TWISTED MASS WILSON χ-PT VERSUS LATTICE DATA: A CASE STUDY*

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We compare lattice data obtained from a dynamical simulation with twisted mass fermions to the analytical predictions of Twisted Mass Wilson χ-PT and extract an estimate for the chiral condensate and the LEC W_8.

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1. Introduction

Twisted mass fermions is a prominent discretization of QCD due to some very advantageous features such as automatic $\mathcal{O}(a)$ improvement, absence of exceptional configurations and fast dynamical simulations [1] in a well-established theoretical framework. Present day simulations are performed in the deep chiral regime with physical value of the pion mass [2]. However, due to the non-commutativity of the chiral and the continuum limit, Wilson fermions exhibit artificial phases with no continuum analogue. Such scenarios include the Aoki phase [3] that one reaches via a second order phase transition or the first order Sharpe–Singleton scenario [4]. One can study the phase diagram of twisted mass fermions in a lattice augmentation of Chiral Perturbation Theory (χ-PT) particularly extended for Wilson fermions. In Wilson χ-PT, one takes explicitly into account the lattice artifacts in the

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chiral expansion and ends with new terms which come with new Low Energy Constants (LECs). These LECs are particular to the lattice action but their knowledge is extremely important in order to gain access to the physical LECs such as the chiral condensate Σ and the pion decay constant F_π. In addition to that, their relative strength and sign determine the potential of the effective theory and thus the phase structure [5–8]. There has been substantial analytical [9–14] and numerical [15–17] effort dedicated to their determination. Methods previously used include, among others, pion mass splittings [16], unitarity violations in a mixed action setup [15] as well as pion scattering in Wilson χ-PT [14]. Here, we follow a different approach first applied in quenched studies [18–20], where one matches analytical results from Wilson χ-PT for a sector with fixed index ν of the Wilson–Dirac operator with results obtained on the lattice. These results were also presented in the Lattice 2015 meeting [21].

2. The theoretical background

We make use of the analytical results for the microscopic spectral density for $N_{\rm f} = 2$ derived in [22], starting from the supersymmetric extension of the chiral Lagrangian in the ϵ regime in a sector of fixed index ν. In the microscopic power counting, $m \propto \epsilon^4$ and $a \propto \epsilon^2$, and terms up to $\mathcal{O}(\epsilon^4)$ are taken into account. In this regime, the partition function factorizes and one ends up with a unitary matrix integral which, in our case, reads

$$\mathcal{Z}^{\nu}_{3|1}\left(\widehat{\mathcal{Z}};\widehat{a}\right) = \int\limits_{Gl(3|1)/U(1)} {\rm d}U\ {\rm Sdet}(iU)^{\nu} e^{+\frac{i}{2}{\rm Str}\left(\widehat{\mathcal{Z}}\left[U+U^{-1}\right]\right)+\widehat{a}^2{\rm Str}\left(U^2+U^{-2}\right)}\,, \quad (2.1)$$

where $\widehat{\mathcal{Z}} \equiv {\rm diag}(i\widehat{z}_{\rm t}, -i\widehat{z}_{\rm t}, \widehat{z}, \widehat{z}\,')$ and the integration manifold is exactly the one that we encounter in continuum partially quenched χ-PT calculations. Here, we have introduced the rescaled variables $\widehat{a} = a\sqrt{W_8 V/2}$ (with W_8 a new LEC parametrizing lattice artifacts) and $\widehat{z}_{\rm t} = z_{\rm t} V\Sigma$, where $z_{\rm t}$ is the twisted mass, a is the lattice spacing, Σ is the chiral condensate and V is the lattice volume. Note that the two other LECs which contribute to LO in a^2 have been ignored in this study.

3. The computational setup and numerical results

In this study, we employ in the fermionic sector the twisted mass action [23] and in the gauge sector, the Iwasaki improved action [24].

We refer the reader to [21] for the technicalities of the lattice action employed in this study. We computed the topological charge of the gauge configurations utilizing the Wilson Flow [25] which is a low-cost method providing a good definition of the topological charge at finite lattice spacing. Since we diagonalize the Dirac operator in sectors of fixed topological charge in order to have high statistics for given ν, we employed a very long ensemble of the ETM Collaboration which actually has $N_{\rm f} = 2+1+1$. The heavy strange and charm quarks are completely quenched from a spectral viewpoint and do not affect the comparison with the $N_{\rm f} = 2$ analytical results. The pion mass of the employed configurations is 390 MeV, $L \sim 2.5$ fm and $M_\pi L \sim 5$, meaning that our results are practically already extrapolated to infinite volume. Note that this is not an ϵ-regime simulation where $M_\pi L \ll 1$, but this is not an issue, since the smallest Dirac eigenvalues can be in the ϵ regime. The scale below which eigenvalues are given by RMT is called the Thouless energy and for QCD, it is $E_{\rm c} = F_\pi^2/\Sigma L^2$ [26]. We measured the topological charge of 5000 independent configurations and we diagonalized the ones with $|\nu| = 0, 1, 2, 3$. In Fig. 1, we show the analytical results for ρ^5 for $|\nu| = 0$, *cf.* [21] for results corresponding to the other topological sectors, *versus* histograms of lattice data. We observed that due to the large value of $\widehat{z}_{\rm t}$, the results are very close to the quenched ones, while due to the large value of $\widehat{a}$, the structure of the former zero modes, that one would expect for $|\nu| = 1, 2$, is completely dissolved. In order to extract the chiral condensate Σ and additionally W_8, we performed constrained fits with $W_6 = W_7 = 0$ in the individual topological sectors as well as a combined fit in the sectors with $|\nu| = 0, 1, 2$. Our results with their associated statistical errors

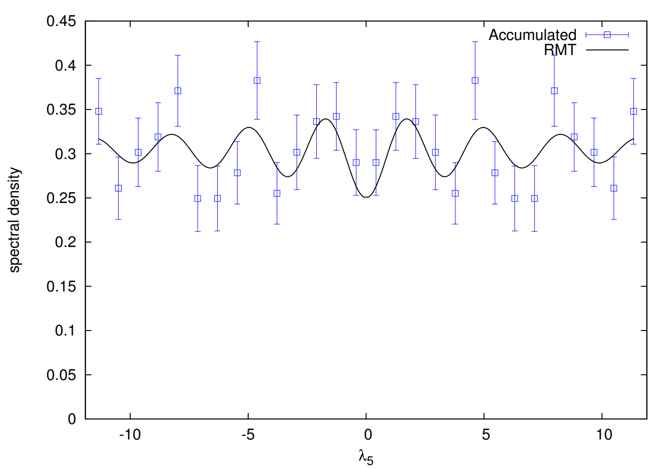

Fig. 1. The microscopic spectral density of the Hermitian Twisted Mass Wilson Dirac operator. The solid curves comprise the analytical result derived in [22], while the data points are the numerical results from a simulation on a $32^3 \times 64$ lattice with $a = 0.0815$ fm [27] and $a\mu = 0.0055$. Fitting results of the topological sector with $\nu = 0$ and fitting parameters $\widehat{z}_{\rm t} = 38.5$ and $\widehat{a} = 0.715$.

are summarized in Table I. The differences between the topological sectors are attributed to cutoff effects and they give a crude estimate of their size. We quote the result of the renormalized condensate having used the value of Z_P in the $\overline{\mathrm{MS}}$ scheme at 2 GeV given by the ETM Collaboration to be $Z_P = 0.509(4)$ [27, 28]. In [29], the continuum extrapolated value of Σ that was quoted can be translated to physical units to $\Sigma^{1/3} = 290 \pm 11$ MeV. We see that this value is very close to the one extracted from this study, which gives us confidence that cutoff effects are taken into account to some extent by the LO Wilson chiral Lagrangian which only includes W_8. The extracted value of W_8 is in complete agreement with [15] but differs by roughly a factor of 2 from the one determined in [16]. This point requires extra clarification. Note that the results of the combined fits are still preliminary and they will be further scrutinized and addressed in an upcoming article.

TABLE I

Extracted values for Σ and W_8.

$\|\nu\|$	0	1	2	Combined
$\Sigma^{1/3}$ [MeV]	289.0(2.7)	272.3(4.1)	270.8(6.8)	271.1(7.3)
W_8 $[r_0^6 W_0^2]$	0.0021(12)	0.0055(19)	0.0064(12)	0.0064(12)

4. Conclusions and outlook

In this study of unquenched twisted mass Dirac spectra, we took the first step towards the extraction of the LECs of Wilson χ-PT from unquenched simulations. We determined the chiral condensate as well as W_8 by comparing analytical results from Wilson χ-PT to numerical data from a lattice simulation. All the errors quoted in this proceeding are at the moment only statistical errors which have been computed with the bootstrap method. Regarding the systematic errors, as it was already mentioned, finite size effects are expected to be under control due to the fact that $m_\pi L \sim 5$. Discretization errors are to a great extent taken into account by the fact that the chiral Lagrangian contains an a^2 term but still there can be contributions by the other two a^2 terms which have been neglected in Ref. [22] and in our study, but also by higher order terms in the chiral expansion. One part of the calculation where discretization effects could creep in is in the computation of the topological charge. In order to minimize this effect, we have taken into account three different discretizations of the topological charge density. The first employs the plaquette definition and has cutoff effects of $\mathcal{O}(a^2)$, the second definition also includes a clover term and also has cutoff effects of $\mathcal{O}(a^2)$, and the last definition contains rectangular clover terms and has cutoff effects of $\mathcal{O}(a^4)$. The agreement among these methods for the given value of the lattice spacing was above 98%, see [30] for a detailed analysis. These discretization errors could potentially lead to a misassignment of the topological charge.

In an upcoming work, we plan to derive the full LO analytical solution including the LECs W_6, W_7, since they could potentially have a large effect in the eigenvalue density. It would be interesting to see how their inclusion will affect the extracted value of Σ and W_8.

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