THE DETERMINATION OF THE BULK SYMMETRY INCOMPRESSIBILITY FROM THE ISOSCALAR GIANT MONOPOLE RESONANCE REVISITED*

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The old problem of whether the coefficients of the leptodermous expansion of the finite nucleus incompressibility (Blaizot's formula) can be fitted using the available experimental data of giant monopole resonances is revisited. Using a mean field model (NL3) as a benchmark, we compute the finite nucleus incompressibility of a large set of nuclei in the scaling approach. These values are fitted to Blaizot's formula and a covariance matrix analysis is performed. From this study, it is seen that some of the fitted coefficients of the leptodermous expansion are strongly contaminated by the neglected terms and differ considerably from the original coefficients which can be directly computed for the given mean field model. As a consequence, it does not seem possible to use the coefficients of Blaizot's formula fitted to experimental information on giant monopole resonances to accurately constrain mean field models available in the literature.

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1. Introduction

A simple way to estimate the excitation energy of the isoscalar Giant Monopole Resonance (GMR) in finite nuclei is provided by the so-called Blaizot model [1, 2], where the finite nucleus incompressibility K_A can be written as a leptodermous expansion

$$K_A = K_{\rm vol} + K_{\rm surf} A^{-1/3} + K_\tau I^2 + K_{\rm Coul} Z^2 A^{-4/3} + \dots\,, \tag{1}$$

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from where the energy of the ISGMR is estimated as

$$E_{\rm GMR} = \sqrt{\hbar^2 K_A/(M\langle r^2\rangle)}, \tag{2}$$

where $\langle r^2\rangle$ is the r.m.s. radius of the nucleus and M the nucleon mass.

Several works, see *e.g.* Refs. [4, 5], have studied the problem of the extraction of the different coefficients in Blaizot's formula for the finite nucleus incompressibility from experimental data on the GMR. Using data consistent with the scaling approach, Ref. [5] performed a fit to obtain the bulk incompressibility $K_{\rm vol}$, which is identified with the incompressibility of infinite symmetric nuclear matter (K_∞), the bulk symmetry incompressibility K_τ, and the ratio of the surface and volume terms $c = K_{\rm surf}/K_{\rm vol}$. An important question that arises is whether the fitted coefficients may or may not be used to constrain mean field models, as *e.g.* in the analysis of Skyrme forces [6] and Relativistic Mean Field (RMF) models [7]. It is to be mentioned that the fit of Blaizot's formula to experimental data on the GMR has been contested by several authors [8–10], who pointed out that the extracted coefficients could not be obtained with enough accuracy, invalidating a direct comparison with the same coefficients predicted by a given mean field model.

Here, we reanalyze this problem from a different point of view. First, we generate the theoretical finite nucleus incompressibility K_A of a large set of nuclei with the scaling method using the Extended Thomas–Fermi (ETF) approximation to the RMF model [12] for the NL3 parameter set [13]. The semiclassical treatment of the breathing mode is in a good agreement with the excitation energies of the GMR obtained using the relativistic Random Phase Approximation (RPA) with the same RMF model, pointing out that this semiclassical approach can be used confidently to estimate K_A [12]. Second, through calculations in infinite and semi-infinite nuclear matter, we compute the coefficients $K_{\rm vol}$, $K_{\rm surf}$, K_τ and $K_{\rm Coul}$ of the leptodermous expansion of K_A with the same NL3 parameter set. Note that $K_{\rm vol}$, K_τ and $K_{\rm Coul}$ are easily evaluated in nuclear matter [14]. To compute $K_{\rm surf}$ requires a scaling calculation in semi-infinite nuclear matter [1]. We perform this calculation fully consistently with the method used to obtain K_A in finite nuclei, *i.e.*, in the ETF approach to RMF [3]. Third, we recalculate the coefficients of the expansion of K_A for NL3 by performing a fit of (1) in the same conditions as in Ref. [5], but using as pseudo-data the theoretical K_A values that we have computed in finite nuclei. Finally, we compare the values of the coefficients obtained in the fit with the original values derived for NL3.

2. Basic theory

The volume $K_{\rm vol}$, K_τ and $K_{\rm Coul}$ coefficients in Blaizot's formula (1) are expressed within the scaling approach in terms of some parameters of the liquid droplet model [15] and can be computed using nuclear matter properties only [1]. They read $K_{\rm vol} = K_\infty$ and

$$K_\tau = K_{\rm sym} + L\left(\frac{K'}{K_{\rm vol}} - 6\right), \qquad K_{\rm Coul} = \frac{3e^2}{5r_0}\left(\frac{K'}{K_{\rm vol}} - 8\right), \tag{3}$$

where L, K' and $K_{\rm sym}$ are defined from the expansion of the energy per particle in asymmetric nuclear matter around saturation [15]. The numerical values of $K_{\rm vol}$, K_τ and $K_{\rm Coul}$ of the NL3 mean field model can be found *e.g.* in Ref. [14]. The surface coefficient can be written as [1, 16]

$$K_{\rm surf} = 4\pi r_0^2\left[\left(22 - 2\frac{K'}{K_{\rm vol}}\right)\sigma(\rho_0) + 9\rho_0^2\ddot{\sigma}(\rho_0)\right], \tag{4}$$

where the surface tension and its second derivative with respect to the central density $\rho_{\rm c}$ evaluated at saturation density are given in the scaling method by [3, 14]

$$\sigma(\rho_{\rm c}) = \int\limits_{-\infty}^{+\infty}[H - e_\infty(\rho_{\rm c})\rho]\,dz\,, \qquad \ddot{\sigma}(\rho_0) = \left.\frac{d^2\sigma(\rho_{\rm c})}{d\rho_{\rm c}^2}\right|_{\rho_0} = \left.\frac{1}{9\rho_0^2}\frac{d^2\sigma}{d\lambda^2}\right|_{\lambda=1}. \tag{5}$$

A detailed description of these expressions and of the ETF-RMF semi-infinite nuclear matter calculations can be found in Ref. [3]. We report the values of the coefficients $K_{\rm vol}$, $K_{\rm surf}$, $K_{\rm Coul}$ and K_τ for the RMF parameter set NL3 in Table I.

TABLE I

Volume, surface, Coulomb and volume-symmetry incompressibility coefficients computed with the RMF parameter set NL3.

$K_{\rm vol}$	$K_{\rm surf}$	$K_{\rm surf}/K_{\rm vol}$	$K_{\rm Coul}$	K_τ
271.5	−313.7	−1.14	−6.45	−698.9

3. Results

We have computed the finite nucleus incompressibility K_A of 750 nuclei from oxygen to uranium in the ETF-RMF approach with the NL3 parameter set (the calculation of the excitation energy of the GMR and K_A of finite nuclei in the ETF-RMF formalism is described in detail in Refs. [11, 12]). These K_A values can be used as a benchmark (pseudo-data) for fitting the parameters of Eq. (1). The extracted coefficients can then be checked against the NL3 original values reported in Table I.

First, we fit the coefficients $K_{\rm vol}$, K_τ and the ratio $K_{\rm surf}/K_{\rm vol}$ and fix $K_{\rm Coul} = -5.2$ MeV as was done in Ref. [5]. The values of the coefficients from this fit are displayed in the first line of Table II. The associated covariance matrix is shown at the bottom left of the table. The rows (from the left to the right) and the columns (from the top to the bottom) of this matrix correspond to the correlations among the parameters $K_{\rm vol}$, $K_{\rm surf}/K_{\rm vol}$ and K_τ, respectively. We have performed the fits with the MINUIT software which provides the uncertainties of the fitted parameters as well as the covariance matrix. We have performed a χ^2 test with an adopted error for our benchmark K_A of 1 MeV. We see in Table II that the fitted coefficients have relatively large error bars and do not reproduce within the error band the NL3 values of Table I. The covariance matrix shows that the fitted value of $K_{\rm vol}$ is strongly correlated with the ratio $K_{\rm surf}/K_{\rm vol}$ and less correlated with K_τ. Next, to test the influence of the number of data points, we repeat the fit using a reduced set of only 18 pseudo-data corresponding to the nuclei where the excitation energy of the GMR is experimentally known [5]. The new results are displayed in the second line of Table II and the corresponding covariance matrix is shown at the bottom right of the table. The fitted parameters are compatible with the results of the previous fit to a larger number of data points. The covariance matrix, again, points out strong correlations among the parameters. We have verified that setting $K_{\rm Coul} = -6.45$ MeV, as predicted by NL3, instead of $K_{\rm Coul} = -5.2$ MeV does not change the global conclusions of our analysis.

TABLE II

Fit of $K_{\rm vol}$, $K_{\rm surf}/K_{\rm vol}$ and K_τ, with $K_{\rm Coul} = -5.2$ MeV, to K_A of 750 nuclei in the first line and of 18 nuclei in the second line. See the text for details.

$K_{\rm vol}$	$K_{\rm surf}/K_{\rm vol}$	K_τ	χ^2	r.m.s.
258 ± 7	-1.26 ± 0.09	-305 ± 50	0.4	0.7
265 ± 16	-1.34 ± 0.21	-328 ± 131	0.24	0.43

$$\begin{pmatrix} 1.000 & -0.940 & -0.468 \\ -0.940 & 1.000 & 0.202 \\ -0.468 & 0.202 & 1.000 \end{pmatrix} \qquad \begin{pmatrix} 1.000 & -0.985 & -0.863 \\ -0.985 & 1.000 & 0.779 \\ -0.863 & 0.779 & 1.000 \end{pmatrix}$$

Following a strategy suggested in [5,8], we repeat the fit but fixing the ratio $K_{\rm surf}/K_{\rm vol}$ which, in turn, is gradually varied between -1.0 and -1.8. Again, we use 18 data points and $K_{\rm Coul} = -5.2$ MeV. The results of this new fit are reported in Table III. We see that the optimal χ^2 value corresponds to a ratio $K_{\rm surf}/K_{\rm vol} = -1.4$ and that $K_{\rm vol}$ and K_τ are compatible with the results obtained from the fits with the three free parameters (Table II), but, again, do not reproduce the NL3 values shown in Table I. From the covariance matrix, we can see that in this case these two parameters are strongly correlated between them.

TABLE III

Fit of $K_{\rm vol}$ and K_τ for different values of $K_{\rm surf}/K_{\rm vol}$ with $K_{\rm Coul} = -5.2$ MeV and using 18 nuclei.

$K_{\rm surf}/K_{\rm vol}$	$K_{\rm vol}$	K_τ	χ^2
-1.0	243 ± 2	-117 ± 83	5.36
-1.1	251 ± 3	-171 ± 84	3.31
-1.2	259 ± 3	-229 ± 84	1.68
-1.3	269 ± 3	-290 ± 85	0.57
-1.4	279 ± 3	-356 ± 86	0.10
-1.5	289 ± 3	-425 ± 86	0.45
-1.6	301 ± 3	-500 ± 87	1.81
-1.7	313 ± 3	-579 ± 88	4.43
-1.8	326 ± 3	-664 ± 88	8.63

$$\begin{pmatrix} 1.000 & -0.900 \\ -0.900 & 1.000 \end{pmatrix}$$

The discrepancy between the coefficients $K_{\rm vol}$, $K_{\rm surf}$ and K_τ obtained in the fit and those derived from the leptodermous expansion should be attributed to the fact that the fitted values include in an effective, but uncontrolled, way effects from higher-order contributions to Eq. (1) neglected in the fit. However, the inclusion of higher-order terms, such as surface-symmetry and/or curvature corrections, does not necessarily solve the problem. Although the additional terms may improve the quality of the fit, they can induce large changes in the other parameters pointing out uncontrolled correlations and that the fit may have converged to a local minimum. When the latter occurs, the confidence intervals and the standard deviations predicted for all or some of the parameters of the new fit may become large (see in this respect, for instance, Table I of Ref. [17]).

The fact that the fit effectively takes into account higher-order terms can also be seen in Table IV. In the fifth column, we display K_A predicted by the truncated leptodermous expansion using the coefficients given in Table I. These predictions are not very accurate and may differ from the selfconsistent K_A data (third column) up to 10 MeV in some cases. As expected, the fit (fourth column) reproduces much more accurately the data, indicating the effective character of the parameters. The lack of accuracy in the predictions using the truncated expansion with the coefficients of Table I suggests that curvature ($K_{\rm curv} A^{-2/3}$) and surface-symmetry ($K_{\tau,\rm surf} I^2 A^{-2/3}$) terms of the K_A expansion may have a non-negligible role. These parameters have been estimated in Ref. [3] in the ETF approach, without fitting to finite nuclei, to be $K_{\rm curv} = -229.8$ MeV and $K_{\tau,\rm surf} = 1754$ MeV for NL3. If they are added to Blaizot's formula (1), the agreement with the self-consistent K_A data improves considerably (*cf.* the rightmost column of Table IV). However, the result still differs from the K_A data. This implies that more terms in the expansion would be eventually needed.

TABLE IV

Self-consistent finite nucleus incompressibility K_A in the ETF-RMF approach with NL3 and different approximations to it, for the 18 nuclei for which the GMR is experimentally known. See the text for details.

A	Z	K_A[Data]	K_A[Fit]	K_A[Lepto]	K_A[Lepto*]
112	50	163.8243	164.0125	168.5296	162.8166
114	50	163.6577	163.9035	167.0916	162.7730
116	50	163.3574	163.6340	165.3782	162.5590
118	50	163.9277	163.2204	163.4197	162.1888
120	50	162.4116	162.6776	161.2434	161.6757
122	50	161.7905	162.0190	158.8736	161.0317
124	50	161.0854	161.2567	156.3325	160.2677
106	48	163.5288	163.6747	169.3719	162.4104
110	48	163.2594	163.5168	166.5119	162.4320
112	48	162.9075	163.1756	164.6311	162.1670
114	48	162.4317	162.6842	162.4951	161.7393
116	48	161.8460	162.0588	160.1386	161.1631
208	82	161.1026	161.9540	152.0887	158.7898
144	62	164.4524	164.7625	165.3193	163.4108
148	62	163.7110	164.1241	162.1450	162.6517
56	26	154.1632	154.7379	165.5884	152.2278
58	28	154.7513	155.1457	167.1043	152.3064
60	28	155.6126	156.1161	166.7352	153.7325

4. Conclusions

The analysis performed in this contribution emphasizes that if the coefficients of Blaizot's formula for the finite nucleus incompressibility are fitted to the existing data of the GMR, their values are, actually, effective as far as they contain higher-order contributions in a rather uncontrolled way. Therefore, the fitted coefficients, excepting maybe K_{vol}, are not able to reproduce the corresponding leptodermous value very accurately. Consequently, it does not seem evident that they may be used as constraints to rule out several of the existing mean field models that, on the other hand, predict other important properties of finite nuclei, such as binding energies and charge radii in a reasonable good agreement with the experimental data. It is to be noted that the conclusions of our analysis have been derived using a single mean field model (NL3), but we think that they are sufficiently general although the numerical details may be different if a similar study is carried out using another mean field model.

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REFERENCES

[1] J.P. Blaizot, *Phys. Rep.* **64**, 171 (1980).

[2] R.C. Nayak *et al.*, *Nucl. Phys. A* **516**, 62 (1990).

[3] S.K. Patra *et al.*, *Phys. Rev. C* **65**, 044304 (2002).

[4] M.M. Sharma, *Nucl. Phys. A* **816**, 65 (2009).

[5] J.R. Stone, N.J. Stone, S.A. Moszkowski, *Phys. Rev. C* **89**, 044316 (2014).

[6] M. Dutra *et al.*, *Phys. Rev. C* **85**, 035201 (2012).

[7] M. Dutra *et al.*, *Phys. Rev. C* **90**, 055203 (2014).

[8] J. Treiner, H. Krivine, O. Bohigas, J. Martorell, *Nucl. Phys. A* **371**, 253 (1981).

[9] J.P. Blaizot, J.F. Berger, J. Dechargé, M. Girod, *Nucl. Phys. A* **591**, 435 (1995).

[10] J.M. Pearson, N. Chamel, S. Goriely, *Phys. Rev. C* **82**, 037301 (2010).

[11] S.K. Patra, M. Centelles, X. Viñas, M. Del Estal, *Phys. Lett. B* **523**, 67 (2001).

[12] S.K. Patra *et al.*, *Nucl. Phys. A* **703**, 240 (2002).

[13] G.A. Lalazisis, J. Köning, P. Ring, *Phys. Rev. C* **55**, 540 (1997).

[14] T.v. Chossy, W. Stocker, *Phys. Rev. C* **56**, 2518 (1997).
[15] W.D. Myers, W.J. Swiatecki, *Ann. Phys. (N.Y.)* **55**, 395 (1969).
[16] J.M. Pearson, M. Farine, *Phys. Rev. C* **26**, 267 (1982).
[17] P. Veselý *et al.*, *Phys. Rev. C* **86**, 024303 (2013).