PARTICLE PRODUCTION AT NLO IN THE HYBRID FORMALISM*

T. Altinoluk[a], N. Armesto[a], G. Beuf[b], A. Kovner[c]
M. Lublinsky[b]

[a]Departamento de Física de Partículas and IGFAE
Universidade de Santiago de Compostela
15706 Santiago de Compostela, Galicia, Spain
[b]Department of Physics, Ben-Gurion University of the Negev
Beer Sheva 84105, Israel
[c]Physics Department, University of Connecticut
2152 Hillside Road, Storrs, CT 06269-3046, USA

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We reconsider the perturbative next-to-leading calculation of single inclusive hadron production in the framework of the hybrid formalism, applied to hadron production in proton–nucleus collisions. We introduce the explicit requirement that fast fluctuations in the projectile wave function which only exists for a short time are not resolved by the target. This Ioffe time cut-off also strongly affects the next-to-leading order terms.

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1. Introduction

Single inclusive hadron production in pA collisions within the hybrid formalism [1] is a very interesting problem that has been addressed in recent years. Within this framework, the wave function of the projectile proton is treated in the spirit of the collinear factorization. The perturbative corrections to this wave function are provided by the usual QCD perturbative splitting processes. The target, on the other hand, is treated as a distribution of strong color fields which during the scattering process transfer transverse momentum to the propagating partonic contributions. The real corrections at next-to-leading order (NLO) to this process were first calculated in [2] which was followed by the full NLO result in [3]. However, the numerical studies [4] indicate very strong effects of NLO corrections, with cross sections even becoming negative at moderate transverse momenta.

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2. Improved NLO calculation

Recently, the single inclusive gluon production cross section in proton–nucleus collisions at NLO have been reconsidered in [5] in order to improve the existing results in the literature. It has been shown that there are certain important elements missing in the existing calculations which are listed and discussed below in detail.

2.1. The choice of frame

Even though the cross section can be calculated in any Lorentz frame, and the results should not depend on the choice of the frame, it is advantageous to perform the calculation in a frame where it is simplest. We work in the frame where most of the energy of the process is carried by the target. We refer to it as PROJ (projectile frame).

In this frame, the target moves very fast and carries almost all the energy of the process. The projectile, on the other hand, moves fast enough to be able to accommodate parts with momentum fraction x_p but not so fast that it develops a large low-x tail.

We introduce the initial energy s_0 as an auxiliary quantity in our set up. Our final results do not depend on it explicitly but it turns out to be a useful concept. This energy is arbitrary, except it is required to be high enough, so that the eikonal approach is valid at $s > s_0$. Starting from this energy, we can evolve the target according to the high-energy evolution. The energy s_0 is achieved by boosting the projectile from its rest frame to rapidity $Y_{\rm P}$, and the target from its rest frame to rapidity $Y_{\rm T}^0$, so that

$$\begin{aligned} s_0 &= 2P^+_{\rm P,PROJ}P^{-0}_{\rm T}\,, \qquad & P^+_{\rm P,PROJ} &= \frac{M_{\rm P}}{\sqrt{2}}\,e^{Y_{\rm P}}\,, \\ P^{-0}_{\rm T} &= \frac{M_{\rm T}}{\sqrt{2}}\,e^{Y^0_{\rm T}}\,, & P^-_{\rm T} &= \frac{M_{\rm T}}{\sqrt{2}}\,e^{Y^0_{\rm T}+Y_{\rm T}}\,. \end{aligned} \tag{1}$$

Starting from this initial energy s_0, the energy of the process is increased further by boosting the target. Thus, in our setup, the projectile wave function at any energy is evolved only to rapidity $Y_{\rm P} = \ln\frac{1}{x_p} + Y_0$, with Y_0 being a fixed number of the order of one and $x_p = p^+/P^+_{\rm P}$.

The target, on the other hand, is evolved by $Y_{\rm T} = \ln\frac{s}{s_0}$, where s is the total center-of-mass squared energy of the process. The initial condition for the evolution of the target wave function has to be specified at $Y^0_{\rm T}$.

2.2. The Ioffe time restriction

One of the most important missing pieces in the previous calculations of single inclusive particle production at NLO is the so-called Ioffe time constraint on the lifetime of the pairs formed in a splitting process. Actually, the Ioffe time restriction provides a consistent description on what will be resolved by the target during the interaction. Only those pairs whose coherence time (Ioffe time) is greater than the propagation time through the target can be resolved by the target.

Let us explain how the Ioffe time constraint enters the calculation, focusing only on the quark channel for simplicity. The parton level cross section of scattering of a projectile quark at leading order is

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = \frac{1}{(2\pi)^2}\int d^2x d^2y\; e^{ip_\perp(x-y)} s_{Y_\mathrm{T}}(x,y)\,, \tag{2}$$

where $s_{Y_\mathrm{T}}(x,y)$ is the fundamental dipole scattering amplitude at rapidity Y_T, defined in terms of the eikonal scattering factors as

$$s(x,y) = \frac{1}{N_\mathrm{c}}\mathrm{tr}\left[S_\mathrm{F}(x)S_\mathrm{F}^\dagger(y)\right] \tag{3}$$

with $S_\mathrm{F}(x)$ the Wilson line for propagation of a high-energy parton in the fundamental representation of $\mathrm{SU}(N_\mathrm{c})$ at transverse position x. At NLO, the quark splits into a quark–gluon pair in the projectile wave function with probability of the order of α_s, and the wave function of the dressed quark state with transverse momentum $n_\perp$ and longitudinal momentum $x_B P^+$ reads

$$\begin{aligned}\big|(q)\,x_BP^+,n_\perp,\alpha,s\big\rangle_\mathrm{D} &= \int_x e^{in_\perp x}\Big\{A^q\,\big|(q)\,x_BP^+,x,\alpha,s\big\rangle\\ &+g\int_{\Omega,y,z} F_{(qg)}\left(x_BP^+,\xi,y-x,z-x\right)_{s\bar{s};j}\,t^a_{\alpha\beta}\,\big|(q)\;p^+,y,\beta,\bar{s};(g)\;q^+,z,a,j\big\rangle\Big\}.\end{aligned} \tag{4}$$

Here, s and $\bar{s}$ are the quark spin indices; j is the gluon polarization index, α,β are fundamental and a are adjoint color indices. We use the notation Ω for the longitudinal phase space in $+$ components for the splitting.

The dressed quark now scatters on the target and produces final state particles. Within the "hybrid" formalism, the scattering of the quark–gluon pair is treated as a completely coherent process where each parton picks an eikonal phase during the interaction with the target. However, this is only possible if the coherence time (Ioffe time) of the configuration is greater than the propagation time through the target. Only the qg pairs that satisfy the relation

$$\frac{2(1-\xi)\xi x_BP^+}{k_\perp^2} > \tau\,, \tag{5}$$

where τ is a fixed time scale determined by the longitudinal size of the target, scatter coherently. This scale enters the calculation via the initial energy: $P^+/\tau = s_0/2$.

Taking into account the Ioffe time constraint for a quark state amounts to modifying the splitting function F_{qg}

$$F_{(qg)} = \frac{-i}{\sqrt{2\xi x_B P^+}} \left\{ \delta_{s\bar{s}} \delta_{ij} (2-\xi) - i\epsilon_{ij} \sigma^3_{s\bar{s}} \xi \right\} \delta^2 \Big(x - [(1-\xi)y + \xi z] \Big) A^i_{\xi,x_B}(y-z) \,, \tag{6}$$

where $A^i_{\xi,x_B}(y-z)$ is the modified Weizsäcker–Williams field which is defined as

$$A^i_{\xi,x_B}(y-z) = -\frac{1}{2\pi} \, \frac{(y-z)^i}{(y-z)^2} \, \left[1 - \mathrm{J}_0 \left(|y-z| \sqrt{2\xi \frac{x_p P^+}{\tau}} \right) \right] . \tag{7}$$

Note that neglecting the Ioffe time constraint amounts to going back to the standard Weizsäcker–Williams field. Moreover, with the Ioffe time constraint, the relative contribution of short distances is suppressed.

2.3. Rapidity evolution

Another important issue to settle is the rapidity up to which the target should be evolved. The procedure set out in [3, 4] is to evolve the target to rapidity $Y_g = \ln \frac{1}{x_g}$ with $x_g = \frac{p_\perp}{\sqrt{s}} e^{-\eta}$. The reason for choosing this particular value of Y_g in [3] is based on the following kinematic argument. At leading order, the incoming projectile parton carries momentum $(p^+, 0, 0)$. The parton measured in the final state has the same + component of momentum but a nonzero transverse momentum $p_\perp$ and is on shell. This means that during the scattering, it picks up − component of momentum $p^- = \frac{p_\perp^2}{2p^+} = e^{-\eta} \frac{p_\perp}{\sqrt{2}}$ from the target. If one assumes that this momentum has been transferred to the projectile parton by a single gluon of the target, the gluon in question must have carried at least this amount of p^-, and therefore had to have the longitudinal momentum fraction of the target

$$x_g = \frac{p^-}{P^-} = e^{-\eta} \frac{p_\perp}{\sqrt{s}} \,. \tag{8}$$

On the other hand, the high-energy evolution (in the dilute regime) has the property that any hadronic wave function is dominated by the softest gluons. One, thus, may conclude that x_g is the longitudinal momentum fraction of the softest gluons in the target wave function, and thus the target has to be evolved to Y_g.

On closer examination, however, it transpires that this argument does not hold water. It overlooks the fact that the target is, in fact, dense. For the dense target, the projectile parton undergoes multiple scatterings, and therefore picks up momentum p^- not from a single target gluon, but from several ones. This means that x_g is actually an upper bound on the momentum fraction of the target gluons, and therefore Y_g only gives a lower bound on the rapidity up to which the target wave function has to be evolved. In fact, it is very natural that the total rapidity Y_{T} should not depend on the transverse momentum of the produced particle, rather than depend on it as in (8). Recall that in the dense scattering regime, the transverse momentum of the scattered parton "random walks" as the parton propagates through the target. Thus, the total transverse momentum is proportional to the square root of the number of collisions with the target gluons, $p_\perp^2 \propto N_g$. On the other hand, the transferred p^- does not random walk, since all the gluons in the target have p^- of the same sign. Thus $p^- \propto N_g$, which is perfectly consistent with the relation between p^- and $p_\perp$ that follows from the onshellness condition of the outgoing parton. Therefore, increasing $p_\perp$ of the observed parton (at fixed p^+), while increasing the total p^- acquired by the projectile parton, does not change the fraction of longitudinal momentum of individual gluons in the target wave function that participate in the scattering, and therefore does not affect the value of Y_{T}. It is thus important to use Y_{T} rather than Y_g.

3. Results

The final expressions are quite long due to the fact that there are multiple production channels. We refer the reader to [5] for details. However, all the channels have the same structure. For simplicity, let us take a closer look to the hadron production from fragmentation of the final state quark that originates from the quark in the projectile wave function

$$\frac{d\sigma^{q\to H}}{d^2p_\perp d\eta} = \left[\frac{d\sigma^{q\to H}}{d^2p_\perp d\eta}\right]_{\mathrm{LO}} + \left[\frac{d\sigma^{q\to H}}{d^2p_\perp d\eta}\right]_{\mathrm{NLO}} . \tag{9}$$

The LO piece of the cross section reads

$$\left[\frac{d\sigma^{q\to H}}{d^2p_\perp d\eta}\right]_{\mathrm{LO}} = \frac{1}{(2\pi)^2}\int \frac{d\zeta}{\zeta^2} D_H^q(\zeta)\frac{x_p}{\zeta} f_{p_\perp}^q\left(\frac{x_p}{\zeta}\right)\int\limits_{y\bar{y}} e^{i\frac{p_\perp}{\zeta}(y-\bar{y})} s_{Y_{\mathrm{T}}}(y,\bar{y})\,, \tag{10}$$

where $s_{Y_{\mathrm{T}}}(y,\bar{y})$ is the dipole cross section that is evolved with the BK equation from the initial rapidity Y_{T}^0 by the rapidity interval $Y_{\mathrm{T}} = \ln(s/s_0)$. The NLO piece of the cross section has two different parts

$$\left[\frac{d\sigma^{q\to H}}{d^2p_\perp d\eta}\right]_{\mathrm{NLO}} = \left[\frac{d\sigma^{q\to H}}{d^2p_\perp d\eta}\right]_{Y_g\to Y_{\mathrm{T}}} + \delta\sigma^q\,. \tag{11}$$

The first term in Eq. (11) is the term that is independent of the Ioffe time restriction and it coincides with the results of [3] if the target is evolved up to rapidity Y_{T} instead of Y_g. The second term is the new term that does not exist in the previous calculations which carries the information about the Ioffe time restriction. Its explicit form reads

$$\delta\sigma^q = \frac{g^2}{(2\pi)^3} N_{\mathrm{c}} x_p f^q_{\mu^2}(x_p) \int_0^1 \frac{d\xi}{\xi} \int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})} \left[A^i_\xi(y-z) - A^i_\xi(\bar{y}-z)\right]^2 \times \left[s(y,z)s(z,\bar{y}) - s(y,\bar{y})\right] , \tag{12}$$

where $A^i_\xi(y-z)$ and $A^i_\xi(\bar{y}-z)$ are the modified Weizsäcker–Williams field defined in Eq. (7). This extra term has been included in a recent numerical analysis [6] and it shows a clear improvement.

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