P_\sigma\right)\left(\boldsymbol{k'}\cdot\boldsymbol{k}\right)\left(\boldsymbol{k'}^2+\mathbf{k}^2\right) \\ & +\tfrac{1}{2}t_1^{(6)}\left(1+x_1^{(6)}P_\sigma\right)\left(\boldsymbol{k'}^2+\boldsymbol{k}^2\right)\left[\left(\boldsymbol{k'}^2+\boldsymbol{k}^2\right)^2+12\left(\boldsymbol{k'}\cdot\boldsymbol{k}\right)^2\right] \\ & +t_2^{(6)}\left(1+x_2^{(6)}P_\sigma\right)\left(\boldsymbol{k'}\cdot\boldsymbol{k}\right)\left[3\left(\boldsymbol{k'}+\boldsymbol{k}^2\right)^2+4\left(\boldsymbol{k'}\cdot\boldsymbol{k}\right)^2\right]. \end{aligned} \tag{1}$$

The notations used here are standard and more details can be found in Ref. [1]. The spin-orbit and tensor terms are here discarded since they do not contribute to the equation of state (EoS), although they do contribute to its multipolar partial wave decomposition, as shown in [18]. The parameters of the Skyrme interaction have been fitted on the results of the Brueckner–Hartree–Fock (BHF) calculations of the Catania group [20] for infinite symmetric nuclear matter. In the present article, we restrict ourselves to the parametrisation named VLyB62 which contains only gradients

up to 4^{th} order [21], whose parameters are given in Table I. More details concerning the N2LO functional can be found in Ref. [22].

TABLE I

The parameters of the central part of the VLyB62 pseudo-potential.

$\alpha = 1/6$, $t_3 = 13763$ MeV fm$^{3(1+\alpha)}$, $x_3 = 1$			
n	i	$t_i^{(n)}$ [MeV fm^{3+n}]	$x_i^{(n)}$
0	0	−2394.15	0.632433
2	1	−194.381	35.182
2	2	513.2670	−1.01914
4	1	9.63577	3.65615
4	2	−654.3664	−1.22006

Furthermore, in the present article, we consider the pairing phenomena between particles of the same species (*i.e.* the two particles are coupled to total isospin $T = 1$), and we limit our study to the case of total angular momentum of the pair $J = 0$ [6], which is the most favourable coupling in the atomic nuclei. In this case, the available couplings are [23]: ${}^1S_0, {}^3P_1, {}^1D_2, {}^3F_3$, where we have adopted the standard spectroscopic notation ${}^{2S+1}L_J$. We refer to Ref. [24] for a more detailed discussion. According to the results of [23], the ${}^1S_0$ channel is the dominant coupling for the description of nuclear superfluidity and we thus limit our analysis to this case. By using the partial wave decomposition of the Skyrme interaction [18], we can write immediately the pairing matrix element of residual interaction in the ${}^1S_0$ channels as

$$\begin{aligned} v(\boldsymbol{k},\boldsymbol{p}) \;=\; & t_0(1-x_0) + \tfrac{1}{6}t_3(1-x_3)\rho^\alpha + \tfrac{1}{2}t_1(1-x_1)\left(k^2+p^2\right) \\ & + \tfrac{1}{12}t_1^{(4)}\left(1-x_1^{(4)}\right)\left(3k^4+10k^2p^2+3p^4\right) \\ & + \tfrac{1}{2}t_1^{(6)}\left(1-x_1^{(6)}\right)\left(k^6+7k^4p^2+7k^2p^4+p^6\right) . \end{aligned} \tag{2}$$

For the particular parametrization used here, the density-dependent term does not contribute to pairing since $x_3 = 1$.

In Fig. 1, we compare the diagonal matrix elements of the VLyB62 interaction and the Gogny D1S [11] as a function of the momentum k. Due to its finite-range nature, the matrix elements of the Gogny interaction vanish at $k \approx 1.5$ fm^{-1}, while the ones of VLyB62 go to infinity since the zero-range nature of the interaction does not provide any natural cut-off on the matrix elements. It follows that we have to introduce a cut-off E_c to avoid divergences (see the next section). In the interval $k \leq 0.7$ fm^{-1}, both interactions have the same shape, with a nearly constant shift between them.

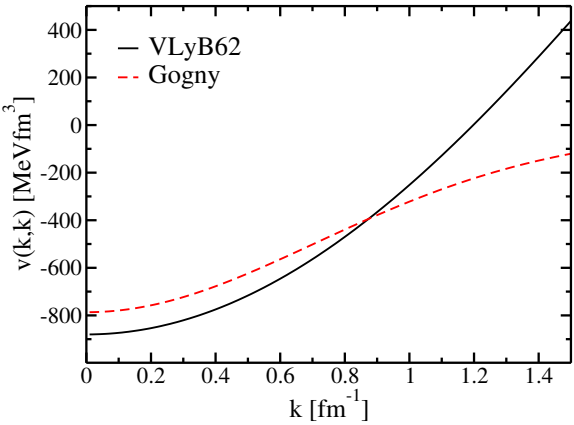

Fig. 1. (Colour on-line) Diagonal matrix elements of the VLyB62 and Gogny D1S interaction as a function of the momentum k. See the text for details.

3. BCS equations

The BCS equations in the infinite nuclear medium read [19]

$$\Delta_{\boldsymbol{k}} = -\sum_{\boldsymbol{p}\geq 0} v(\boldsymbol{k},\boldsymbol{p}) \frac{\Delta_{\boldsymbol{p}}}{2E_{\boldsymbol{p}}}\,, \tag{3}$$

$$N = 2\sum_{\boldsymbol{p}\geq 0} v_{\boldsymbol{p}}^2\,. \tag{4}$$

These two coupled equations are usually referred to the *gap* and *number* equations [6], respectively. $\Delta_{\boldsymbol{k}}$ is the so-called pairing gap. For simplicity, we have dropped the isospin index since we do not mix neutrons and protons. The occupation factor $v_{\boldsymbol{k}}^2$ and the quasi-particle energy $E_{\boldsymbol{k}}$ are related by the equations

$$v_{\boldsymbol{k}}^2 = \frac{1}{2}\left[1 - \frac{\varepsilon_{\boldsymbol{k}} - \mu}{E_{\boldsymbol{k}}}\right], \tag{5}$$

$$E_{\boldsymbol{k}} = \sqrt{(\varepsilon_{\boldsymbol{k}} - \mu)^2 + \Delta_{\boldsymbol{k}}^2}\,, \tag{6}$$

where μ is the chemical potential, which is obtained by keeping constant the average number of particle (per unit volume) along the calculations. $\varepsilon_{\boldsymbol{k}}$ corresponds to the Hartree–Fock (HF) single particle spectrum and it is defined as

$$\varepsilon_{\boldsymbol{k}} = \frac{\hbar^2 \boldsymbol{k}^2}{2m} + U(\boldsymbol{k})\,. \tag{7}$$

For simplicity, we consider the free spectrum, that is $U(\boldsymbol{k}) = 0$. The use of a contact interaction for the pairing channel implies that the BCS equations diverge [25], and a cut-off parameter is required to overcome the problem. Following Ref. [26], we solve the BCS equations by restricting the integration area to

$$k_{\min} = \sqrt{k_{\mathrm{F}} - \frac{2m}{\hbar^2} E_{\mathrm{c}}}\,, \tag{8}$$

$$k_{\max} = \sqrt{k_{\mathrm{F}} + \frac{2m}{\hbar^2} E_{\mathrm{c}}}\,. \tag{9}$$

In Fig. 2, we show the value of the pairing gap at the Fermi surface as obtained by Eq. (3) as a function of the density of the system. Since the choice of the cut-off parameter is arbitrary, we present the results as a function of different values of E_{c} within a range of values which are commonly

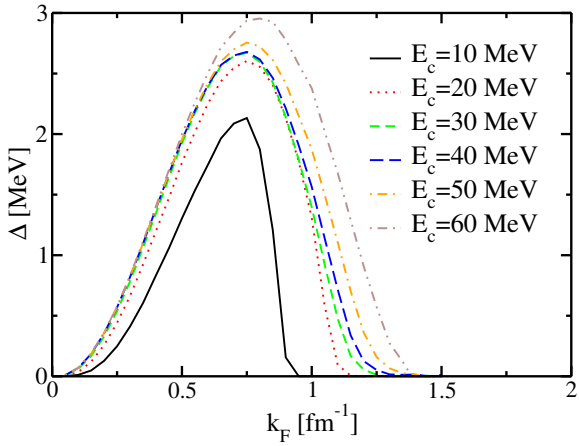

Fig. 2. (Colour on-line) Pairing gap at the Fermi surface as a function of the density of the system for the VLyB62 interaction and different values of the cut-off parameter E_{c}.

used for nuclear structure calculations in finite nuclei (see Ref. [27] for details). We leave the possibility of introducing a regularisation procedure as done in Ref. [25] in a future work.

We have observed that for values of $E_c \geq 60$ MeV, the pairing gap is suppressed. This can be understood since we include in the BCS equation strongly repulsive matrix elements (see Fig. 1). The magnitude of the pairing gap is not very sensitive to the choice of the cut-off parameter within a range of $E_c \in [20–50]$ MeV. The authors of Ref. [26] have shown that the pairing gap obtained using several Skyrme interactions increases with the cut-off up to a *plateau* where it stays rather constant despite the increase of the available space. Beyond such an interval, the calculation starts to diverge. The position of the *plateau* changes with the interaction, but in the case of the VLyB62, it is located in a region suitable for calculations in finite nuclei.

In Fig. 3, we show for the VLyB62 interaction with $E_c = 40$ MeV the evolution of the pairing gap Δ_k as a function of the momentum k. Contrary to the standard case of a simple contact interaction [7], the pairing gap has an explicit dependence on the momentum k. The pairing gap is attractive in the low-momentum region up to $k \approx 1.3$ fm^{-1}, then it becomes repulsive.

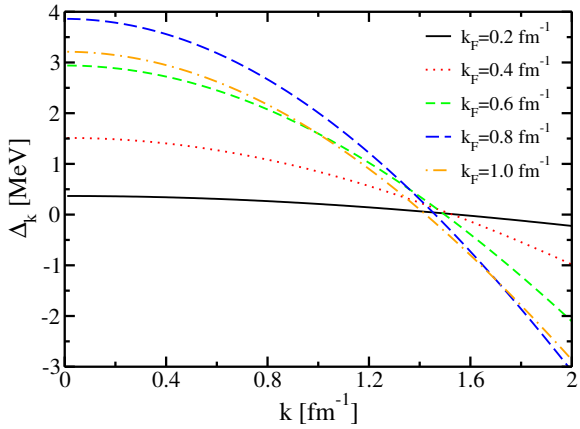

Fig. 3. (Colour on-line) Pairing matrix elements Δ_k as a function of the momentum k. See the text for details.

4. Conclusions

We have studied the pairing properties of the extended Skyrme interaction. By generalising the work done in Ref. [26], we have calculated the matrix element in the $^1S_0$ channel for the Skyrme N3LO interaction [21]. We have solved the BCS equations in SNM and for the free single particle spectrum. Due to the ultraviolet divergence [25], we have studied the

dependence of our results on the value of the cut-off parameter E_c. We have observed that the VLyB62 interaction gives reasonable pairing gaps compatible with the existing literature for several values of the cut-off parameter. Since the interaction has not been explicitly constrained in the pairing channel, we can consider such results quite encouraging, although a more detailed investigation is necessary.

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