IN-MEDIUM $\eta \to 3\pi$ DECAY WIDTH AND CHIRAL RESTORATION IN NUCLEAR MEDIUM*

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We investigate the effect of nuclear medium on $\eta \to 3\pi$ decay using linear sigma model with a particular attention to the role of the σ meson. We find that the decay width is enhanced in the nuclear medium in association with the softening of the σ meson. The in-medium $\eta \to 3\pi$ decay width can be a possible probe for the chiral restoration in the nuclear medium.

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1. Introduction

Understanding of $\eta \to 3\pi$ decay is a longstanding problem of hadron physics. This process is a G parity-violating process, and this indicates the necessity of the isospin-symmetry breaking originating from the electromagnetic or strong interaction in the decay process. Taking account of the smallness of the electromagnetic contribution [1], we expect that the isospin-symmetry breaking in quantum chromodynamics, *i.e.*, the current-quark mass difference of u and d quarks should be responsible for the process. However, the contribution from this isospin-symmetry breaking is found insufficient for the quantitative description of the observed $\eta \to 3\pi$ decay width [2]. Instead, the final-state interaction (FSI) between pions in the $I = J = 0$ channel was found to give a significant contribution to the decay width [3]. In particular, the effect of FSI would be significant if one takes into account the existence of the resonance pole of $f_0(500)$ in the channel, which is confirmed by the recent phase shift analyses [4]. Indeed, such an effect in the $\eta \to 3\pi$ process have been studied in Ref. [5].

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The investigation of hadron properties in various environments characterized by temperature, density and so on is an interesting subject in hadron physics. Chiral symmetry and its spontaneous breaking (Spontaneous Breaking of chiral Symmetry: SBχS) are essential ingredients to describe low-energy hadron phenomena [6]. In such an environment, the vacuum (or the ground state) is changed so that the broken symmetry is partially restored. Such a change of the vacuum would affect various hadron phenomena [7–11]. For a latest review, see, for example, Ref. [12].

In our previous work [13], we investigated the decay width of $\eta \to 3\pi$ in the nuclear medium with a non-zero isospin asymmetry $\delta\rho \equiv \rho_n - \rho_p$, where ρ_n and ρ_p are the neutron and proton number densities, respectively. It was found that the decay width is enhanced by the total nuclear density $\rho = \rho_n + \rho_p$ as well as $\delta\rho$. The enhancement by ρ originates from the 4-meson–N–N vertex. The importance of such a vertex had been pointed out in the study of the softening of the σ meson in the nuclear medium [14]; the 4π–N–N vertex significantly enhances the in-medium $\pi\pi$ cross section in the $I = J = 0$ channel. As mentioned above, the correlation of pions in this channel plays an important role in the $\eta \to 3\pi$ decay process. Then, one may expect that the enhancement of the correlation of the pions through the softening of the σ meson would modify the $\eta \to 3\pi$ decay width. In the present article, we study the $\eta \to 3\pi$ decay in the nuclear medium using a linear sigma model with a special attention to the role of the σ meson. The details of this study are given in Ref. [15].

2. Model

In this study, we investigate the decay width of the η meson into 3π in the symmetric nuclear medium. We use a linear σ model so that it is readily possible to include the correlation of pions in the sigma channel; we suppose that the pole of the σ meson mimics the strong correlation between two pions in the s-wave state. The Lagrangian used in this study, $\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}$, is given as follows:

$$\begin{aligned}\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} &= \frac{1}{2}\mathrm{tr}\left[\partial_\mu M \partial^\mu M^\dagger\right] - \frac{\mu^2}{2}\mathrm{tr}\left(MM^\dagger\right) - \frac{\lambda}{4}\mathrm{tr}\left[\left(MM^\dagger\right)^2\right] \\ &-\frac{\lambda'}{4}\left[\mathrm{tr}\left(MM^\dagger\right)\right]^2 + \frac{B}{2}\left(\det M + \det M^\dagger\right) + \frac{A}{2}\left(\chi M^\dagger + M\chi^\dagger\right) \\ &+\bar{N}\left[i\partial\!\!\!/ - g\left(\frac{\sigma_0}{\sqrt{3}} + \frac{\vec{\tau}\cdot\vec{a}_0}{\sqrt{2}} + \frac{\sigma_8}{\sqrt{6}} + i\gamma_5\left(\frac{\eta_0}{\sqrt{3}} + \frac{\vec{\tau}\cdot\vec{\pi}}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}}\right)\right)\right] N\,, \end{aligned} \quad (1)$$

$$M = M_s + iM_{ps} = \sum_{a=0}^{8}\frac{\sigma_a\lambda_a}{\sqrt{2}} + i\sum_{a=0}^{8}\frac{\pi_a\lambda_a}{\sqrt{2}}\,, \qquad N = {}^t(p,n)\,, \quad (2)$$

where M and N are the meson and nucleon fields, respectively, and λ_a and τ_a are the Gell-Mann and Pauli matrices with the normalization $\mathrm{tr}(\lambda_a \lambda_b) = 2\delta_{ab}$ and $\mathrm{tr}(\tau_a \tau_b) = 2\delta_{ab}$, respectively. The hadron fields belong to the $(3, \bar{3}) + (\bar{3}, 3)$ representation of $\mathrm{SU(3)_L \times SU(3)_R}$. The meson (baryon) field $M(B)$ is transformed into $LM(B)R^\dagger$ under the chiral transformation. The relevant nucleon doublet is shown in the baryon field. The order parameter of SBχS is expectation value of the σ field $\langle\sigma\rangle$. We determine the value so as to minimize the effective potential obtained from the $\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}$ within the tree-level approximation in this study. The σ meson is explicitly contained as the fluctuation mode of the order parameter in this model. The expectation value of σ and parameters contained in the Lagrangian in the meson part μ^2, λ, λ', A, and B, are fixed to reproduce the observed meson decay constants and masses in the isospin-symmetric limit, and the effect of the isospin-symmetry breaking is taken into account perturbatively. The parameter g in the nucleon part which determines the strength of the coupling of a meson and the nucleon is fixed so that $\langle\sigma\rangle$ is reduced by a factor 30% at the normal nuclear density; such an amount of reduction is suggested from the analyses of the pion–nucleus system [16, 17]. Here, we note that the mass of the σ meson, m_σ, is an input parameter in this calculation. We present the results in Sec. 3 with the values of m_σ varied.

The finite-density effect is taken into account using the following nucleon propagator $G(p; k_f)$

$$iG\left(p; k_f\right) = \frac{i(\not{p} + m_N)}{p^2 - m_N^2 + i\epsilon} - 2\pi\delta\left(p^2 - m_N^2\right)\left(\not{p} + m_N\right)\theta(p_0)\theta\left(k_f - |\vec{p}|\right) , \tag{3}$$

where the first and second terms are responsible for the free-space propagation of nucleon and the hole state in the nuclear Fermi sea, respectively. Diagrammatically, a nucleon loop has the contribution from the nuclear medium effect. We take account of the nuclear medium effect up to the nucleon one-loop level; the diagrams are shown in Figs. 1 and 2 which give the nuclear medium modification of the masses and couplings of mesons, respectively. We take into account the leading-order contribution in the Fermi-momentum expansion. Hence, the validity of our calculation is limited in the range of the small nuclear density.

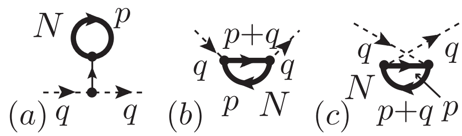

Fig. 1. Diagrams contributing to the meson self-energy in the nuclear medium. The dashed and solid lines represent the propagation of the meson and nucleon, respectively.

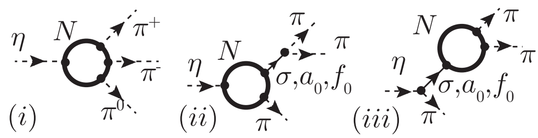

Fig. 2. Diagrams which have the nuclear medium effect on $\eta \to 3\pi$. The dashed, thin solid, and thick solid lines represent the pseudoscalar meson, scalar meson, and nucleon propagation, respectively.

3. Results

By evaluating the diagrams shown in Figs. 1 and 2, we obtain the matrix element of the in-medium $\eta \to \pi^+\pi^-\pi^0$ decay, $\mathcal{M}_{\eta\to\pi^+\pi^-\pi^0}$, as follows:

$$i\mathcal{M}_{\eta\to\pi^+\pi^-\pi^0} = ig_{\eta\pi^0\pi^+\pi^-} - \sum_{a=\sigma,f_0,a_0^0} \frac{ig_{a\eta\pi^0}g_{a\pi^+\pi^-}}{s-m_a^2+i\epsilon} - \frac{ig_{a_0^+\eta\pi^-}g_{a_0^-\pi^0\pi^+}}{t-m_{a_0^-}^2+i\epsilon} - \frac{ig_{a_0^-\eta\pi^+}g_{a_0^+\pi^0\pi^-}}{u-m_{a_0^+}^2+i\epsilon}, \tag{4}$$

where the Mandelstam variables s, t, u are defined as $s=(p_\eta - p_{\pi^0})^2$, $t=(p_\eta - p_{\pi^+})$, $u=(p_\eta - p_{\pi^-})^2$, respectively, m_a $(a=\sigma,\ f_0,\ a_0^{0,\pm})$ is the meson mass, and g_{abc}, g_{abcd} $(a,b,c,d = \eta,\pi^{0,\pm},\sigma,f_0,a_0^{0,\pm})$ are the couplings of mesons. These masses and couplings of mesons have the density dependence caused by the diagrams shown in Figs. 1 and 2. The matrix element of $\eta \to 3\pi^0$ is obtained from that of $\eta \to \pi^+\pi^-\pi^0$ by the use of Bose symmetry of $3\pi^0$ in the final state. The correlation of the pions is concisely described as the pole of the σ meson which couples to the s-wave two pions in this study. Here, we include the decay width of the σ to 2π in the tree-level approximation; we add the imaginary part $-i\Theta(s)$ to the mass of σ, m_σ^2, where $\Theta(s) = \frac{g_{s\pi\pi}^2}{16\pi}\sqrt{1-\frac{4m_\pi^2}{s}}\theta(s-4m_\pi^2)$.

The decay width $\Gamma_{\eta\to3\pi}$ is obtained by integrating the matrix element over the three-body phase space

$$\Gamma_{\eta\to3\pi} = \frac{1}{32\pi^3 m_\eta^3 n!}\int \mathrm{d}s \int \mathrm{d}t\, |\mathcal{M}_{\eta\to3\pi}|^2\,, \tag{5}$$

where n is the number of the identical particles in the final state. In the following calculation, the phase-space integrals are evaluated with the meson masses being approximated by those in the free space.

The plots of the $\eta \to \pi^+\pi^-\pi^0$ and $3\pi^0$ decay width in the nuclear medium are shown in Fig. 3.

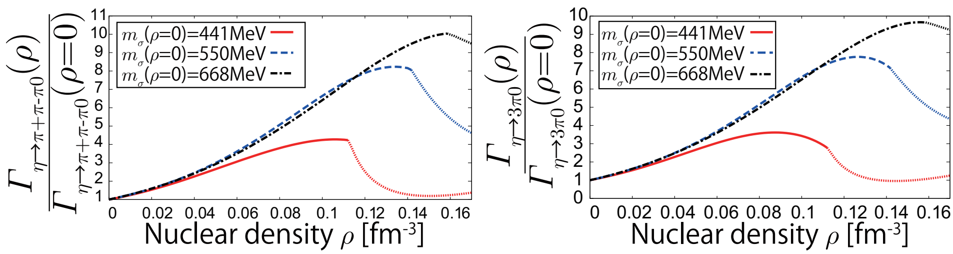

Fig. 3. Plots of the in-medium decay width of the $\eta \to \pi^+\pi^-\pi^0$ $\Gamma_{\eta\to\pi^+\pi^-\pi^0}(\rho)$ (left) and $3\pi^0$ $\Gamma_{\eta\to3\pi^0}(\rho)$ (right) normalized by the values in the free space. The solid (red), dashed (blue), and dash-dotted (black) lines are the plot of the decay width with $m_\sigma = 441$, 550, and 668 MeV, respectively.

One can see in Fig. 3 that the decay width increases monotonically along with the nuclear density until some high density depending on the sigma meson mass m_σ in the free space used as an input parameter: We note that our calculation is reliable only for relatively low densities. At small density like half of the normal nuclear density, m_σ dependence is small, and the enhancement of the decay width becomes about four times larger than that in the free space. The highest value of the decay width is four to ten times of that in the free space. This value largely depends on m_σ in the free space again. Here, we note that the contribution from the terms containing the σ meson as an intermediate state is particularly large due to the softening of the σ meson in the nuclear medium, although not shown explicitly.

Finally, we show the plot of the ratio of the in-medium decay width of $\eta \to 3\pi^0$ to $\eta \to \pi^+\pi^-\pi^0$ in Fig. 4: One can see that after keeping a rough constancy, the ratio decreases as the density increases, although the steepness of the slope is largely dependent on m_σ in the free space. This happens due to the cancellation of the terms in the second line of Eq. (4) due to the Bose symmetry in the final state of the $\eta \to 3\pi^0$ process and the softening of σ. Then, the decrease of the ratio of the decay widths would also become an indicator of the softening of the σ meson in the nuclear medium.

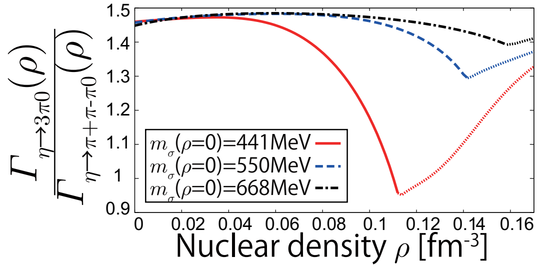

Fig. 4. Plot of the ratio of the $\eta \to 3\pi^0$ to the $\eta \to \pi^+\pi^-\pi^0$ decay width in the nuclear medium. The meanings of the lines are the same as in Fig. 3.

4. Conclusion

In this study, we have investigated the $\eta \to 3\pi$ decay width in the nuclear medium using a linear σ model focusing on the possible relevance of the softening of the σ meson in the nuclear medium. We have found an enhancement of the decay width in the nuclear medium in association with the softening of the σ meson. Hence, we expect that the $\eta \to 3\pi$ decay width in the nuclear medium is a possible probe for the chiral restoration in nuclear medium. We have evaluated the ratio of the $\eta \to \pi^+\pi^-\pi^0$ and $3\pi^0$ decay width in the nuclear medium, and found the decrease of the ratio in some density region due to the Bose symmetry of the $3\pi^0$ and the softening of the σ meson, which may thus become also an indicator of the partial restoration of chiral symmetry in the nuclear medium.

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