

ARE DEFORMED NUCLEI STIFF AGAINST QUADRUPOLE VIBRATIONS?*

J.F. SHARPEY-SCHAFFER

University of the Western Cape, Department of Physics
P/B X17, Bellville 7535, South Africa

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Over several decades doubt has been cast on the “ β vibration” interpretation of the first excited 0_2^+ rotational bands in deformed nuclei. Experimental evidence is presented to show that low-lying 0_2^+ bands in deformed nuclei are $2p$ – $2h$ seniority zero pairing isomers lowered into the pairing gap by configuration-dependent pairing. Doubts then arise about the interpretation of the lowest $K^\pi = 2^+$ rotational bands observed in all deformed nuclei as “ γ vibrations”. Experimental evidence for these $K^\pi = 2^+$ rotational bands existing as a consequence of the lack of axial symmetry is addressed. We are forced to conclude that “phonon excitation” models of even–even deformed nuclei are deeply flawed.

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1. Introduction

The degrees of freedom that determine the structure of nuclei at low energies have been a subject of controversy for many decades. With the wealth of experimental spectroscopic data that now exist throughout the nuclear chart, it is time to address some of the long-held assumptions that are generally treated as “gospel” in the textbooks. One of the basic degrees of freedom, that has this status, is the ability of both spherical and deformed nuclei to vibrate. In quadrupole deformed even–even nuclei, the lowest excited 0_2^+ has been taken to be a vibration along the axis of symmetry, a “ β vibration”. The lowest $K^\pi = 2^+$ band is identified as the orthogonal vibration of the nuclear shape perpendicular to the symmetry axis, a “ γ vibration”. As the 0_2^+ and $K^\pi = 2^+$ band heads are always found to be

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within the pairing gap of about 2.0 MeV, it is assumed that they must arise from collective motions of the nucleons, shape vibrations being the obvious candidates.

The liquid drop model of the nucleus had considerable success in the 1930s in giving a useful account of fission and a helpful picture of compound nuclear reactions. The vibrations of a spherical liquid drop had been studied long before the advent of quantum mechanics by Lord Raleigh [1]. In 1879, by considering an inviscid incompressible liquid sphere, he obtained

$$\omega^2 = \frac{(\lambda - 1)\lambda(\lambda + 2)\gamma}{\rho R^3}, \quad (1)$$

where ω is the frequency of the λ pole oscillation of the drop, γ is the surface energy per unit area due to the surface tension, ρ is the density of the liquid and R is the radius of the spherical drop. That $\omega^2 \propto \gamma/\rho R^3$ is easily obtained by elementary dimensional analysis. The application of this to a charged spherical nucleus, also assuming no viscosity and irrotational flow, gives

$$\omega^2 = \frac{(\lambda - 1)\lambda(\lambda + 2)}{3} \frac{C_s}{R_A^2 m A} - \frac{2(\lambda - 1)\lambda}{2\lambda + 1} \frac{e^2 Z^2}{4\pi\epsilon_0 R_A^3 m_p A^2}, \quad (2)$$

where the radius of the nucleus $R_0 = R_A A^{1/3}$, C_s is the surface energy term in the Weizsäcker mass formula ~ 18 MeV [2], m is the nucleon mass, A is the atomic mass number, the second term is due to a uniform charge Ze^+ spread throughout the nucleus and m_p is the mass of the proton. The second term has little effect on ω for nuclei with $Z \ll 90$. For $A = 150$, the Rayleigh term gives $E_x = \hbar\omega \approx 2.4$ MeV.

However, we know from the Strutinski [3, 4] shell corrections to the liquid drop that the shell corrections oscillate much more rapidly than the liquid drop energy with increasing quadrupole deformation. Hence, for any deformed nucleus, the potential it sees, with respect to quadrupole deformation, will be more constrained than the liquid drop which will increase any vibrational frequency and the Rayleigh vibrational excitation energy. But we also know that the moments-of-inertia $\mathcal{I}$, deduced from the measured quadrupole moments and the energy spacing of rotational bands, is between the irrotational and the rigid body values. The increase of $\mathcal{I}$ from irrotational towards rigid will surely increase any vibrational frequency of the shape. Doubts about the identification of 0_2^+ states with β vibrations have continued for several decades. An authoritative review of the properties of these states has been made by Garrett [5] who concludes that *... most of the 0_2^+ states are not β vibrations. (...) microscopic calculations of 0_2^+ states underscores the need to consider the role of pairing in the description of these states.* So what are these states?

2. Key experimental data

The $B(E2)$ transition rates from 0_2^+ bands were comprehensively reviewed by Garrett [5] and will not be repeated here. In their iconic textbook, Bohr and Mottelson are clear about the meaning of a vibration: *A vibrational mode of excitation is characterized by the property that it can be repeated a large number of times. The n th excited state of a specified mode can thus be viewed as consisting of n individual quanta. The quanta obey Bose statistics ...* [6] (page 330). Bohr and Mottelson themselves point out [7] that the $0_{2,3}^+$ states in ^{168}Er are not “ β vibrations”. A very beautiful experiment by Kulp *et al.* [8] clearly demonstrates that there are no candidate two phonon 0_3^+ states in the $N = 90$ nucleus ^{152}Sm . The 0_2^+ states themselves, in the $N = 88$ and 90 nuclei, lie at low excitation energies of less than 900 keV. They are populated strongly in two-neutron transfer reactions when the transfer is across $N = 89$. An example of two particle transfer is given by Shahabuddin *et al.* [9] for the $^{152}\text{Gd}(t, p)^{154}\text{Gd}$ reaction. The 0_2^+ state is very strongly populated and the 0_3^+ state and 0_4^+ states are also fairly strongly populated. A similar 0_2^+ state in ^{150}Sm is only very weakly populated in the $^{148}\text{Nd}(3\text{He}, n)^{150}\text{Sm}$ two-proton transfer reaction [10]. This demonstrates that these 0_n^+ states near $N = 90$ have significant amounts of paired two neutrons outside the target core in their wavefunctions. Unfortunately, two nucleon stripping only gives information on the spin and parity of the final state and not on the specific single particle orbital involved. Single particle transfer does not populate these 0_2^+ states with sufficient strength to give any information on the angular momentum of the transferred nucleon.

However, the intrinsic configuration of any core excitation of an even-even nucleus can be coupled to by the odd nucleon, or nucleon hole, in the neighbouring odd nuclei as long as that nucleon is not Pauli blocked by the core excitation having a time-reversed pair of nucleons in the same orbital. We, therefore, have to look for the orbital that does not couple to the core excitation. Schmidt *et al.* [11] have made a very comprehensive study of the low-spin states of ^{155}Gd using the (n, γ) reaction and (d, p) and (d, t) neutron transfer reactions. Candidates for the coupling of the low- K orbitals to the ^{154}Gd 0_2^+ state are found. Their assignment of the 592 keV level as the $[521]3/2^-$ neutron coupled to the 0_2^+ state in ^{154}Gd is elegantly supported by the data from the $^{157}\text{Gd}(p, t)^{155}\text{Gd}$ reaction [12]. Our $^{154}\text{Sm}(\alpha, 3n)^{155}\text{Gd}$ experiment at iThemba LABS using the AFRODITE γ -ray spectrometer gives a very complete decay scheme for the lower- to mid-spin states in ^{155}Gd . If the $[505]11/2^-$ neutron can couple to the 0_2^+ state in ^{154}Gd , then there should be an $11/2^-$ level at about 802 keV excitation energy, 681 keV above the $[505]11/2^-$ isomer. We find no sign of such a state or of the rotational band expected to be built upon it [13]. This indicates that the core 0_2^+ state in ^{154}Gd has two $[505]11/2^-$ neutrons as a major component

of its configuration. The $[505]11/2^-$ neutron is Pauli blocked from coupling to the core 0_2^+ state. Clearly the same blocking should occur for all the $[505]11/2^-$ neutrons outside even-even cores in the $N = 88$ and 90 nuclei. Indeed, this blocking situation exists in all other neighbouring odd neutron nuclei.

Recent measurements at iThemba LABS have identified the $K^\pi = 0_2^+$ bands in $^{158}\text{Er}_{90}$ [14] and $^{160}\text{Yb}_{90}$ [15] to higher spins than seen in β -decay experiments. These measurements allow the comparison of the behaviour of these bands with spin with that of the ground state yrast $K^\pi = 0_1^+$ bands, as the proton number increases and the deformation decreases. This comparison is shown in Fig. 1 where the excitation energies are plotted against spin for $^{150}\text{Nd}_{90}$ to $^{160}\text{Yb}_{90}$. It is immediately obvious that the ground state $K^\pi = 0_1^+$ bands are decreasing in deformation, as the proton number Z increases after $^{156}\text{Dy}_{90}$, while the $K^\pi = 0_2^+$ bands maintain an almost constant moment-of-inertia. This is in contrast to the predictions of all IBA and similar models where the moments-of-inertia of the excited $K^\pi = 0_2^+$ bands are always less than the moments-of-inertia of the ground state $K^\pi = 0_1^+$ bands. To quote [16] *While the IBA calculations using the most common form of the IBA Hamiltonian reproduce the energetics of the 0_2^+ mode, they fail to account for the properties of the states built upon it.*

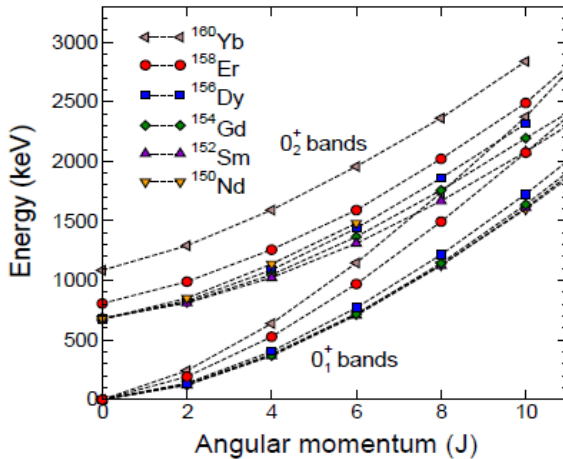


Fig. 1. Excitation energy as a function of angular momentum for members of the ground state 0_1^+ bands and the ex-“ β vibrations” 0_2^+ for the $N = 90$ isotones [14, 15]. The deformation of 0_1^+ ground state bands decreases as the proton number Z increases, whereas the deformation of the excited 0_2^+ ex-“ β bands” does not. This is, in total, contradiction to the predictions of all IBM calculations [16].

It is not very usual for γ bands to be identified much above spin 12^+ as they are usually about 1.0 MeV above the yrast line. This makes it difficult to populate such states in fusion-evaporation (HI, xn) reactions as they are embedded in other structures which compete for intensity. The use of very heavy ion beams to Coulomb excite the most deformed nuclei has, in favourable cases, allowed γ bands to be traced to much higher spins. A notable feature of γ bands is that they track the intrinsic configuration, usually the ground state that they are based on. An example of this is shown in Fig. 2 for the γ band in ^{156}Dy [17] (and references therein). Here, both the even and odd spin members of the γ band track the ground state configuration up to spins of 32^+ and 31^+ respectively.

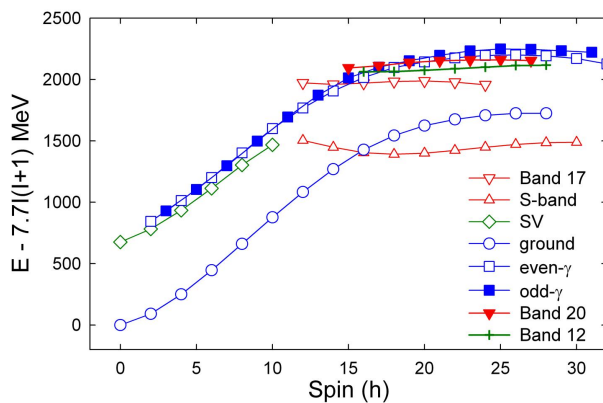


Fig. 2. Plot of the excitation energy, minus a rigid rotor, for the positive-parity bands in $^{156}\text{Dy}_{90}$ [17]. The Second Vacuum (SV) band is the 0_2^+ ex-“ β band” which we claim is a “pairing isomer”.

3. Pairing isomers

Clearly, the 0_2^+ states in $N = 88$ and 90 nuclei are not due to β vibrations of the nuclear shape. But if they are 2 neutron 2 neutron-hole states, how is it that they can be lowered so far into the pairing gap? The solution was found in the early 1970s and applied to the 0_2^+ states in Th, U and Pu nuclei. These had been observed in (p, t) two neutron pick-up reactions by Maher *et al.* [18] but not in (t, p) two neutron stripping reactions by Casten *et al.* [19]. The solution put forward by Griffin, Jackson and Volkov [20] was that simple monopole pairing was too crude an approximation to explain excited 0_n^+ states. With monopole pairing all the two particle transfer strength is decanted into the ground state [21]. Clearly, this is not the case for actinide nuclei or for the $N = 88$ and 90 nuclei discussed above. Reference [20] postulates that, in the pairing model, scattering

from one pair of time-reversed orbits to another pair of time-reversed orbits the probability increases with the minimisation of the momentum transfer in the process or, indeed, with the overlap of the wavefunctions of the initial paired orbit with the wavefunctions of the final paired orbit. Thus nucleons in low- Ω Nilsson orbits are inhibited from scattering into high- Ω orbits and *vice versa*. Nucleons in low- Ω Nilsson orbits have positive (prolate) single particle quadrupole moments, whereas nucleons in high- Ω orbits have negative (oblate) quadrupole moments. A better approximation to the pairing interaction is one that depends on the quadrupole moment of the nucleons involved: hence “quadrupole pairing”.

To demonstrate the effect of this improvement in the pairing interaction, Ref. [20] has a convincing toy model: suppose that $\Delta_{pp} \approx \Delta_{oo} \gg \Delta_{op}$, where Δ_{pp} , Δ_{oo} , and Δ_{op} are the pairing interactions between nucleons scattering between prolate–prolate, oblate–oblate and prolate–oblate Nilsson orbitals. Also suppose there are n prolate and n oblate orbitals at the Fermi surface. Assume that each pairing matrix element is the same for the same type $-a$, but the prolate–oblate matrix elements are very weak $-\epsilon a$. Then if the prolate $n \times n$ matrix is A , the oblate matrix is also A , the matrix for the total system is

$$\begin{pmatrix} A & \epsilon A \\ \epsilon A & A \end{pmatrix}. \quad (3)$$

Then, there are $(2n - 2)$ states with zero energy and 2 states with spin 0^+ and energies $E_{1,2} = -(1 \pm \epsilon)na$ and separated by an energy of $2\epsilon na$. Obviously, there is mixing between the two lowered 0^+ states depending on the size of ϵ . It was pointed out by Abdulvagabova, Ivanova and Pyatov [22] that, in reality, the matrix (3) is not symmetric in the density of prolate and oblate states. High- Ω oblate states are extruded to the Fermi surface at the onset of deformation but have a much lower density of states than the prolate low- Ω states that are driving the deformation. Hence, the pairing interaction for oblate orbitals will be much reduced compared to the pairing involving prolate orbitals. Hence, the prolate paired state will be the 0_1^+ ground state and the oblate paired state will be the excited 0_2^+ state. These authors [22] pointed out that these conditions also applied in the $N \sim 90$ nuclei as well as in the actinides. Other authors have also developed this quadrupole pairing model [23–25]. The latter authors coined the term “pairing isomers” for these 0_2^+ states. (We are reliably informed by Ingmar Ragnarsson that Ricardo Broglia insisted on this label in spite of these states being nowhere near “isomeric”!)

It becomes glaringly obvious that near $N = 90$ the $[505]11/2^-$ neutrons will not partake in the monopole pairing if the conjecture of Ref. [20] is correct. Indeed this is found to be the case. In 1982, Garrett *et al.* [26] pointed out that the $[505]11/2^-$ neutron bands in the odd neutron nuclei

near $N \sim 90$ “back-bend” ($i_{13/2}$ “AB” alignment) at a critical frequency of $\hbar\omega_c \approx 0.28$ MeV, which is the same unblocked frequency as the “back-bends” in the neighbouring even–even and odd proton nuclei yrast bands. In contrast, in odd neutron nuclei, the neutrons in low- Ω Nilsson orbits block some of the monopole (prolate) pairing giving a “back-bending” critical frequency for the energy required to align a pair of $i_{13/2}$ neutrons of $\hbar\omega_c \approx 0.23$ MeV [27]. The systematics of current data on AB alignment frequencies for nuclei with $N = 88$ to 98 are shown in Fig. 3. The horizontal broken lines in Fig. 3 indicate the average critical frequencies for even–even, odd proton and odd neutron nuclei. The few examples of the critical frequencies in $[505]11/2^-$ odd neutron bands are marked with hour-glass symbols. It is clear that the AB critical frequency is not blocked by the $[505]11/2^-$ neutron orbital. The two known examples of AB alignments in 0_2^+ bands are marked in Fig. 3 with a star symbol. Again, there is no reduction of the critical alignment frequency which means that, whatever the configuration of the 0_2^+ state is, the neutrons involved in the configuration do not partake in

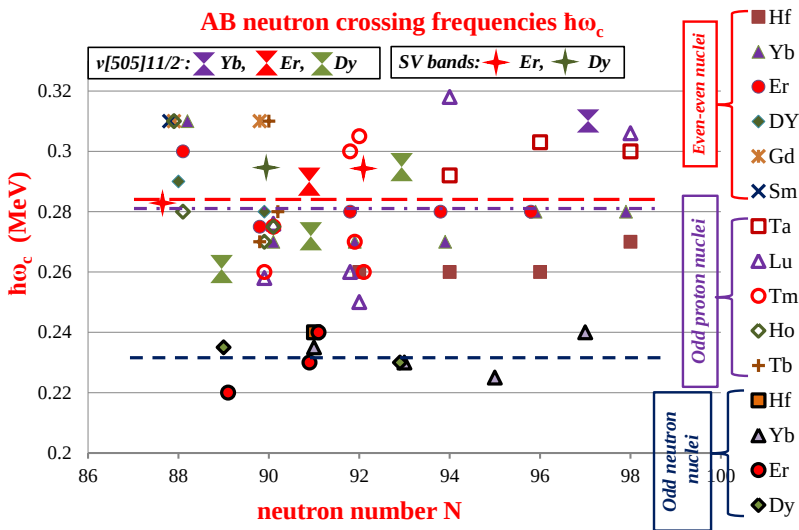


Fig. 3. Critical frequencies $\hbar\omega_c$ for the alignments of the AB “back-bends” due to $i_{13/2}$ neutrons in a variety of rotational bands in nuclei with neutron number between $N = 88$ and 98. The long-dashed line is the average $\hbar\omega_c$ for even–even nuclei, the dash-dotted line is the average $\hbar\omega_c$ for odd proton nuclei and the short-dashed line is the average $\hbar\omega_c$ for the odd neutron nuclei. Clearly, the odd neutron $[505]11/2^-$ nuclei and the Second Vacuum (SV) 0_2^+ alignments do not suffer the same reduction in $\hbar\omega_c$ that the other odd neutron bands do. Errors on the data points are between 5 and 10 keV. Data with larger errors have not been included.

the monopole pairing. Figure 4 (a) shows the alignments of the + and – signatures of the $[505]11/2^-$ band compared with those of the $[521]3/2^-$ ground state band in ^{159}Er . Figure 4 (b) shows the alignments of the yrast band and the 0_2^+ ex-“ β vibration” in ^{156}Er [28].

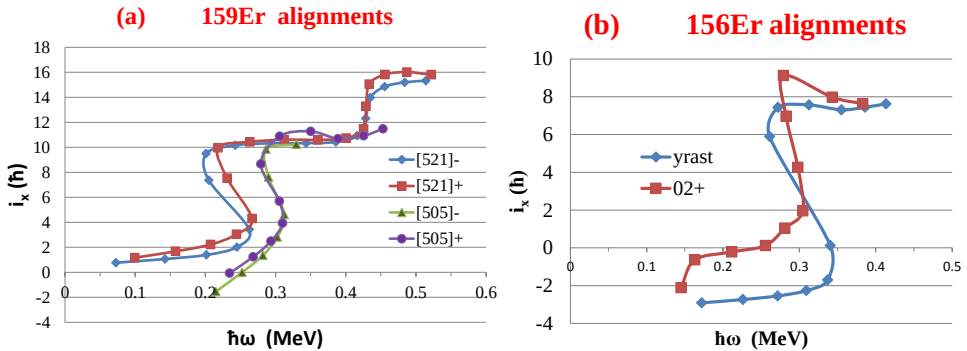


Fig. 4. Alignments of bands in (a) the odd neutron nucleus $^{159}\text{Er}_{91}$ showing the reduction of $\hbar\omega_c$ due to blocking of the monopole pairing in both signatures (+ and –) of the $[521]3/2^-$ band compared to the lack of blocking in both signatures of the $[505]11/2^-$ band. (b) The even–even nucleus $^{156}\text{Er}_{88}$ showing that the ex-“ β vibrational” band 0_2^+ has the same unblocked critical alignment frequency $\hbar\omega_c$ as the yrast band.

I draw an analogy between the trajectory of the $[505]11/2^-$ neutron orbital, with respect to the Fermi surface, and that of a “flying fish”! The $[505]11/2^-$ orbital is chased to the surface by the increasing nuclear deformation as the neutron number is increased. It then flops back into the Fermi sea as orbitals get filled by further increases in the neutron number. As the $[505]11/2^-$ orbital retreats from the Fermi surface, the excitation energy of the 0_2^+ states increase, driving them nearer the point where the p – h states start. This will lead to an increase in mixing with other configurations.

4. Axial asymmetry

This year 2015 is the 40th anniversary of the Nobel Prize given to Aage Bohr and Ben Mottelson. It is therefore appropriate to turn to their iconic textbook for advice. On page 166 they say [6] . . . *but it might be expected that the zero-point oscillations in the γ direction would be of similar magnitude as those in the β direction.* As the data indicate that any “ β vibrations” are at higher excitation energies than formerly imagined, then we might well expect any “ γ vibrations” to also be well above the pairing gap? In that case, we are left with the other explanation given by Bohr and Mottelson that the plethora of $K^\pi = 2^+$ bands found in deformed nuclei throughout

the nuclear chart are due to these nuclei not being axially symmetric [6] (page 166). The way of deciding between γ vibrations and axial asymmetry is to look experimentally for the predicted two-phonon states for vibrations or the lone $K^\pi = 4^+$ band predicted for axial asymmetry. This is difficult, as these structures are even further from the yrast line and will be embedded in a high density of p - h states with which they will mix. The experimental data to date is not very convincing and is too voluminous to be presented in this short talk. Reference [29] deals interestingly with axial asymmetry and an overview of experimental data may be found in Ref. [30].

5. Short discussion

The vast expansion of the experimental data now available to us has in some ways simplified our approach to the structure of quadrupole deformed nuclei. It is no longer fruitful to postulate the existence of “phonon” or “boson” excitations and then expand these in some convenient basis or other, inventing an interaction with parameters determined by a variational calculation and “fitting” the available experimental data using some preconceived view of the physics. We now know that the ex-“ β vibrations” are seniority zero states that are not handled by over simplified monopole pairing. The assumptions needed to support the idea of “ γ vibrations” are unnecessary and the observation of many $K^\pi = 2^+$ bands in even-even nuclei can adequately be described by the axial asymmetry of the nuclear mean field. Hopefully, this understanding of the basic physics will improve our theoretical descriptions of the structure of nuclei. In spherical nuclei, a doubt has also been cast on the interpretation of the standard “vibrational” even-even Cd isotopes [31]. If we take the Raleigh formula seriously, then octupole vibrations should also lie well above the pairing gap. This would leave the low-lying negative parity states in deformed nuclei arising from the breaking of reflection symmetry. A major challenge for the future is to identify experimentally where the actual vibrational excitations of the nuclear shape are located.

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