MODEL-INDEPENDENT ANALYSIS OF NEARLY LÉVY CORRELATIONS*

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A model-independent method for the analysis of two-particle short-range correlations is presented. It can be utilized to describe such Bose–Einstein (HBT), dynamical (ridge) and other correlation functions which have a nearly Lévy or "stretched exponential" shape. For the special case of Lévy exponent $\alpha = 1$, earlier Laguerre expansions are recovered, while for $\alpha = 2$, a new expansion is obtained for correlations which are nearly Gaussian in shape. Multi-dimensional Lévy expansions are also introduced and their potential application to analyse ridge correlation data is discussed.

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1. Introduction

The detailed shape analysis of the two-particle Bose–Einsten Correlations (BEC) is an important topic in high-energy particle and nuclear physics because the shape of the correlation function carries information about the space-time structure of the particle emission process [1, 2].

With a few assumptions [2], the two-particle correlation function is related to the Fourier transformed source distribution. In this article, however, we do not assume such a relationship between Fourier transformed source distributions, and measured two-particle correlations, because in some cases, these assumptions are experimentally shown to be invalid even if they were expected to hold [3]. Instead, we continue the development of a *model-independent* method of analysing short-range correlations, developing further the ideas suggested in Ref. [4]. This general, model-independent characterization of short-range correlation functions [4] depends on the following experimental conditions:

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(i) The correlation function tends to a constant for large values of the relative momentum Q.

(ii) The correlation function has a non-trivial structure at a certain value of its argument, here assumed to be around $Q = 0$.

The first applications of this method investigated nearly Gaussian and nearly exponential correlations [4, 5]. Here, we continue the investigations started in Ref. [6] to develop a model-independent technique to analyse short-range correlations that have a stretched exponential or Lévy shape in zeroth order approximation; hence, we add a third experimental precondition:

(iii) The short-range behaviour of the correlation function has a form which is close to the stretched exponential *i.e.* an exponential in the stretched variable Q^α with $0 \leq \alpha \leq 2$.

We compare the resulting Lévy expansion series to the earlier results for the $\alpha = 1$ and 2 special cases, and extend this analysis in a natural manner to the case of multi-variate, nearly symmetric Lévy distributions.

2. Univariate Lévy expansions

In order to characterize the deviation of the correlation shape from the approximate Lévy shape, we apply the general expansion method of Ref. [4] for the special case of the Lévy weight function, $t = QR$, $w(t|\alpha) = \exp(-t^\alpha) = \exp(-Q^\alpha R^\alpha)$. The expansion is based on a set of polynomials which are orthonormal with respect to the weight function $w(t|\alpha)$. This expansion is uniquely defined by a Gram–Schmidt process if the order n terms are order n polynomials, with convergence criteria specified in Ref. [4].

The Lévy expansion of short-range correlation functions results in the following formula which can be easily fitted to a given data set as

$$t = QR\,, \tag{1}$$

$$C_2(t) = N\left\{1 + \lambda \exp(-t^\alpha) \sum_{n=0}^{\infty} c_n L_n(t|\alpha)\right\}\,, \tag{2}$$

where N is a normalization coefficient, λ measures the strength of the correlation function, $\exp(-t^\alpha)$ is the weight function and zeroth order approximation for the experimentally measured correlation function and α is the Lévy index of stability. The expansion coefficients are denoted by c_n and $\{L_n(t|\alpha)\}_{n=0}^{\infty}$ denote the Lévy polynomials, a complete set of polynomials which are orthogonal with respect to the Lévy weight function $\exp(-t^\alpha)$.

These Lévy polynomials were introduced in Ref. [6]; the first three are:

$$\begin{aligned} L_0(t\,|\,\alpha) &= 1\,, \\ L_1(t\,|\,\alpha) &= \det\begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & t \end{pmatrix}, \\ L_2(t\,|\,\alpha) &= \det\begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & t & t^2 \end{pmatrix}, \qquad \textit{etc.} \end{aligned} \tag{3}$$

where

$$\mu_{n,\alpha} = \int\limits_0^\infty \mathrm{d}t\, t^n \exp(-t^\alpha) = \frac{1}{\alpha}\,\Gamma\left(\frac{n+1}{\alpha}\right)$$

and Euler's gamma function is defined as $\Gamma(z) = \int_0^\infty \mathrm{d}t\ t^{z-1}e^{-t}$ as usual. The lowest-order Lévy polynomials are:

$$L_0(t\,|\,\alpha) = 1\,, \tag{4}$$

$$L_1(t\,|\,\alpha) = \frac{1}{\alpha}\left\{\Gamma\left(\frac{1}{\alpha}\right)t - \Gamma\left(\frac{2}{\alpha}\right)\right\}, \tag{5}$$

$$\begin{aligned} L_2(t\,|\,\alpha) = \frac{1}{\alpha^2}\Bigg\{&\left[\Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(\frac{3}{\alpha}\right) - \Gamma^2\left(\frac{2}{\alpha}\right)\right]t^2 \\ &-\left[\Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(\frac{4}{\alpha}\right) - \Gamma\left(\frac{3}{\alpha}\right)\Gamma\left(\frac{2}{\alpha}\right)\right]t \\ &+\left[\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(\frac{4}{\alpha}\right) - \Gamma^2\left(\frac{3}{\alpha}\right)\right]\Bigg\}\,. \end{aligned} \tag{6}$$

For $\alpha = 1$, these reduce to Laguerre polynomials and the Lévy expansion reduces to the Laguerre expansion of Ref. [4]

$$L_0(t\,|\,\alpha=1) = 1\,, \tag{7}$$

$$L_1(t\,|\,\alpha=1) = t-1\,, \tag{8}$$

$$L_2(t\,|\,\alpha=1) = t^2-4t+2\,. \tag{9}$$

The $\alpha = 2$ case provides a new expansion around a Gaussian shape that is defined for non-negative values of t only

$$L_0(t\,|\,\alpha=2) = 1\,, \tag{10}$$

$$L_1(t\,|\,\alpha=2) = \frac{1}{2}\left\{\sqrt{\pi}t-1\right\}, \tag{11}$$

$$L_2(t\,|\,\alpha=2) = \frac{1}{16}\left\{2(\pi-2)t^2 - 2\sqrt{\pi}t + (4-\pi)\right\}. \tag{12}$$

3. Multi-variate Lévy expansions

Multi-variate short-range correlations, in particular Bose–Einstein or HBT correlations, as well as dynamical "ridge" correlations are frequently studied in high-energy particle and heavy-ion physics. If they are symmetric [5, 7], a new variable can be introduced that reduces the multi-variate problem to an effective one-dimensional problem. For the multi-variate Bose–Einstein or HBT correlation measurements, a dimensionless scaling variable has already been introduced [5] as follows,

$$t = \left(\sum_{i,j=\mathrm{side,out,long}} R_{i,j}^2 q_i q_j \right)^{1/2} , \tag{13}$$

where q_i stands for the relative momentum component in the given direction. For multi-variate angular or ridge correlation measurements that result in a nearly Lévy shape [8], a similar, dimensionless scaling variable can be introduced,

$$t = \left(\sum_{i,j=\eta,\phi} \sigma_{i,j}^2 \mathit{\Delta}_i \mathit{\Delta}_j \right)^{1/2} , \tag{14}$$

where $\mathit{\Delta}_i$ stands for the angular difference in the (η, ϕ) lego plot. In both cases, the fit function for a multi-variate, nearly Lévy correlation functions becomes identical to Eq. (2).

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