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IRRELEVANCE OF $f_0(500)$ IN BULK THERMAL PROPERTIES*

Francesco Giacosa[a,b], Viktor Begun[a], Wojciech Broniowski[a,c]

[a]Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland
[b]Institute for Theoretical Physics, Goethe-University
60438, Frankfurt am Main, Germany
[c]The Henryk Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences
Radzikowskiego 152, 31-342, Kraków, Poland

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We discuss why the scalar–isoscalar resonance $f_0(500)$ should, in practice, not be included in thermal models describing the freeze-out of heavy-ion collisions. Its contribution to pion multiplicities is, in principle, relevant since it is light and that it decays only to pions. However, it is cancelled to a very good numerical precision by the non-resonant scalar–isotensor repulsion among pions. Our approach is an application of a well-known theorem relating spectral function to phase shifts. The numerical results are solely based on pion–pion scattering data and thus model independent.

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1. Introduction

The scalar–isoscalar resonance $f_0(500)$ is now firmly established [1]. The Particle Data Group (PDG) reports the position of its pole in the range of (400–550)$-i$(200–350) [2]. Investigations based on dispersive analysis show even smaller errors: $(400 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$ in Ref. [3] and $(457^{+14}_{-13}) - i(279^{+11}_{-7})$ in Ref. [4].

The resonance $f_0(500)$ is the lightest scalar state; moreover, it decays only into pions. Then, one is lead to think that $f_0(500)$ is important for the determination of pion multiplicities in thermal models for relativistic heavy-ion collisions (see *e.g.* Refs. [6, 7] and references therein). Indeed, a simple inclusion of $f_0(500)$ as a Breit–Wigner resonance would lead to a sizable increase (about 3–5% [8]) of pions. However, in these proceedings (based on the findings of Ref. [5]), we show that this conclusion is not correct. In fact,

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when the repulsion of pion–pion interaction in the scalar–isotensor channel is properly taken into account using the formalism developed in Refs. [9–11], the effect of $f_0(500)$ cancels to a very good numerical accuracy. We show this cancellation in a model-independent way, since the only input is given by the well-known pion–pion scattering data in these two scalar channels.

As a net result, one can neglect in thermal models both the scalar–isoscalar attraction due to $f_0(500)$ and the non-resonant scalar–isotensor repulsion.

2. Cancellation of $f_0(500)$

A successful description of hadron emissions at the freeze-out of relativistic heavy-ion collisions is achieved with the help of thermal models. For simplicity, we restrict our presentation to a gas which includes stable pions ($I=0$, $J^{PC}=0^{-+}$, where I stays for isospin, J for the total spin, and P and C for parity and charge-conjugation), the ρ-resonance ($I=1$, $J^{PC}=1^{--}$), the resonance $f_0(500)$ ($I=0$, $J^{PC}=0^{++}$), and the non-resonant contribution of the repulsion in the $I=2$, $J^{PC}=0^{++}$ channel. (Other contributions with different I and J^{PC} correspond to heavier mesons and are neglected here.)

The logarithm of the partition function Z is given by the sum of contributions of all channels

$$\ln Z = \ln Z_\pi + \ln Z_{(1,1^{--})} + \ln Z_{(0,0^{++})} + \ln Z_{(2,0^{++})}\,.$$

All other thermodynamic quantities follow: $P=-T\ln Z/V$, $\varepsilon=-\partial_\beta \ln Z/V$, *etc.* For what concerns stable pions (we do not include chemical potentials), one has

$$\ln Z_\pi = 3V\int\limits_p \ln\left[1-e^{-\frac{\sqrt{\vec{p}^{\,2}+M_\pi^2}}{T}}\right]^{-1}, \qquad \int\limits_p = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\,,$$

where V is the volume, $\vec{p}$ the pion momentum, M_π the pion mass, and the factor 3 the isospin degeneracy. Following Refs. [9, 10], we can express the contribution in the channel $I=1$, $J^{PC}=1^{--}$ as

$$\ln Z_{(1,1^{--})} = 3\times 3\int\limits_0^{\Lambda_1} \mathrm{d}m \frac{\mathrm{d}\delta_{(1,1)}(m)}{\pi\mathrm{d}m}\int\limits_p \ln\left[1-e^{-\frac{\sqrt{p^2+m^2}}{T}}\right]^{-1}, \qquad (1)$$

where $\delta_{(1,1)}$ is the measured $\pi\pi$ phase shift as a function of $m=\sqrt{s}$. We set $\Lambda_1 = 1$ GeV as maximal energy in the integral, then only the ρ-meson is present in this range. The spectral function of the ρ-meson can be approximated as

$$\mathrm{d}_\rho(m) = \frac{\mathrm{d}\delta_{(1,1)}(m)}{\pi \mathrm{d}m} \,. \tag{2}$$

Thus, one can take into account the ρ-meson in a thermal gas in a model-independent way by introducing the well-known scattering data in Eq. (1). For a small width, $\mathrm{d}_\rho(m)$ can be well-approximated by a Breit–Wigner function, $\mathrm{d}_\rho(m) \simeq \frac{\Gamma}{2\pi}\left[(m-M_\rho)^2 + \Gamma^2/4\right]^{-1}$, and, in the limit of zero width, one correctly obtains $\mathrm{d}_\rho(m) = \delta(m - M_\rho)$. Thus, the example of the ρ-meson shows quite general features of a thermal gas. The approximation of using a Breit–Wigner expression — typically used in practice — emerges.

We now turn to the main topic of the present work. For $I = J = 0$, the contribution of $f_0(500)$ is included in the integral

$$\ln Z_{(0,0^{++})} = \int_0^{\Lambda_0} \mathrm{d}m \frac{\mathrm{d}\delta_{(0,0)}}{\pi \mathrm{d}m} \int_p \ln\left[1 - e^{-\frac{\sqrt{p^2+m^2}}{T}}\right]^{-1} \,, \tag{3}$$

where $\Lambda_0 \simeq 0.8$ GeV (far above the average mass of $f_0(500)$ but below $f_0(980)$, which is not considered here). The spectral function of the $f_0(500)$ is approximated by $\mathrm{d}_{f_0(500)}(m) = \frac{1}{\pi}\mathrm{d}\delta_{(0,0)}/\mathrm{d}m$. The form of $\mathrm{d}_{f_0(500)}(m)$ is far from being a Breit–Wigner [5] and is even not normalized to unity. This is in agreement with the fact that the resonances $f_0(500)$ is not the chiral partner of the pion and is not a quark–antiquark field [1] (the chiral partner of π, the 'true' σ of linear Sigma Models, should be identified with the heavier scalar resonance $f_0(1370)$ [12]).

As a last step, we consider the joint contribution of both $I = 0$ and $I = 2$ channels

$$\ln Z_{(0,0^{++})} + \ln Z_{(2,0^{++})} = \int_0^{\Lambda_0} \mathrm{d}m \left[\frac{\mathrm{d}\delta_{(0,0)}}{\pi \mathrm{d}m} + 5\frac{\mathrm{d}\delta_{(2,0)}}{\pi \mathrm{d}m}\right] \int_p \ln\left[1 - e^{-\frac{\sqrt{p^2+m^2}}{T}}\right]^{-1} \,, \tag{4}$$

where the factor 5 in front of $\mathrm{d}\delta_{(2,0)}/\mathrm{d}m$ is the degeneracy $2I+1$. *Data* on pion–pion scattering show the following peculiar fact [5]:

$$\frac{\mathrm{d}\delta_{(0,0)}}{\pi \mathrm{d}m} + 5\frac{\mathrm{d}\delta_{(2,0)}}{\pi \mathrm{d}m} \simeq 0 \quad \text{for} \quad 2M_\pi \leq m \lesssim 0.8 \text{ GeV} \,, \tag{5}$$

which is valid to a very good numerical accuracy. Then, $\ln Z_{(0,0^{++})} + \ln Z_{(2,0^{++})} \simeq 0$! The contribution of $f_0(500)$ cancels.

3. Conclusions

In these proceedings, we have shown that the resonance $f_0(500)$ can, in practice, be neglected in all isospin-averaged quantities of a thermal hadronic gas, *e.g.* for pion multiplicities. Then, the proton-to-pion puzzle becomes even stronger, leaving other explanations open [13]. On the other hand, in correlation studies of pion-pair production, the cancellation does not occur, hence $f_0(500)$ may play a relevant role [14].

A similar cancellation occurs for the not yet confirmed $K_0^*(800)$ ($I = 1/2, J^{PC} = 0^{++}$, *e.g.* Ref. [15] and references therein), whose contribution is (only partly) compensated by the $I = 3/2, J^{PC} = 0^{++}$ channel [5, 16].

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