IMPROVED EFFECTIVE TMD FACTORIZATION FOR FORWARD DIJET PRODUCTION IN p–A COLLISIONS* **

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We propose a new factorization formula for forward dijet production in dilute–dense collisions which generalizes the hybrid high energy factorization to the regime of the gluon saturation. The approach uses several unintegrated gluon distributions convoluted with appropriate gauge invariant off-shell hard factors calculated in two independent ways, in particular using the color decomposition and the spinor helicty method. The new formula extrapolates the Transverse Momentum Dependent (TMD) factorization approach beyond the leading-twist and towards the small x.

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1. Introduction

One of the approaches to proton–nucleus collisions in the gluon saturation [1] regime is the Color Glass Condensate (CGC) [2]. In particular, when the observed final states consist in particles produced in the forward direction, the underlying partonic kinematics is highly asymmetric, *i.e.* the longitudinal fractions of the parent hadrons satisfy $x_A \ll x_B$, where x_B is the fraction of the proton momentum p_B and x_A is the fraction of the nucleus momentum p_A (see Fig. 1). Often, for such a setup, it is assumed that the large-x parton is treated using the collinear factorization, *i.e.* it evolves according to the DGLAP evolution, while for the small-x parton all the complicated machinery of the CGC is used. Such an approach is often called the *hybrid approach* [3]. Let us recall that in CGC, a process is described by partonic wave functions propagating through the dense color field of nucleus

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expressed in terms of various correlators of the Wilson lines. In general, those correlators are process-dependent and thus there is no factorization. Despite the complexity, the CGC approach has been successfully applied to the phenomenology at RHIC [4].

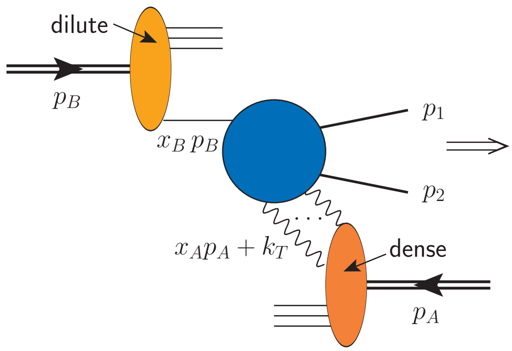

Fig. 1. Forward dijet production in the dilute–dense collision within the hybrid factorization approach. The upper blob undergoes the DGLAP evolution, while the bottom blob corresponds to small-x type of evolution.

It was argued in [5] that at the LHC energies, one can establish an 'effective' factorization for dijets, which is capable of describing processes in the saturation domain in terms of several universal building blocks (unintegrated gluon distributions). This approach utilized a power counting in CGC to establish a connection with the Transverse Momentum Dependent (TMD) factorization approach and thus the definition of gluon distributions was possible. However, by removing the higher twists, the link to the High Energy Factorization (HEF) [1, 6, 7] was lost. It is not desired, as HEF is quite successful in describing LHC proton–proton dijet data, provided suitable unintegrated gluon distributions are used (see *e.g.* [8–10]). Here, we report on a new approach which extrapolates the result of [5] to include the higher twist contributions and thus makes it possible to apply in the dijet decorrelation regime.

2. Scale regimes

Let us start by a short summary of the potentially large scales present in our problem and the available approaches.

The process under consideration is depicted in Fig. 1. The jets are produced with a certain average transverse momentum of the order of P_{T} which by definition sets the largest scale. The jets with momenta $p_{\mathrm{T}1}$, $p_{\mathrm{T}2}$ are decorrelated by an amount $k_{\mathrm{T}} = |\vec{k}_{\mathrm{T}}| = |\vec{p}_{\mathrm{T}1} + \vec{p}_{\mathrm{T}2}|$ due to untagged emissions. In the spirit of the hybrid formalism, the transverse momentum $\vec{k}_{\mathrm{T}}$

enters the nonperturbative matrix elements parameterizing the nucleus. The momentum disbalance $k_{\rm T}$ sets another, possibly large, scale in our problem. Finally, the third scale is given by the saturation scale $Q_{\rm s}$ which increases with x_A decreasing. Below, we list the scale regimes depending on the relative magnitude of this three scales involved:

— *Color Glass Condensate*. This is the best suited approach when all the three scales are of the same order $Q_{\rm s} \sim k_{\rm T} \sim P_{\rm T}$.

— *Collinear Factorization*. If x_A, x_B are both large, we are well outside the saturation regime (the saturation scale is small, $Q_{\rm s} \sim \Lambda_{\rm QCD}$). Suppose, in addition, that the jets are produced back-to-back, $k_{\rm T} \ll P_{\rm T}$. Then, essentially, we are in the standard leading twist collinear factorization regime. The cross section is given by the convolution of collinear (or 'integrated') PDFs and on-shell amplitude squared.

— *High Energy Factorization* (HEF). If x_A is small but we are still outside the saturation regime, $Q_{\rm s} \ll k_{\rm T} \sim P_{\rm T}$, the BFKL-type resummation [11] is needed. This leads to the High Energy or $k_{\rm T}$-factorization [1, 6, 7] where the cross section is expressed in terms of *Unintegrated Gluon Distribution* (UGD) (for nucleus) and off-shell gauge invariant amplitude squared. This approach takes into account higher twist corrections $\mathcal{O}(k_{\rm T}/P_{\rm T})$.

— *Generalized TMD Factorization*. When we are in the saturation regime and the dijet decorrelation region, *i.e.* $k_{\rm T} \sim Q_{\rm s} \ll P_{\rm T}$, one can establish an 'effective' factorization [5]. First, studying the CGC approach in that limit, one can show that at large $N_{\rm c}$ the cross section involves two distinct UGDs: the so-called 'dipole' UGD G_2 and the Weizsacker–Williams UGD G_1. They are universal, *i.e.* can be accessed in different processes. Second, these two UGDs can be recognized also within the *Transverse Momentum Dependent* (TMD) factorization approach, *i.e.* when constructing TMD gluon distributions using [12] and taking the large $N_{\rm c}$ and small x limits. This approach provides a tool to describe dijets in proton–nucleus collisions in terms of factorization.

— When we are in the saturation regime and the dijet decorrelation region, the HEF is no longer valid. It turns out that one needs more UGDs than one (on CGC side, it can bee seen as a need of correlators of more than two Wilson lines). In [13], an extension of the previous approach has been proposed, which generalizes HEF to the saturation regime. Below, we refer to this approach as *Improved TMD Factorization*, although one may call it an improved high energy factorization as well.

3. Improved TMD factorization

Our main result [13] can be summarized by the following factorization formula for the cross section for forward dijet production in dilute–dense collisions:

$$\frac{d\sigma^{pA\to\text{dijets}+X}}{d^2P_{\mathrm{T}}d^2k_{\mathrm{T}}dy_1dy_2} = \frac{\alpha_{\mathrm{s}}^2(\mu)}{(x_Ax_BS)^2}\sum_{a,c,d}x_Bf_a(x_B,\mu)$$
$$\times\sum_{i=1}^{2}K^{(i)}_{ag^*\to cd}\Phi^{(i)}_{ag\to cd}(x_A,k_{\mathrm{T}},\mu)\,\frac{1}{1+\delta_{cd}}\,, \quad (1)$$

where y_1, y_2 are the rapidities of the outgoing parton momenta p_1, p_2, $P_{\mathrm{T}}=(1-z)p_{\mathrm{T}1}-zp_{\mathrm{T}2}$ with $z=p_1^+/(p_1^++p_2^+)$. The f_a is the collinear PDF for parton a, while $\Phi^{(i)}_{ag\to cd}$, $i=1,2$ are the TMD gluon distributions in a nucleon corresponding to subprocess $ag\to cd$. Finally, $K^{(i)}_{ag^*\to cd}$, $i=1,2$ are the hard factors calculated from the gauge invariant color-ordered amplitudes corresponding to off-shell $ag^*\to cd$ subprocess. When calculating the off-shell matrix elements with gluons, we have used methods of [14–16] (corresponding also to the Lipatov's effective vertices [17]). The hard off-shell factors are:

$$K^{(1)}_{gq\to gq} = -\frac{\overline{u}\left(\overline{s}^{\,2}+\overline{u}^{\,2}\right)}{2\overline{t}\hat{t}\hat{s}}\left(1+\frac{\overline{s}\hat{s}-\overline{t}\hat{t}}{N_{\mathrm{c}}^2\,\overline{u}\hat{u}}\right), \quad (2)$$

$$K^{(2)}_{gq\to gq} = -\frac{C_F}{N_{\mathrm{c}}}\,\frac{\overline{s}\left(\overline{s}^{\,2}+\overline{u}^{\,2}\right)}{\overline{t}\hat{t}\hat{u}}\,, \quad (3)$$

$$K^{(1)}_{gg\to q\overline{q}} = \frac{1}{2N_{\mathrm{c}}}\,\frac{\left(\overline{t}^{\,2}+\overline{u}^{\,2}\right)\left(\overline{u}\hat{u}+\overline{t}\hat{t}\,\right)}{\overline{s}\hat{s}\hat{t}\hat{u}}\,, \quad (4)$$

$$K^{(2)}_{gg\to q\overline{q}} = \frac{1}{4N_{\mathrm{c}}^2C_F}\,\frac{\left(\overline{t}^{2}+\overline{u}^{2}\right)\left(\overline{u}\hat{u}+\overline{t}\hat{t}-\overline{s}\hat{s}\right)}{\overline{s}\hat{s}\hat{t}\hat{u}}\,, \quad (5)$$

$$K^{(1)}_{gg\to gg} = \frac{N_{\mathrm{c}}}{C_F}\,\frac{\left(\overline{s}^{\,4}+\overline{t}^{\,4}+\overline{u}^{\,4}\right)\left(\overline{u}\hat{u}+\overline{t}\hat{t}\,\right)}{\overline{t}\hat{t}\overline{u}\hat{u}\overline{s}\hat{s}}\,, \quad (6)$$

$$K^{(2)}_{gg\to gg} = -\frac{N_{\mathrm{c}}}{2C_F}\,\frac{\left(\overline{s}^{\,4}+\overline{t}^{\,4}+\overline{u}^{\,4}\right)\left(\overline{u}\hat{u}+\overline{t}\hat{t}-\overline{s}\hat{s}\right)}{\overline{t}\hat{t}\overline{u}\hat{u}\overline{s}\hat{s}}\,, \quad (7)$$

where various Mandelstam variables are defined as

$$\hat{s} = (k_A+k_B)^2=(p_1+p_2)^2\,, \qquad \bar{s}=(x_Ap_A+k_B)^2\,, \quad (8)$$
$$\hat{t} = (p_2-k_B)^2=(p_1-k_A)^2\,, \qquad \bar{t}=(x_Ap_A-p_1)^2\,, \quad (9)$$
$$\hat{u} = (p_1-k_B)^2=(p_2-k_A)^2\,, \qquad \bar{u}=(x_Ap_A-p_2)^2 \quad (10)$$

with $k_A = x_A p_A + k_{\rm T}$ and $k_B = x_B p_B$ being the momenta of the incoming partons, with the first one being off-shell. The unintegrated gluon distributions corresponding to hard factors are:

$$\begin{aligned}
\Phi^{(1)}_{gq\to gq} &= \mathcal{F}^{(1)}_{qg}\,, &(11)\\
\Phi^{(2)}_{gq\to gq} &= \frac{1}{N_{\rm c}^2-1}\left(-\mathcal{F}^{(1)}_{qg}+N_{\rm c}^2\mathcal{F}^{(2)}_{qg}\right), &(12)\\
\Phi^{(1)}_{gg\to q\bar{q}} &= \frac{1}{N_{\rm c}^2-1}\left(N_{\rm c}^2\mathcal{F}^{(1)}_{gg}-\mathcal{F}^{(3)}_{gg}\right), &(13)\\
\Phi^{(2)}_{gg\to q\bar{q}} &= -N_{\rm c}^2\mathcal{F}^{(2)}_{gg}+\mathcal{F}^{(3)}_{gg}\,, &(14)\\
\Phi^{(1)}_{gg\to gg} &= \frac{1}{2N_{\rm c}^2}\left(N_{\rm c}^2\mathcal{F}^{(1)}_{gg}-2\mathcal{F}^{(3)}_{gg}+\mathcal{F}^{(4)}_{gg}+\mathcal{F}^{(5)}_{gg}+N_{\rm c}^2\mathcal{F}^{(6)}_{gg}\right), &(15)\\
\Phi^{(2)}_{gg\to gg} &= \frac{1}{N_{\rm c}^2}\left(N_{\rm c}^2\mathcal{F}^{(2)}_{gg}-2\mathcal{F}^{(3)}_{gg}+\mathcal{F}^{(4)}_{gg}+\mathcal{F}^{(5)}_{gg}+N_{\rm c}^2\mathcal{F}^{(6)}_{gg}\right), &(16)
\end{aligned}$$

where the $\mathcal{F}^{(i)}_{ag}$ objects are matrix elements of bilocal gluon operators with various Wilson line insertions (appearing due to resummation of collinear gluons). For example

$$\mathcal{F}^{(1)}_{qg}=2\!\int\!\frac{d\xi^+d^2\xi_{\rm T}}{(2\pi)^3p_A^-}e^{\,ix_Ap_A^-\xi^+-i\vec{k}_{\rm T}\cdot\vec{\xi}_{\rm T}}\,\langle p_A|\,{\rm Tr}\left\{F^{+i}\left(\xi\right)\mathcal{U}^{[-]\dagger}F^{+i}\left(0\right)\mathcal{U}^{[+]}\right\}|p_A\rangle \tag{17}$$

with the gluon field strength tensors in the fundamental representation and the gauge links defined as $\mathcal{U}^{[\pm]}=U(0,\pm\infty;0_{\rm T})U(\pm\infty,\xi^+;\xi_{\rm T})$, where the path exponential is $U(a,b;x_{\rm T})=\mathcal{P}\exp\left[ig\int_a^b dx^+A_a^-(x^+,x_{\rm T})t^a\right]$. The rest of the matrix elements are (with Fourier transforms omitted and matrix element replaced by an average for compactness):

$$\begin{aligned}
\mathcal{F}^{(2)}_{qg} &\sim \left\langle{\rm Tr}\left\{F\left(\xi\right)\frac{{\rm Tr}\,\mathcal{U}^{[\Box]}}{N_{\rm c}}\mathcal{U}^{[+]\dagger}F\left(0\right)\mathcal{U}^{[+]}\right\}\right\rangle, &(18)\\
\mathcal{F}^{(1)}_{gg} &\sim \left\langle{\rm Tr}\left\{F\left(\xi\right)\frac{{\rm Tr}\,\mathcal{U}^{[\Box]}}{N_{\rm c}}\mathcal{U}^{[-]\dagger}F\left(0\right)\mathcal{U}^{[+]}\right\}\right\rangle, &(19)\\
\mathcal{F}^{(2)}_{gg} &\sim \frac{1}{N_{\rm c}}\left\langle{\rm Tr}\left\{F\left(\xi\right)\mathcal{U}^{[\Box]\dagger}\right\}{\rm Tr}\left\{F\left(0\right)\mathcal{U}^{[\Box]}\right\}\right\rangle, &(20)\\
\mathcal{F}^{(3)}_{gg} &\sim \left\langle{\rm Tr}\left\{F\left(\xi\right)\mathcal{U}^{[+]\dagger}F\left(0\right)\mathcal{U}^{[+]}\right\}\right\rangle, &(21)
\end{aligned}$$

$$\mathcal{F}_{gg}^{(4)} \sim \left\langle \mathrm{Tr}\left\{ F\left(\xi\right) \mathcal{U}^{[-]\dagger} F\left(0\right) \mathcal{U}^{[-]} \right\} \right\rangle , \tag{22}$$

$$\mathcal{F}_{gg}^{(5)} \sim \left\langle \mathrm{Tr}\left\{ F\left(\xi\right) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} F\left(0\right) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right\} \right\rangle , \tag{23}$$

$$\mathcal{F}_{gg}^{(6)} \sim \left\langle \mathrm{Tr}\left\{ F\left(\xi\right) \mathcal{U}^{[+]\dagger} F\left(0\right) \mathcal{U}^{[+]} \right\} \left(\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_{\mathrm{c}}} \right)^2 \right\rangle . \tag{24}$$

Above, the gauge loops are defined as $\mathcal{U}^{[\square]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$.

Note, that the above formulae are all finite-N_{c} expressions. In order to obtain the 'effective' factorization *i.e.* to restore the universality, one needs to take the large N_{c} limit. In that limit, the surviving gluon distributions are $\mathcal{F}_{qg}^{(1)}$ and $\mathcal{F}_{gg}^{(3)}$ which correspond to 'dipole' UGD G_2 and the Weizsacker–Williams UGD G_1.

4. Summary

The factorization formula (1) is not a theorem of perturbative QCD, it however, has limiting cases which are solid results of QCD. The formula is valid in the saturation regime and in the whole range of dijet momentum disbalance. It can be considered as a generalization of the High Energy Factorization to the saturation regime. Its main property is a presence of several unintegrated gluon distributions which at large N_{c} limit are present in other processes like inclusive DIS or photon–jet production in hadron–hadron collisions, thus being (to certain extent) universal.

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