

## TWIST EXPANSION OF FORWARD DRELL–YAN PROCESS\*

TOMASZ STEBEL, LESZEK MOTYKA, MARIUSZ SADZIKOWSKI

The Marian Smoluchowski Institute of Physics, Jagiellonian University  
Łojasiewicza 11, 30-348 Kraków, Poland

*(Received November 12, 2015)*

We present a twist expansion of differential cross sections of the forward Drell–Yan process at the high energies. The expansion of all invariant form factors is performed assuming Golec-Biernat and Wüsthof (GBW) saturation model and the saturation scale plays the role of the hadronic scale of Operator Product Expansion (OPE). Some explicit predictions for LHC experiments are given. It is shown also how the Lam–Tung relation is broken at twist 4 what provides a sensitive probe for searching of higher twists.

DOI:10.5506/APhysPolBSupp.8.1011

PACS numbers: 13.85.Qk, 12.38.Cy

### 1. Introduction

The Large Hadron Collider (LHC) opens new kinematic regions in high energy physics. The most promising process at the LHC for investigating QCD effects for small Bjorken- $x$  ( $\simeq 10^{-6}$ ) and moderate energy scales is a forward Drell–Yan (DY) scattering. Such a small  $x$  at parton density scale  $\mu^2 > 6 \text{ GeV}^2$  is about two orders of magnitude smaller than in measurements at HERA. Due to the forward kinematics of the process, the LHCb detector is the most suitable for those measurements [1].

The angular distribution of DY lepton–antilepton pairs may be parametrized in terms of four independent structure functions. There are two possible choices of these functions [2]. Lorentz-invariant structure functions are basic ones. For our purpose, more convenient are so-called helicity structure functions  $W_L, W_T, W_{TT}, W_{LT}$ . In this approach, one contracts both hadronic and leptonic tensors with virtual photon polarization vectors (PPVs). Then, the leptonic degrees of freedom reduce to an angles  $\Omega = (\theta, \phi)$  in lepton pair center-of-mass frame.  $W$  structure functions correspond to the hadronic degrees of freedom.

---

\* Presented at EDS Blois 2015: The 16<sup>th</sup> Conference on Elastic and Diffractive Scattering, Borgo, Corsica, France, June 29–July 4, 2015.

The inclusive DY cross section is given by the formula

$$\frac{d\sigma}{dx_F dM^2 d\Omega d^2q_\perp} = \frac{\alpha_{em}^2 \sigma_0}{2(2\pi)^4 M^4} \left[ W_L (1 - \cos^2 \theta) + W_T (1 + \cos^2 \theta) + W_{TT} (\sin^2 \theta \cos 2\phi) + W_{LT} (\sin 2\theta \cos \phi) \right], \quad (1)$$

where  $x_F$  is a fraction of projectile’s longitudinal momentum taken by virtual photon,  $M$  is an invariant mass of leptons pair,  $q_\perp$  is transverse momentum of virtual photon and the constant  $\sigma_0$  gives the dimension.

Helicity structure functions, in contrast to invariant ones, depend on the choice of axes in lepton pair center-of-mass frame. Here, the most convenient one is the so-called  $t$ -channel helicity frame (see [2] for the detailed definition).

### 2. Inclusive cross section for forward Drell–Yan

In Fig. 1, we plot leading diagrams for the forward Drell–Yan scattering in hadron–hadron collisions. They are dominant due to the large gluon density at small longitudinal momentum fraction of one hadron ( $x_1 < 10^{-5}$ ). The most convenient frame for calculating these diagrams is such that target (proton with small  $x$ ) is at rest. In this frame, the energy of projectile,  $E$ , is much larger than other scales (such as  $M$  or  $q_\perp$ ) and we can drop in calculations non-leading terms in  $1/E$  expansion.

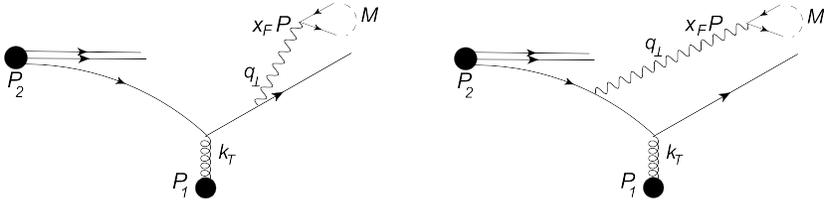


Fig.1. Dominant diagrams for forward Drell–Yan process in target rest frame. Photon carries  $x_F$  part of projectile’s momentum.  $M^2$  is photon’s virtuality and the square of invariant mass of lepton–antilepton pair. Transverse momenta of photon and gluon are denoted as  $q_\perp$  and  $k_T$ , respectively.

To calculate the diagrams from Fig. 1, we apply  $k_T$ -factorization framework, then the differential cross-section can be written it the following way:

$$\frac{d\sigma}{dx_F dM^2 d\Omega d^2q_\perp} = \frac{\alpha_{em} \alpha_s}{6\pi(P_1 P_2)^2 M^2} \int_{x_F}^1 dz \frac{1}{1-z} \frac{\wp(x_F/z)}{x_F^2} \times \int d^2k_T/k_T^4 f(\bar{x}_g, k_T^2) L^{\tau\tau'}(\Omega) \tilde{\Phi}_{\tau\tau'}(q_\perp, k_T, z), \quad (2)$$

where  $\wp(x_F/z)$  is a pdf for projectile  $P_2$ ,  $f(\bar{x}_g, k_T^2)$  is an unintegrated gluon density of target  $P_1$ ,  $L^{\tau\tau'}(\Omega)$  is a lepton tensor contracted with PPV which reduces to angular coefficients from (1), the impact factor  $\tilde{\Phi}_{\tau\tau'}(q_\perp, k_T, z)$  is a square of hard amplitudes of diagrams (describing emission of virtual photon).

For our purpose, it is convenient to use the color dipole model in which the unintegrated gluon density  $f(\bar{x}_g, k_T^2)$  in (2) is replaced by an (equivalent in the leading logarithmic approximation) color dipole cross section  $\hat{\sigma}(r)$  [3–5]

$$\int d^2k_T/k_T^4 f(\bar{x}_g, k_T^2) \tilde{\Phi}(q_\perp, k_T, z) \rightarrow \int d^2r \hat{\sigma}(r) \Phi(q_\perp, r, z). \quad (3)$$

To perform twist expansion of helicity structure functions, we follow methods developed in Refs. [6–8] and apply the Mellin transformation. Then, (1) is given by

$$W_i = \int_{x_F}^1 dz \wp(x_F/z) \int_C \frac{ds}{2\pi i} \left( \frac{z^2 Q_0^2}{M^2(1-z)} \right)^s \tilde{\sigma}(-s) \hat{\Phi}_i(q_\perp, s, z), \quad (4)$$

where  $\tilde{\sigma}(-s)$  and  $\hat{\Phi}_i(q_\perp, s, z)$  are Mellin transforms of dipole cross section and impact factor. For the details, see [9], in particular expressions for  $\hat{\Phi}_i(q_\perp, s, z)$ .

### 3. Twist expansion

In formula (4), we have explicitly three energy scales: saturation scale  $Q_0$  (coming from dipole cross section  $\hat{\sigma}$ ) which is a soft scale and two semi-hard scales:  $M$  and  $q_\perp$ . OPE is here given in terms of positive powers of the  $Q_0$ .

In order to perform twist expansion, one should choose a model of the dipole cross section  $\hat{\sigma}$ . We adopt the Golec-Biernat and Wüsthoff model [10] which was proven to be successful in description of Deep Inelastic Scattering (DIS) and diffractive DIS data from HERA.

Mellin transform of such function is particularly simple  $\tilde{\sigma}(-s) = -\sigma_0 \Gamma(-s)$  and integral (4) is a sum of infinite number of residues which are proportional to  $Q_0^{2k}$  with  $k = 1, 2, \dots$ . Expressions for twist two  $W_i^{(2)}$  and twist four  $W_i^{(4)}$  for all structure functions are given in [9].

To compare the twists, it is useful to introduce averaged cross section over the angles (at the leading twist)

$$\langle \sigma \rangle = \frac{1}{4\pi} \int d\Omega \frac{d\sigma}{dx_F dM^2 d\Omega d^2q_\perp} \equiv \frac{2}{3} \left( W_L^{(2)} + 2W_T^{(2)} \right). \quad (5)$$

In Fig. 2, we show twists 2 (left) and 4 (right) divided by  $\langle\sigma\rangle$ . They are plotted as functions of lepton-pair transverse momentum  $q_T$  for fixed pair mass  $M^2 = 6 \text{ GeV}^2$ , which might be reachable at the LHCb experiment [1]. It can be seen that the  $W_T$  is a dominant structure function at twist 2. The same is at twist 4 for large  $q_T$ , however, for small transverse momentum  $W_L^{(4)}$  is larger. One should note that around  $q_T \approx 2 \text{ GeV}$  twists 4 start to be very large. This is, however, a region where  $q_T \approx Q_0$  and twist expansion breaks.

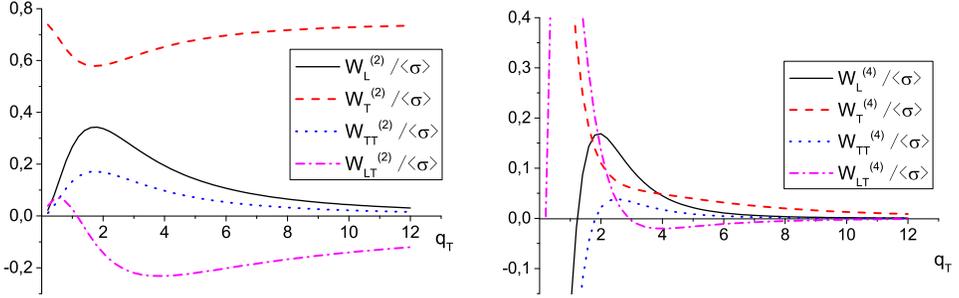


Fig. 2. Left: twist 2 divided by averaged cross section  $\langle\sigma\rangle$  as a function of photon transverse momentum  $q_T$  for all helicity structure functions. Right: the same for twist 4. Both plots for  $M^2 = 6 \text{ GeV}^2$  and  $x_F = 0.1$ .

In Fig. 3, we plot ratios of twist 4 to twist 2. It can be seen that the largest contribution to non-leading twist is for  $W_L$  — up to 20% for small  $q_T$ .

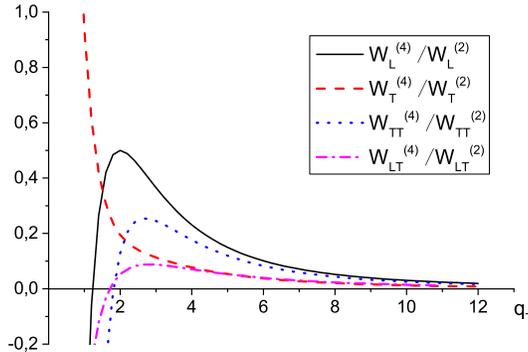


Fig. 3. Twist 4 divided by twist 2 as a functions of photon transverse momentum  $q_T$  for all helicity structure functions.  $M^2 = 6 \text{ GeV}^2$  and  $x_F = 0.1$ .

#### 4. Lam–Tung relation

For experimental searches of higher twists, the most interesting are quantities that vanish at the leading twist and are nonzero at higher twists. In the DY process, such a quantity might be constructed using the Lam–Tung relation [11, 12]

$$\Delta_{\text{LTT}}^{(2)} \equiv W_{\text{L}}^{(2)} - 2W_{\text{TT}}^{(2)} = 0. \quad (6)$$

At the next-to-leading twist, namely twist 4, relation is broken (see [9]). It is also violated by higher order QCD corrections, however, at the very small  $x$ , the contribution coming from higher twists is sizable comparing to them.

In Fig. 4, we plot  $\Delta_{\text{LTT}}^{(4)} \equiv W_{\text{L}}^{(4)} - 2W_{\text{TT}}^{(4)}$  divided by  $\langle\sigma\rangle$  (5) as a function of  $q_{\text{T}}$  for different masses  $M^2$ . For the comparison, we plot also leading twist of  $W_{\text{L}}$  (black dotted line). We see that for small transverse momentum, the ratio  $\Delta_{\text{LTT}}^{(4)}/W_{\text{L}}^{(2)}$  could be around 20%.

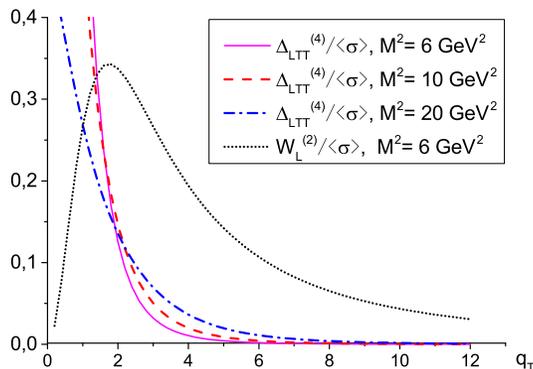


Fig. 4. Ratio  $\Delta_{\text{LTT}}^{(4)}/\langle\sigma\rangle$  as a function of  $q_{\text{T}}$  for three masses  $M^2$ . For comparison: leading twist of  $W_{\text{L}}$  divided by  $\langle\sigma\rangle$ .

#### 5. Conclusions and outlook

The forward Drell–Yan scattering is a promising process for searching of higher twists at the LHC. Here, we presented several plots with predictions for higher twists based on the GBW model. In particular, the quantity  $W_{\text{L}} - 2W_{\text{TT}}$  should be very useful for searches of higher twists since it is nonzero only at the next-to-leading twist. For the experimental searches, it is essential to measure precise angular distribution at low mass and transverse momentum of lepton pair. Then, as we showed, non-leading twists contribution could be around 20%.

Authors would like to thank the organizers of the EDS Blois 2015, Corsica, France, for the very interesting meeting. Support of the Polish National Science Centre grant No. DEC-2014/13/B/ST2/02486 is gratefully acknowledged. T.S. acknowledges the support from the scholarship of Marian Smoluchowski Scientific Consortium “Matter-Energy-Future” from the KNOW funding.

## REFERENCES

- [1] J. Anderson [LHCb Collaboration], Proc. of 40<sup>th</sup> International Symposium on Multiparticle Dynamics (ISMD 2010), Sep 21–25, 2010, Antwerp, Belgium.
- [2] C.S. Lam, Wu-Ki Tung, *Phys. Rev. D* **18**, 2447 (1978).
- [3] N.N. Nikolaev, B.G. Zakharov, *Z. Phys. C* **49**, 607 (1991).
- [4] S.J. Brodsky, A. Hebecker, E. Quack, *Phys. Rev. D* **55**, 2584 (1997).
- [5] B.Z. Kopeliovich, J. Raufeisen, A.V. Tarasov, *Phys. Lett. B* **503**, 91 (2001).
- [6] J. Bartels, K.J. Golec-Biernat, K. Peters, *Eur. Phys. J. C* **17**, 121 (2000).
- [7] J. Bartels, K.J. Golec-Biernat, L. Motyka, *Phys. Rev. D* **81**, 054017 (2010).
- [8] K.J. Golec-Biernat, E. Lewandowska, A.M. Stasto, *Phys. Rev. D* **82**, 094010 (2010).
- [9] L. Motyka, M. Sadzikowski, T. Stebel, *J. High Energy Phys.* **1505**, 087 (2015).
- [10] K.J. Golec-Biernat, M. Wüsthof, *Phys. Rev. D* **59**, 014017 (1998); **60**, 114023 (1999).
- [11] C.S. Lam, Wu-Ki Tung, *Phys. Lett. B* **80**, 228 (1980).
- [12] F. Gelis, J. Jalilian-Marian, *Phys. Rev. D* **76**, 074015 (2007).