

DESCRIPTION OF EM STRUCTURE OF NONET
OF PSEUDOSCALAR MESONS BY UNITARY AND
ANALYTIC MODEL LEADS TO MORE ACCURATE
MUON $g - 2$ ANOMALY AND QED $\alpha(M_Z^2)$ EVALUATION*

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It is demonstrated that the description of electromagnetic structure of pseudoscalar meson nonet by the sophisticated Unitary and Analytic (U&A) model leads to a more precise evaluation of muon $g - 2$ anomaly and QED $\alpha(M_Z^2)$.

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1. Introduction

The anomalous magnetic moment of the muon, $a_\mu = \frac{g-2}{2}$, provides an extremely clean test of the Standard Model (SM) of elementary particle physics.

Therefore, it is important to achieve in its evaluation the inequality $(a_\mu^{\text{exp}} - a_\mu^{\text{th}}) < \Delta(a_\mu^{\text{exp}} - a_\mu^{\text{th}})$, where $\Delta(a_\mu^{\text{exp}} - a_\mu^{\text{th}})$ means the error of the difference between experimentally measured value a_μ^{exp} and its theoretical evaluation a_μ^{th} .

Another quantity, the running QED fine structure coupling constant $\alpha(s)$, is also very important to be known with very high precision, since the overwhelming majority of the SM predictions of observable quantities depend on its value.

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In both, a_μ and $\alpha(M_Z^2)$, the dominant sources of the total uncertainties in theoretical predictions are hadronic contributions, which can be reduced to the calculation of dispersion integrals through $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$.

Up to the present, almost all evaluations of these integrals have been carried out by other authors by the integration through existing experimental data points on $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$, joining them by straight lines, *i.e.* by using the so-called “trapezoidal rule”.

In this paper, we demonstrate how the errors of the evaluated integrals through two-body total cross sections $\sigma(e^+e^- \rightarrow M\bar{M})$ and $\sigma(e^+e^- \rightarrow \gamma M)$ can be reduced if the corresponding form factors of the nonet of pseudoscalar mesons

$$\pi^+, \quad \pi^0, \quad \pi^-, \quad K^+, \quad K^0, \quad \bar{K}^0, \quad K^-, \quad \eta, \quad \eta'$$

are expressed by the Unitary and Analytic model [1] in comparison with evaluation of the same integrals and at the same energy intervals by the numerical integration through experimental points.

2. Muon $g - 2$ anomaly

The dominant hadronic contribution to a_μ is given by the Feynman diagram in Fig. 1 which can be represented by the dispersion integral

$$a_\mu^{(2)\text{had}} = \frac{1}{3} \left(\frac{\alpha(0)}{\pi} \right)^2 \left(\int_{4m_\pi^2}^{s(\text{cut})} \frac{ds}{s} R^{\text{data}}(s) K(s) + \int_{s(\text{cut})}^{\infty} \frac{ds}{s} R^{\text{PQCD}}(s) K(s) \right), \quad (1)$$

with $R(s) = \sigma_{\text{tot}}(e^+e^- \rightarrow \text{had}) / \frac{4\pi\alpha^2(0)}{3s}$ and $K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\mu^2}}$.

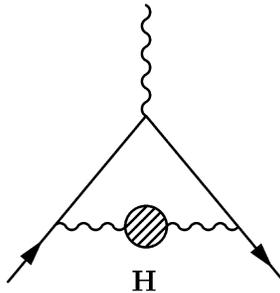


Fig. 1. The lowest-order hadronic vacuum-polarization contributions.

Just in the first integral of (1) instead of integration through data on $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{had})$, we evaluate explicitly contributions separately of

$$\begin{aligned}\sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+\pi^-) &= \frac{\pi\alpha(0)^2}{3s} (1 - 4m_\pi^2/s)^{\frac{3}{2}} |F_{\pi^\pm}(s)|^2, \\ \sigma_{\text{tot}}(e^+e^- \rightarrow K^+K^-) &= \frac{\pi\alpha(0)^2}{3s} (1 - 4m_{K^\pm}^2/s)^{\frac{3}{2}} |F_{K^\pm}(s)|^2, \\ \sigma_{\text{tot}}(e^+e^- \rightarrow K^0\bar{K}^0) &= \frac{\pi\alpha(0)^2}{3s} (1 - 4m_{K^0}^2/s)^{\frac{3}{2}} |F_{K^0}(s)|^2, \\ \sigma_{\text{tot}}(e^+e^- \rightarrow \pi^0\gamma) &= \frac{\pi\alpha(0)^2}{6} (1 - m_{\pi^0}^2/s)^3 |F_{\pi^0\gamma}(s)|^2, \\ \sigma_{\text{tot}}(e^+e^- \rightarrow \eta\gamma) &= \frac{\pi\alpha(0)^2}{6} (1 - m_\eta^2/s)^3 |F_{\eta\gamma}(s)|^2, \\ \sigma_{\text{tot}}(e^+e^- \rightarrow \eta'\gamma) &= \frac{\pi\alpha(0)^2}{6} (1 - m_{\eta'}^2/s)^3 |F_{\eta'\gamma}(s)|^2,\end{aligned}$$

where $F_{\pi^\pm}(s)$, $F_{K^\pm}(s)$, $F_{K^0}(s)$, $F_{\pi^0\gamma}(s)$, $F_{\eta\gamma}(s)$ and $F_{\eta'\gamma}(s)$ are EM form factor of charged pions, EM form factor of charged kaons, EM form factor of neutral kaons, neutral pion–photon transition form factor, eta–photon transition form factor and eta prime–photon transition form factor, respectively, provided that these are represented by the U&A model [2].

In construction of the U&A model, the $F_{\pi^\pm}(s)$, $F_{K^\pm}(s)$, $F_{K^0}(s)$, $F_{\pi^0\gamma}(s)$, $F_{\eta\gamma}(s)$ and $F_{\eta'\gamma}(s)$ form factors, which represent a consistent unification of finite number of complex conjugate pairs of vector-meson pole contributions and just continua contributions represented by the cuts on the positive real axis, are split into isoscalar and isovector parts

$$\begin{aligned}F_{\pi^\pm}(s) &= F_\pi^{I=1}[W(s)], \\ F_{K^\pm}(s) &= F_K^{I=0}[V(s)] + F_K^{I=1}[W(s)], \\ F_{K^0}(s) &= F_K^{I=0}[V(s)] - F_K^{I=1}[W(s)], \\ F_{\pi^0\gamma}(s) &= F_{\pi^0\gamma}^{I=0}[V(s)] + F_{\pi^0\gamma}^{I=1}[W(s)], \\ F_{\eta\gamma}(s) &= F_{\eta\gamma}^{I=0}[V(s)] + F_{\eta\gamma}^{I=1}[W(s)], \\ F_{\eta'\gamma}(s) &= F_{\eta'\gamma}^{I=0}[V(s)] + F_{\eta'\gamma}^{I=1}[W(s)].\end{aligned}$$

In order to take into account the experimental fact of the creation of ρ , ω , ϕ , ρ' , ω' , ϕ' , *etc.* in $e^+e^- \rightarrow \text{hadrons}$, first, the iso-scalar parts of these form factors are represented by the standard VMD model with stable iso-scalar vector mesons ω , ϕ , ω' , ϕ' , *etc.* and the corresponding iso-vector parts are represented by the standard VMD model with stable iso-vector vector mesons ρ , ρ' , *etc.* Then, they both are unitarized by an incorporation of two-cut approximation of the analytic properties of EM FFs with the help of the

special non-linear transformations [1] and by an introduction of instabilities of vector mesons.

As a result, every $F^{I=1}[W(s)]$ and $F^{I=0}[V(s)]$ represents one analytic function in the whole complex s -plane, besides two cuts on the positive real axis, and depends only on physically interpretable parameters, such as inelastic thresholds and coupling constant ratios (f_{MMV}/f_V) or ($f_{M\gamma V}/f_V$), which are determined by a comparison of form factors with existing data.

More or less successful description of all existing data on the whole complete nonet of pseudoscalar mesons $\pi^-, \pi^0, \pi^+, K^-, K^0, \bar{K}^0, K^+, \eta, \eta'$ has been achieved in space-like and time-like regions simultaneously [2].

Substituting U&A models of $F^{I=1}[W(s)]$ and $F^{I=0}[V(s)]$ with corresponding numerical values of parameters into two-body total cross sections $\sigma(e^+e^- \rightarrow MM)$ and $\sigma(e^+e^- \rightarrow \gamma M)$, one is ready to evaluate LO contributions to muon $g - 2$ anomaly.

In order to have an opportunity to compare our results with other authors, we have carried out both, direct data integration and also integration by exploiting the U&A model of the corresponding FFs, at the interval of energies $s_0 < s < 2.0449 \text{ GeV}^2$.

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| 1. | $a_\mu^{\text{had,LO}}(e^+e^- \rightarrow \pi^+\pi^-) \times 10^{-11}$ | for $4m_\pi^2 < s < 2.0449 \text{ GeV}^2$ [3] |
| | U&A model integration | $5128.22_{-0.67}^{+0.73}$ |
| | direct data integration | $5031.22_{-16.43}^{+28.94}$ |
| | Hagiwara <i>et al.</i> (2007) | 5008.2 ± 28.70 |
| 2. | $a_\mu^{\text{had,LO}}(e^+e^- \rightarrow K^+K^-) \times 10^{-11}$ | for $4m_K^2 < s < 2.0449 \text{ GeV}^2$ [4] |
| | U&A model integration | $224.67_{-1.28}^{+1.23}$ |
| | direct data integration | $235.76_{-5.00}^{+9.07}$ |
| | Hagiwara <i>et al.</i> (2004) | 216.2 ± 7.6 |
| 3. | $a_\mu^{\text{had,LO}}(e^+e^- \rightarrow K^0\bar{K}^0) \times 10^{-11}$ | for $4m_K^2 < s < 2.0449 \text{ GeV}^2$ [4] |
| | U&A model integration | $128.38_{-0.76}^{+0.76}$ |
| | direct data integration | $135.40_{-0.96}^{+1.66}$ |
| | Hagiwara <i>et al.</i> (2004) | 131.6 ± 3.1 |
| 4. | $a_\mu^{\text{had,LO}}(e^+e^- \rightarrow \pi^0\gamma) \times 10^{-11}$ | for $m_{\pi^0}^2 < s < 2.0449 \text{ GeV}^2$ [5] |
| | U&A model integration | 53.72 ± 0.36 |
| | Davier <i>et al.</i> (2011) | 44.2 ± 1.94 |
| 5. | $a_\mu^{\text{had,LO}}(e^+e^- \rightarrow \eta\gamma) \times 10^{-11}$ | for $m_{\pi^0}^2 < s < 2.0449 \text{ GeV}^2$ [5] |
| | U&A model integration | 11.55 ± 0.08 |
| | Davier <i>et al.</i> (2011) | 6.40 ± 0.24 |
| 6. | $a_\mu^{\text{had,LO}}(e^+e^- \rightarrow \eta'\gamma) \times 10^{-11}$ | for $m_{\pi^0}^2 < s < 2.0449 \text{ GeV}^2$ [5] |
| | U&A model integration | 20.69 ± 9.65 |
| | Davier <i>et al.</i> (2011) | no data (*) |

Note: (*) there are no data on the corresponding total cross section at the region of evaluation of the integral.

3. QED $\alpha(M_Z^2)$ running coupling constant

The running fine structure coupling constant of QED $\alpha(s)$ can be expressed as

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}; \quad \alpha(0) = 1/137,036, \quad (2)$$

where $\Delta\alpha(s)$ is compound of three independent contributions

$$\Delta\alpha(s) = \Delta\alpha_l(s) + \Delta_{\text{had}}^{(5)}(s) + \Delta\alpha_{\text{top}}(s), \quad (3)$$

- $\Delta\alpha_l(s)$ from leptons (e, μ, τ),
- $\Delta_{\text{had}}^{(5)}(s)$ from 5 light quarks u, d, c, s, b (mass < 5 GeV),
- $\Delta\alpha_{\text{top}}(s)$ from “top” quark t (mass ≈ 175 GeV).

While the leptonic contributions $\Delta\alpha_l(s)$ are calculable in perturbation theory

$$\Delta\alpha_l(s) = \frac{\alpha(0)}{3\pi} \sum_{f=e,\mu,\tau} \left[\ln \frac{s}{m_f^2} - \frac{5}{3} \right] \quad (4)$$

and numerically at the mass of Z boson yield $\Delta\alpha_l(M_Z^2) \approx 0.031498$, and since the t quark is heavy ($m_t \gg M_Z \approx 91$ GeV), one cannot use the light fermion approximation for it and it decouples like

$$\Delta\alpha_{\text{top}}(s) \approx -\frac{\alpha(0)}{3\pi} \frac{4}{15} \frac{M_Z^2}{m_t} \rightarrow 0, \quad (5)$$

the most problematic is an evaluation of the light-quark contribution $\Delta_{\text{had}}^{(5)}(s)$. Due to the light masses of these five quarks, it cannot be calculated in the framework of the “perturbative” QCD (pQCD).

Fortunately, one can evaluate it from $e^+e^- \rightarrow$ hadrons total cross-section data, like in the muon $g - 2$ anomaly, by exploiting dispersion integral

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = -\frac{\alpha(0)M_Z^2}{3\pi} \text{Re} \int_{4m_\pi^2}^{\infty} \frac{R(s')}{s'(s' - M_Z^2 - i\varepsilon)} ds', \quad (6)$$

which gives $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = 0.027896 \pm 0.000395$.

If we add this result to the calculated value of $\Delta_{\text{leptons}}(M_Z^2) = 0.031498$, then for $\alpha(s)$ at the mass of the Z boson, one obtains

$$\alpha(m_Z^2) = \frac{\alpha(0)}{1 - 0.059394} = 1/128,897,$$

which clearly demonstrates that QED fine structure constant is really running.

4. Conclusions

It has been clearly demonstrated that the description of electromagnetic structure of pseudoscalar meson nonet by the sophisticated Unitary and Analytic model in space-like and time-like regions simultaneously leads to a more precise evaluation of muon $g - 2$ anomaly and QED $\alpha(M_Z^2)$ running fine structure constant.

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