

HBT RADII FROM THE MULTIPOLE BUDA–LUND MODEL*

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The Buda–Lund model describes an expanding hydrodynamical system with ellipsoidal symmetry and fits the observed elliptic flow and oscillating HBT radii successfully. The ellipsoidal symmetry can be characterized by the second order harmonics of the transverse momentum distribution, and observed in the azimuthal oscillation of the HBT radii measured *versus* the second order reaction plane. The model can be changed to describe the experimentally indicated higher order azimuthal asymmetries. In this paper, we detail an extension of the Buda–Lund hydro model to investigate higher order flow harmonics and triangular azimuthal oscillations of the HBT radii.

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1. Introduction

The investigation of Bose–Einstein correlation functions and their widths, the so-called HBT radii, is a useful tool to measure properties of the strongly interacting quark–gluon plasma (sQGP) such as the size or the asymmetries

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of the medium. Originally, the method had been discovered by Hanbury Brown and Twiss [1] in radioastronomy, and Goldhaber and collaborators developed it to measure the size of the soft particle emitting source in heavy-ion reactions [2]. If the source is not azimuthally symmetric, then the HBT radii also depend on the azimuthal angle. To observe this dependence of the HBT radii, the corresponding reaction plane has to be taken into account. The experiments show that (*e.g.* in [3]) besides a second order anisotropy, triangular (3rd order) oscillations can also be observed. In our paper, we utilize the Buda–Lund model, the elliptical version of which [4] can describe the elliptic flow (v_2) and the second order oscillations of the HBT radii [5]. In the extended Buda–Lund model presented here, the triangular flow (v_3) and third order HBT oscillations are also non-zero, in addition to the elliptical version of the model.

2. Description of the geometry

Generally, the source function assumed in a hydrodynamical model is

$$S(x, p) d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{B(x, p) + s_q} \quad \text{with} \quad B(x, p) = \exp \left[\frac{p_\mu u^\mu(x) - \mu}{T(x)} \right], \quad (1)$$

where g is the degeneracy factor, $p^\mu d^4\Sigma_\mu(x)$ is the Cooper–Frye factor, $B(x, p)$ is the Boltzmann distribution and s_q is the usual quantum statistic term. In the temperature profile and in the fugacity term, there is the scale parameter s which ensures the asymmetry of the space-time distribution. The inverse temperature profile and the fugacity term in the Buda–Lund model can be written as

$$\frac{1}{T(x)} = \frac{1}{T_0} (1 + a^2 s) \quad \text{and} \quad \frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s, \quad (2)$$

where $a^2 = \frac{T_0 - T_s}{T_s}$ and T_0 (T_s) means the temperature of the center (surface) of the expanding fireball at the freeze-out. Let us write up s in cylindrical coordinate system instead of Cartesian with $r^2 = r_x^2 + r_y^2$ and $\cos(\varphi) = \frac{r_x}{r}$, and prescribe the azimuthal angle dependence. In the elliptic version of the model, the scale parameter can be written as

$$s = \frac{r^2}{R^2} (1 + \epsilon_2 \cos(2\varphi)) + \frac{r_z^2}{Z^2}, \quad (3)$$

where R is the radial and Z is the longitudinal scale, ϵ_2 is the parameter controlling the 2nd order asymmetry. The scale parameter s can be generalized

to n^{th} order asymmetries as

$$s = \frac{r^2}{R^2} \left(1 + \sum_n \epsilon_n \cos(n(\varphi - \Psi_n)) \right) + \frac{r_z^2}{Z^2}, \quad (4)$$

where Ψ_n is the angle of the n^{th} order reaction plane and ϵ_n is the parameter controlling the n^{th} order azimuthal flow anisotropy. The velocity field distribution u^μ of the original Buda–Lund model is also elliptically asymmetric. We can generalize this velocity field in the present framework with the generalization of the velocity field potential Φ , defined as $u_\mu = \gamma(1, \nabla\Phi)$. This potential of the original elliptical model with cylindrical coordinates is

$$\Phi = Hr^2(1 + \chi_2 \cos(2\varphi)) + \frac{H_z}{2} r_z^2, \quad (5)$$

where χ_2 is the parameter of the 2nd order asymmetry of the velocity field, H is the radial and H_z the longitudinal Hubble-parameter. The n^{th} order generalized case can be expressed as

$$\Phi = Hr^2 \left(1 + \sum_n \chi_n \cos(n(\varphi - \Psi_n)) \right) + H_z r_z^2, \quad (6)$$

where χ_n is the parameter of the n^{th} order reaction plane. From these potential, the velocity field can be derived. With this way of generalization of velocity field and space-time distribution, in principle, any azimuthal asymmetry can be modeled. Let us note that there is a known solution of the relativistic hydrodynamics with similarly generalized scale parameter in [6], however, in that solution, the velocity field remains spherically symmetric.

3. Observables

Let us discuss here how the invariant transverse momentum distribution, the n^{th} order flows and the azimuthally sensitive HBT radii, can be derived from the model. The asymmetries and their effects on the observables have to be investigated *versus* the proper reaction plane. If we explore quantities in respect to the 2nd order reaction plane, we have to average on the angle between the 2nd and 3rd order reaction plane and *vice versa*. The invariant momentum distribution $N_1(p)$ can be calculated by simply integrating on the space-time variable x in $S(x, p)$ of Eq. (1). The flow coefficients can be obtained from the Fourier-series of the invariant momentum distribution as $v_n(p_t) = \langle \cos(n\alpha) \rangle_{N_1(p)}$, where α is the azimuthal angle of the emitted particle and the averaging is performed over $N_1(p)$. The investigations show that the n^{th} order asymmetry coefficients affect only the n^{th} order flow,

however, both of the spatial and the velocity field asymmetry have effects. For example, ϵ_2 and χ_2 have effect on the v_2 but they do not affect v_3 . The effect of ϵ_n and χ_n on v_n is illustrated in Fig. 1. There is an entanglement of the spatial and the velocity field asymmetry so the values of these cannot be extracted only from the measurements of the flows.

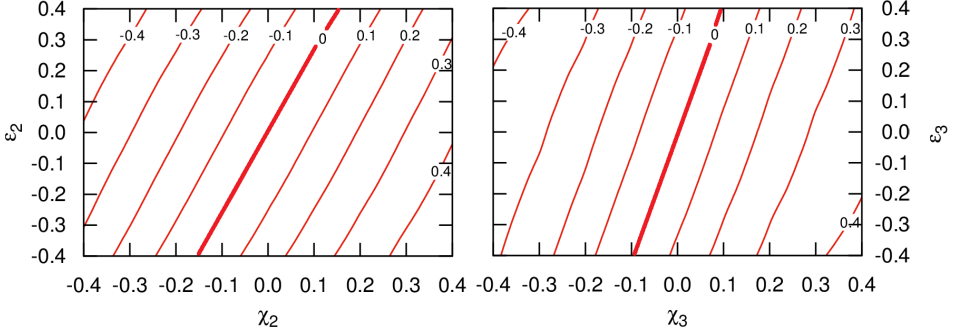


Fig. 1. Elliptic flow v_2 at $p_t = 1000$ MeV/ c as a function of χ_2 and ϵ_2 (left). Triangular flow v_3 at $p_t = 1000$ MeV/ c as a function of χ_3 and ϵ_3 (right).

The azimuthally sensitive HBT radii are important in the survey of the geometry and the size of the source. Generally, the two-particle momentum correlation function can be expressed as the Fourier-transform of the source function. Similarly to Ref. [8], the HBT radii can be calculated as

$$R_{\text{out}}^2 = \langle r_{\text{out}}^2 \rangle - \langle r_{\text{out}} \rangle^2 \quad \text{and} \quad R_{\text{side}}^2 = \langle r_{\text{side}}^2 \rangle - \langle r_{\text{side}} \rangle^2, \quad (7)$$

where $r_{\text{side}} = r \sin(\varphi - \alpha)$, $r_{\text{out}} = r \cos(\varphi - \alpha) - \beta_t t$ with $\beta_t = p_t / \sqrt{m^2 + p_t^2}$ of the given pair, φ is the spatial angle and α is the momentum angle. The averaging in $\langle \cdot \rangle$ is understood as $\langle f(x) \rangle = \int f(x) S(x, p) d^4x / \int S(x, p) d^4x$, where, in our case, $S(x, p)$ is defined in Eq. (1). We can calculate the azimuthal angle dependence of the HBT radii from Eq. (7) and parametrize them with the following functions:

$$\text{Elliptical case: } R_{\text{out}}^2 = R_{\text{o},0}^2 + R_{\text{o},2}^2 \cos(2\alpha) + R_{\text{o},4}^2 \cos(4\alpha) + R_{\text{o},6}^2 \cos(6\alpha), \quad (8)$$

$$\text{Triangular case: } R_{\text{out}}^2 = R_{\text{o},0}^2 + R_{\text{o},3}^2 \cos(3\alpha) + R_{\text{o},6}^2 \cos(6\alpha) + R_{\text{o},9}^2 \cos(9\alpha). \quad (9)$$

The higher order terms are caused by the rotation to the (*out*, *side*, *long*) system as it is detailed in Ref. [9]. These amplitudes are affected by both of the asymmetry parameters χ_3, ϵ_3 , so there is also a mixing in this case. These are illustrated in Figs. 2, 3. Yielding the value of the parameters

from only the azimuthally sensitive HBT radii measurements is not possible but combined with the flow measurements, the value of the asymmetry coefficients can be determined.

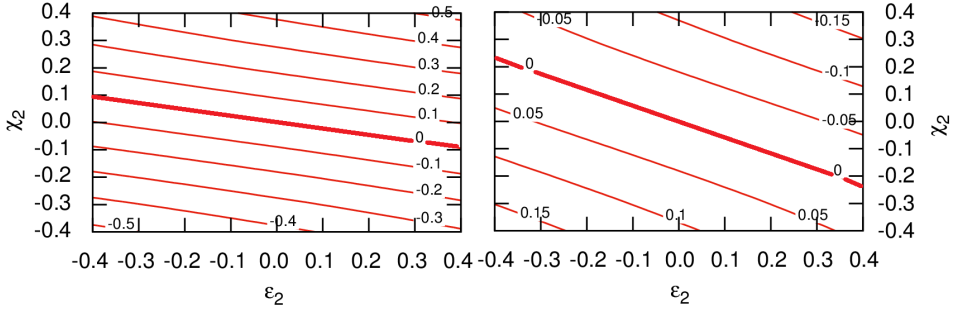


Fig. 2. The parameter dependence of the second order oscillations of the $R_{\text{out},2}^2/R_{\text{out},0}^2$ (left) and the $R_{\text{side},2}^2/R_{\text{side},0}^2$ (right) at $p_t = 300$ MeV/ c .

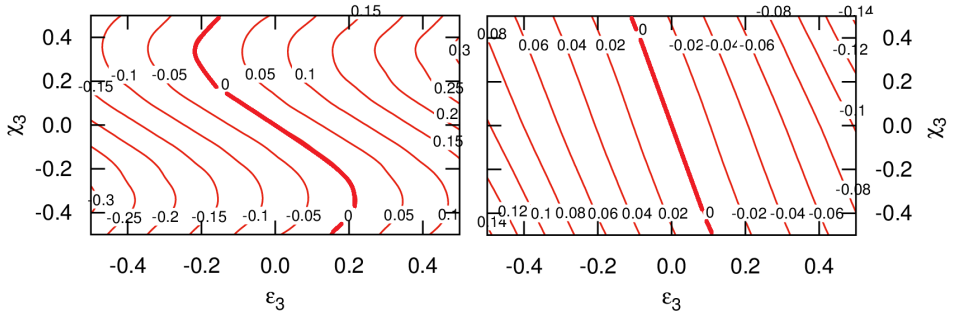


Fig. 3. The parameter dependence of the third order oscillations of the $R_{\text{out},3}^2/R_{\text{out},0}^2$ (left) and the $R_{\text{side},3}^2/R_{\text{side},0}^2$ (right) at $p_t = 300$ MeV/ c .

4. Summary

The spatial density distribution and the velocity field can be generalized in the framework of the Buda–Lund hydrodynamical model. The hadron momentum distributions and Bose–Einstein correlations can be derived from the extended model, and the effects of the higher order azimuthal asymmetries can be investigated. The density and flow asymmetries (ϵ_n and χ_n , respectively) affect these observables together (Figs. 1, 2, 3). Thus, the value of these parameters can be disentangled from the experiments if the flows and the azimuthally sensitive HBT radii are measured simultaneously.

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