LOW-x PHYSICS: SELECTED TOPICS IN THEORY AND PHENOMENOLOGY*

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A selected set of topics in the low-x physics is discussed. Among them, recent developments in the next-to-leading corrections to the small-x nonlinear evolution with saturation, resummation of higher order corrections, forward inclusive production beyond lowest order and impact parameter dependence.

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1. Introduction: high energy limit

The high energy, or Regge limit, pre-dates the Quantum Chromodynamics and was developed as the limit of the S-matrix theory for the description of the strong interactions in high energy collisions. In this limit, the center-of-mass energy s of the scattering is taken to be very large, greater then any other scales in the process, such that $s \gg |t|$, where t is the momentum transfer. By the analysis in the complex angular momentum plane, one can then demonstrate that the scattering amplitude can be expressed as

$$\mathcal{A}(s,t) \sim \tilde{\beta}(t) s^{\alpha(t)} \, , \tag{1}$$

where $\alpha(t) = \alpha(0) + \alpha' t$ is the Regge trajectory, and $\alpha(0)$ is the intercept. Thus, the scattering amplitude is dominated (for negative t) in the Regge limit by the exchange of the Regge trajectory. Such behavior for the scattering amplitude can be translated onto the energy dependence of the total

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cross section which can be obtained from the optical theorem

$$\sigma_{\text{tot}} = \frac{1}{s}\text{Im}\mathcal{A}(s,0) \sim s^{\alpha(0)-1}\,, \tag{2}$$

thus implying that the intercept of the Regge trajectory determines the behavior of the cross section. A special type of the Reggeon with the even signature and intercept greater than unity is called Pomeron. It corresponds to the exchange of the vacuum quantum numbers and dominates the behavior of the cross section at asymptotically high energies. The data on total proton–proton cross section suggest that the rise at high energies can be effectively described by a small power $\sim 0.08-0.1$, see [1]. Such rise, however, is incompatible with the Froissart bound [2] which states that the hadronic cross section cannot grow faster than the $\ln^2 s$, otherwise it would violate unitarity. The situation is, however, more complicated due to the fact that the numerical coefficient in front of the Froissart bound is rather large and, therefore, quantitatively the current data are far from this bound. On the other hand, if the numerical coefficient is different, then indeed the current data from the Large Hadron Collider and cosmic ray observations could indicate the logarithmic behavior consistent with the Froissart bound [3] and excluding the power-like behavior, and thus the dominance of the simple Pomeron pole. The Pomeron within the Quantum Chromodynamics has been computed in a series of seminal papers by Balitskii–Fadin–Kuraev–Lipatov [4, 5]. The dominant contribution at the lowest order which sums up terms of $\alpha_{\text{s}} \ln s \sim 1$ is due to the gluon emissions, as they are the elementary quanta of the highest spin. The gluon emissions are summed assuming the strong ordering in the longitudinal components of their momenta, while the transverse momenta are not ordered along the emission chain. This leads to the leading logarithmic summation of powers $(\alpha_{\text{s}} \ln s)^n$. Effectively, this summation is performed through the integro-dfferential BFKL evolution equation for the gluon Green's function. The solution to this equation was shown to exhibit very strong power growth [6] with the decreasing x, $x^{-\lambda}$, where the hard Pomeron intercept $\lambda = \frac{\alpha_{\text{s}} N_{\text{c}}}{\pi} 4 \ln 2 \simeq 0.5$. Subsequently, this growth was shown to be incompatible with the phenomenology calling for the necessity to evaluate the next-to-leading logarithmic corrections to the BFKL [7, 8]. In addition, such power-like growth also violates the unitarity bounds as mentioned above. In the pioneering paper [9], it was suggested that the strong rise of the gluon density should be eventually tamed by the nonlinear corrections. To be more precise, in addition to the gluon emissions which are enhanced at high energies, there should be a competing mechanics, a gluon recombination which will start to play an increasingly important role when the density of the gluons is very high. This will lead to the modification of the evolution equations, which should be supplemented with the nonlinear terms in the density. The dense regime in which such corrections are important is characterized by the dynamically generated scale, so-called saturation scale which is defined by the condition

$$\frac{A \times xg\left(x, Q_{\mathrm{s}}^2\right)}{\pi A^{2/3}} \times \frac{\alpha_{\mathrm{s}}\left(Q_{\mathrm{s}}^2\right)}{Q_{\mathrm{s}}^2} \sim 1\,, \tag{3}$$

where A is the mass number if nuclei are present in the process. The saturation scale grows very fast with the decreasing value of x, its behavior can be roughly approximated as $Q_{\mathrm{s}}^2(x) \sim A^{1/3} Q_0^2 \left(\frac{1}{x}\right)^{\lambda}$, with $\lambda \simeq 0.3$, which is compatible with the growth of the gluon density extracted from the experimental data. During last decades, several approaches to saturation were developed at small x, all of which lead to some form of the nonlinear equation or coupled set of equations. The most widely used is the Balitsky–Kovchegov equation which was derived in the leading logarithmic approximation in [10, 11]. Later, NLL corrections have been completed to this equation in [12]. The BK equation is the equation for the dipole amplitude which can be related to the unintegrated gluon density. In the coordinate space, this dipole amplitude takes value between 0 and 1, thanks to the saturation effects resummed through the nonlinear term present in the BK equation. When the nonlinear term can be neglected, the BK equation reduces to the BFKL equation. At small values of the dipole sizes, which correspond to the large transverse momenta, the dipole amplitude behaves approximately like a power of r^2 which is consistent with the phenomenon of color transparency. For large values of r, the dipole amplitude saturates to unity, thanks to the nonlinear term. With the decreasing x, the transition region between small and large values of the dipole amplitude moves to the smaller values of the dipole size. This border between the dilute and dense regime defines the saturation scale.

2. Small-x evolution with saturation beyond LO

It has been known since many years that the linear BFKL equation suffers from very large NLL corrections [7,8] which lead to the instabilities. Improvement schemes were proposed, which were based on the resummation of the so-called collinearly enhanced terms. By imposing the appropriate kinematical constraints and including the nonsingular terms of the DGLAP splitting functions as well as the running coupling, one can demonstrate that the resulting solution to such improved equation is stable and that the effective power which governs the small-x behavior is reduced to better match the experimental data [13, 14]. The natural question then arises whether the same sort of the instability is present when the nonlinear BK evolution equation is evaluated at NLL. First indications that nonlinearities will not cure the problems at NLL were demonstrated in [15] where the linear BFKL equation at NLL was solved by imposing the saturation boundary which mimics the solution to the nonlinear equation. The solution even turned to be negative for some values of the transverse momenta, and the saturation scale in some cases could not be defined. The exact numerical solution to the BK equation at NLL order has been recently achieved in [16]. The results confirm earlier findings using the boundary method and indicate that indeed in the NLL order the solution to the BK equation is unstable, and can even reach unphysical, negative values. Thus, the NLL instability is similar to that observed in the linear BFKL case and is not cured by the nonlinear saturation corrections. Subsequently, it was proposed [17] that similar collinear resummation should be performed by imposing the kinematical constraint onto the nonlinear evolution equation. The results show that the resulting solution is stable and that the effective power of the saturation scale is reduced with respect to the LL value.

3. Forward inclusive production

An inclusive production of hadrons in proton–nucleus collisions in the forward rapidity is one of the key processes to test the effects of high parton density in the nucleus. Due to the asymmetric kinematics, the incoming proton projectile is probed at large values of x, whereas the nucleus is probed at small values of x. Thus one can expect that the higher twist effects may become important in the kinematic regime when $p_{\mathrm{T}} \sim Q_{\mathrm{s}}(x_g) \gg \Lambda_{\mathrm{QCD}}$ and that the collinear factorization will not be sufficient and will need to be supplemented by the calculations which include the saturation effects in the target nucleus. The formalism to describe this process taking into account saturation effects at small x has been developed in [18, 19]. The leading order cross section for the production of the hadron at rapidity y and with transverse momentum p_{T} is given by the hybrid factorization formula which involves integrated parton distribution functions for the incoming projectile proton and the unintegrated gluon distribution functions in the nucleus. In this formalism, the transverse momentum of the hadron comes entirely from the transverse momentum of the unintegrated gluon distribution. At lowest order, this formalism was successfully used to describe the single inclusive production of hadrons in deuteron–nucleus collisions at RHIC. The full NLO calculation was performed in [20]. It took into account real and virtual corrections to the production of the final state parton, but most importantly it was shown that the hybrid factorization holds in this limit. That is the appropriate divergencies are factorized into the corresponding distribution functions. The collinear divergencies are factorized into the parton distribution functions of the incoming projectile and the hadron fragmentation function and the rapidity divergence is factorized into the small-x evolution of the target unintegrated distribution function. The corresponding hard factor remains finite. The numerical calculations using this formalism at NLO were performed in [21] and compared with the experimental data. The NLO calculation has a reduced dependence on the renormalization and factorization scale, as expected. What was, however, very surprising was that the cross section turned negative for certain value of the transverse momentum. The point at which this breakdown of the calculation occurred depended on the rapidity. The higher the rapidity, the wider range of the applicability of this calculation. In subsequent works, the exact matching of this calculation to the collinear calculation was performed as well as some kinematical improvements [22, 23], which, however, do not cure the problem of the negativity completely and thus further research is needed in this direction. More research is needed to determine the correct matching of the non-perturbative effects together with the perturbative saturation.

4. Impact parameter dependence

One of the most challenging problems in the high energy scattering of hadrons is the issue of the impact parameter dependence. So far, the small-x calculations typically do not include the impact parameter profile, and the solutions are integrated over this variable. On the other hand, we know that the profile in impact parameter can provide the important information about the unitarity limit as well as is vital to understand the correlations and the multi-parton interactions. The solution to the nonlinear BK equation with impact parameter has been obtained in [24, 25]. The solutions in the LL approximation demonstrate that the amplitude suffers from the unphysical Coulomb tails in impact parameter, the consequence of the lack of the confinement scale in the problem. In order to make this result more physical, a cutoff is necessary in the form of the mass parameter, which will effectively change the functional behavior of the amplitude at large values of the impact parameter. This was performed in [25, 26], where it was shown that indeed such cutoffs lead to the exponential tails in impact parameter. Preliminary phenomenology with inclusive cross section and with diffractive production of vector mesons in deep inelastic scattering was also performed, where it was demonstrated that such evolution is compatible with the experimental data.

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