HEAVY QUARK ENTROPY SHIFT: FROM THE HADRON RESONANCE GAS TO POWER CORRECTIONS* **

E. Megías

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)
Föhringer Ring 6, 80805 Munich, Germany

E. Ruiz Arriola, L.L. Salcedo

Departamento de Física Atómica, Molecular y Nuclear
and
Instituto Carlos I de Física Teórica y Computacional
Universidad de Granada, 18071 Granada, Spain

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A heavy quark placed in the medium modifies its specific heat. Using a renormalization group argument, we show a low-energy theorem in terms of the defect in the trace of the energy-momentum tensor which allows the unambiguous determination of the corresponding entropy shift after imposing the third principle of thermodynamics for degenerate states. We show how recent lattice QCD data can be understood in the confined phase in terms of a singly-heavy hadronic spectrum and above the phase transition through power corrections which are analysed by means of a dimension-2 gluon condensate of the dimensionally reduced theory.

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1. Introduction

The Polyakov loop has been a major ingredient in the development of QCD at finite temperature, where lattice calculations routinely used the bare Polyakov loop as an order parameter for the crossover between the

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confined hadronic phase and the quark–gluon plasma. Its renormalization was first accomplished in perturbative QCD (pQCD) by Gava and Jengo [1] and non-perturbatively by the Bielefeld group [2]. In a series of works [3–6], we have pointed out that, instead of pQCD features, unexpected inverse power corrections in temperature dominate above the phase transition, a non-perturbative feature which finds a natural explanation in terms of dimension-2 condensates of the dimensionally reduced theory. On the other extreme, we have also found a hadronic representation in the confined phase both for the Polyakov loop [7, 8] and its correlators [9] (see also [10]). For a pedagogical review, see *e.g.* [11].

Perturbative calculations of the renormalized Polyakov loop have been pursued to NNLO [12–14]. The mysterious power corrections have been confirmed on the lattice for $N_\mathrm{c} = 3, 4, 5$ [15]. Quite recently, the TUM lattice calculation for physical quark masses in 2+1 flavours has been carried out in the temperature range of $125\,\mathrm{MeV} \le T \le 6000\,\mathrm{MeV}$ [16] concluding that at earliest pQCD to NNLO [14] might set in at 5800 MeV. Here, we display the power correction pattern over a huge range of temperatures and argue on the missing singly-heavy hadronic states below the phase transition.

2. Renormalization group

The Polyakov loop is a local operator which in the static gauge reads $\mathrm{Tr}(\Omega(\vec{x})) = \mathrm{Tr}(e^{igA_0(\vec{x})/T})$ ($\mathrm{Tr}(\mathbf{1}) = N_\mathrm{c}$). Its expectation value is a ratio of two partition functions (we take conventionally $\vec{x} = 0$)

$$\langle \mathrm{Tr}(\Omega) \rangle = \frac{Z_Q}{Z_0} = \frac{\int DAD\bar{q}Dq\, e^{-\int \mathrm{d}^4x\, \mathcal{L}(x)}\, \mathrm{Tr}(\Omega)}{\int DAD\bar{q}Dq\, e^{-\int \mathrm{d}^4x\, \mathcal{L}(x)}} \equiv e^{-\Delta F_Q/T}\,, \tag{1}$$

where the QCD Lagrangian for $N_\mathrm{f} = 3$ flavours u, d, s reads, in terms of the re-scaled gluon field $\bar{A}_\mu = \sum_a g A_\mu^a T_a$ with $\mathrm{Tr}(T_a T_b) = \delta_{ab}/2$,

$$\mathcal{L}(x) = \frac{1}{4g^2}\left(\bar{G}_{\mu\nu}^a\right)^2 + \sum_{q=u,d,s} \bar{q}(i\not{D} + m_q)q\,. \tag{2}$$

The renormalized Polyakov loop is uniquely defined up to a constant factor which corresponds to $\Delta F_Q \to \Delta F_Q + c_\Omega$. There is no natural way to fix the ambiguity, but it can be removed by using the corresponding entropy

$$\Delta S_Q = -\frac{\partial \Delta F_Q}{\partial T} = \frac{\partial}{\partial T}\left[T \log\langle \mathrm{Tr}(\Omega)\rangle\right]\,. \tag{3}$$

As ΔS_Q is dimensionless, one should have $\Delta S_Q = \varphi(g(\mu), \log\frac{\mu}{T}, \log\frac{\mu}{m_q(\mu)})$, with μ the renormalization scale. Renormalization group invariance requires

$$0 = \mu\frac{\mathrm{d}\Delta S_Q}{\mathrm{d}\mu} = \beta(g)\frac{\partial \Delta S_Q}{\partial g} - \sum_q m_q(1+\gamma_q)\frac{\partial \Delta S_Q}{\partial m_q} - T\frac{\partial \Delta S_Q}{\partial T}\,, \tag{4}$$

where the beta function and the mass anomalous dimension are

$$\beta(g) = \mu \frac{\mathrm{d}g}{\mathrm{d}\mu}\,, \qquad \gamma_q(g) = -\frac{\mathrm{d}\log m_q}{\mathrm{d}\log\mu}\,, \tag{5}$$

respectively. Direct evaluation yields the shift in the specific heat

$$\Delta c_Q = T\frac{\partial \Delta S_Q}{\partial T} = \frac{\partial}{\partial T}\left\{T\int \mathrm{d}^4x \left[\frac{\langle \mathrm{Tr}(\Omega)\,\Theta(x)\rangle}{\langle \mathrm{Tr}(\Omega)\rangle} - \langle\Theta(x)\rangle\right]\right\} \equiv \frac{\partial \Delta U_Q}{\partial T}\,, \tag{6}$$

where Θ is the trace of the energy momentum tensor[1],

$$\Theta \equiv \Theta^\mu_\mu = \frac{\beta(g)}{2g}\mathrm{Tr}\left(G^2_{\mu\nu}\right) + \sum_q m_q(1+\gamma_q)\bar{q}q\,. \tag{7}$$

The entropy shift can be obtained by integrating with suitable boundary conditions featuring the dimensions of the Hilbert space with and without Polyakov loop. At low temperatures, $Z_Q \sim 2N_\mathrm{f}e^{-M_0/T}$ for N_f degenerate flavours and $Z_0 \sim 1$, whereas at high temperatures $Z_Q \sim N_\mathrm{c}Z_0$, thus

$$\Delta S_Q(0) = \log\left(2N_\mathrm{f}\right)\,, \qquad \Delta S_Q(\infty) = \log N_\mathrm{c}\,. \tag{8}$$

The first condition is the third principle of thermodynamics for degenerate states. The recent TUM lattice calculations directly provide the entropy in the range of $125\,\mathrm{MeV} \le T \le 6000\,\mathrm{MeV}$ [16] taking the convention $S_Q^{\mathrm{TUM}}(\infty) = 0$ in harmony with their normalization $e^{-F_Q^{\mathrm{TUM}}/T} = \langle\mathrm{Tr}(\Omega)\rangle/N_\mathrm{c}$ which, unlike ours, *is not* a partition function at low temperatures. The critical temperature was found to be $T_\mathrm{c} = 150\,\mathrm{MeV}$.

3. Singly-heavy hadron resonance gas

In the confined phase, we expect a hadronic representation of the Polyakov loop [7,8]. There, the ambiguity comes from the heavy quark mass which is subtracted from the hadron total mass. In [7], we explicitly reconstructed the entropy $\mathrm{d}(T\log L(T))/\mathrm{d}T$ although the available lattice data were noisier than the recent ones [16].

In Fig. 1, we show the lattice entropy results [16] and compare them with the hadron resonance gas using either the bag model (centred at the

[1] Here, we extend to ΔS_Q the argument of Ref. [17,18]. The entropy shift is not a true entropy. For instance, the true entropy must be a monotonous function of the temperature since from $Z = \mathrm{Tr}\,e^{-H/T}$, it follows $c = T\partial_T S = \langle(H-\langle H\rangle)^2\rangle/T^2 > 0$. Thus, *both* $c_Q > 0$ and $c_0 > 0$ but the sign of Δc_Q is not fixed. The exact relations for thermodynamics of heavy quarks [19] are still subjected to ambiguities which are removed in the specific heat.

heavy quark source), the PDG or the RQM of Isgur, Godfrey and Capstick for mesons and baryons [20, 21] taking either the charm or the bottom as the putative heavy quark. We have noted in previous works that these RQM singly-heavy states follow a Hagedorn-like pattern with a Hagedorn–Polyakov temperature of about $T_{\rm H} = 200\,{\rm MeV}$ for b-quarks. Results from a simple constituent quark model (CQM) are also shown

$$L = \sum_{q=u,d,s} g_q e^{-\left(M_{\bar{Q}q}-m_Q\right)/T} + \sum_{q,q'=u,d,s} g_{q,q'} e^{-\left(M_{\bar{Q}qq'}-m_Q\right)/T} + \ldots \qquad (9)$$

with $M_{\bar{Q}q} = 2M + m_q + m_Q$ and $M_{\bar{Q}qq'} = 3M + m_q + m_{q'} + m_Q$ and spin degeneracies $g_q = 2$, $g_{qq'} = 4 - \delta_{q,q'}$.

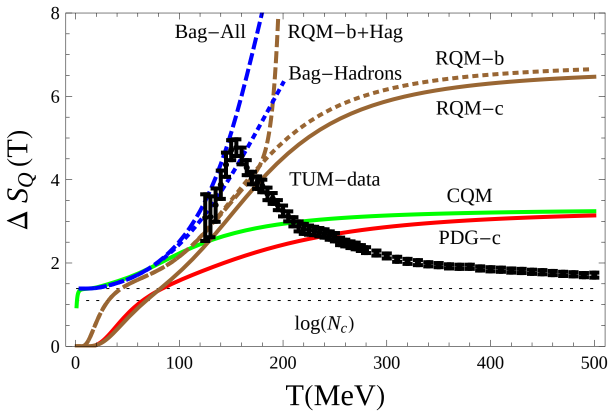

Fig. 1. The entropy as a function of temperature. We show results from various hadronic models: the bag model including all ($Q\bar{q}$, Qqq, $Q\bar{q}g$ and $Qqqg$) states and just hadrons, the RQM with one c- or b-quark and the PDG states with one c-quark. The Hagedorn extrapolation of the b-spectrum is also displayed. We also plot the CQM with uds quarks and constituent mass $M = 300\,{\rm MeV}$ and the bare $m_u = 2.5\,{\rm MeV}$, $m_d = 5\,{\rm MeV}$, $m_s = 95\,{\rm MeV}$ masses. Horizontal lines mark $\Delta S_Q(0) = \log 2N_{\rm f}$, with $N_{\rm f} = 2$ the number of light degenerate flavours, and $\Delta S_Q(\infty) = \log(N_{\rm c})$. Lattice data for 2+1 flavours are taken from Ref. [16].

4. Dimension-2 condensate and power corrections

At high temperatures, the Polyakov loop can be expanded in powers of $\bar{A}_0$ [3]

$$\langle {\rm Tr}(\Omega)\rangle = N_{\rm c} - g^2 \frac{\left\langle (A_0^a)^2 \right\rangle}{4T^2} + O\left(g^6\right) \sim N_{\rm c} \exp\left[-\frac{\left\langle {\rm Tr}\left(\bar{A}_0^2\right)\right\rangle}{2N_{\rm c}T^2} \right]. \qquad (10)$$

$\langle (A_0^a)^2 \rangle$ has both perturbative (pert) and non-perturbative (NP) contributions, whence the entropy reads

$$\Delta S_Q(T) = \log(N_c) + \Delta S_{\rm pert}(T) + \frac{\langle A^2 \rangle^{\rm NP}}{2T^2}\,, \qquad \langle A^2 \rangle^{\rm NP} \equiv \frac{1}{N_c} \left\langle {\rm Tr}\left(\bar{A}_0^2\right)\right\rangle^{\rm NP}\,, \tag{11}$$

where $\Delta S_{\rm pert}(T)$ is a slowly varying function with temperature. In Fig. 2, we display the recent TUM data [16] as a function of $(T_c/T)^2$. A clear straight line behaviour emerges over a huge range of temperatures. A fit for $T_c \le T \le 5000\,\text{MeV}$, neglecting the T dependence in $\Delta S_{\rm pert}(T)$, gives

$$\frac{\langle A^2 \rangle^{\rm NP}}{2T_c^2} = 3.40(2)\,, \qquad \Delta S_{\rm pert}(T) = 0.195(3) \tag{12}$$

with $\chi^2 = 108$ for $N_{\rm dat} = 92$. This gives $\chi^2/\nu = 108/(92-2) = 1.2$ which is within the expected $1 \pm \sqrt{2/\nu}$.

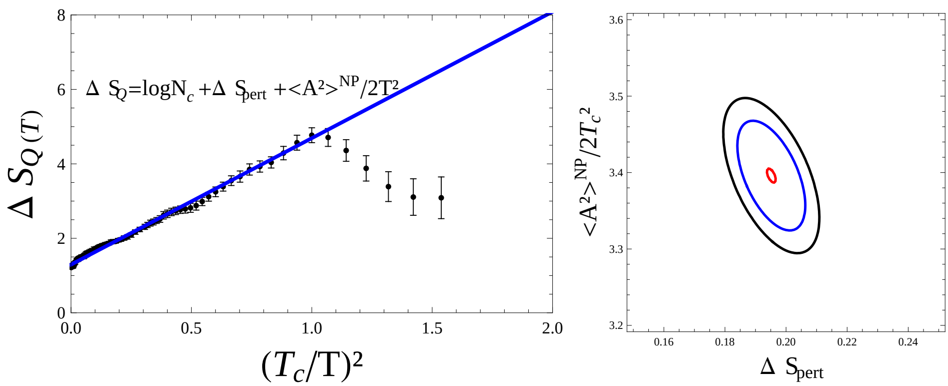

Fig. 2. Left panel: TUM lattice data for the entropy [16] as a function of the inverse squared temperature in units of the critical temperature. The straight line is the fit using the dim-2 condensate. Right panel: Correlation plot between the dim-2 condensate and the perturbative entropy.

5. Conclusions

When a heavy colour source in the fundamental representation of the SU(N_c) group is placed into the hadronic vacuum, there arises an entropy shift as a measurable and unambiguous observable. We noted long ago that lattice data for the corresponding free energy display corrections to the perturbative result in the unequivocal form of an inverse second power of temperature. This behaviour has been corroborated by subsequent lattice studies. The present analysis improves on previous ones thanks to the quality of the TUM lattice data, and the fact that the unambiguous entropy is used for the comparison. At low temperatures, the quick increase in the number of active states, $N = e^{\Delta S}$, suggests that there are missing states in the singly-heavy hadronic spectrum. At high temperatures, power corrections dominate over the perturbative contributions, in harmony with our previous findings. Taken at face value, the impressive agreement calls for a deeper understanding on the nature of these power corrections above the phase transition.

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