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CALCULATION OF REGGE TRAJECTORIES OF STRANGE RESONANCES AND IDENTIFICATION OF THE $K_0^*(800)$ AS A NON-ORDINARY MESON*

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We review how the Regge trajectory of an elastic resonance can be obtained just from its pole position and coupling, using a dispersive formalism. This allows us to deal correctly with the finite widths of resonances in Regge trajectories. In this way, we can calculate the Regge trajectories for the $K^*(892)$, $K_1(1400)$ and $K_0^*(1430)$, obtaining ordinary linear Regge trajectories, expected for $q\bar{q}$ resonances. In contrast, for the $K_0^*(800)$ meson, the resulting Regge trajectory is non-linear and with much smaller slope, strongly supporting its non-ordinary nature.

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1. Introduction

In recent works [1,2], the analytic properties of amplitudes in the complex angular momentum plane have been used to study the Regge trajectories of resonances decaying predominantly to one channel. One of the main features of this calculation is the correct relation of the resonance width to the imaginary part of the trajectory. In principle, the form of these trajectories can be used to discriminate between the underlying QCD mechanisms that generate these resonances. Ordinary linear (J, M^2) trajectories relating the angular momentum J and the mass squared are intuitively interpreted in terms of $q\bar{q}$ states, since they can be easily obtained using a rotating flux tube model. Strong deviations from this linear behavior would suggest a rather different nature, and the scale of the trajectory would also indicate the scale of the mechanism responsible for the formation of the resonance.

In particular, in [1] and [2] several resonances appearing in $\pi\pi$ or $K\bar{K}$ scattering have been used, including the $\rho(770)$, $f_2(1275)$, $f_2'(1525)$, whose the resulting Regge trajectory is an ordinary linear (J, M^2) trajectory,

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whereas the $f_0(500)$ Regge trajectory does not follow the ordinary pattern [3]. Actually, at a very low energy, its trajectory resembled that of the familiar Yukawa potentials [4]. Here, we include our preliminary results on strange elastic resonances, which can be studied almost in the same way. We will see that the resulting trajectories for the $K^*(892)$, $K_1(1400)$ and $K_0^*(1430)$ are linear and with ordinary slopes, whereas the $K_0^*(800)$ shows almost the same behavior as the $f_0(500)$ in the low-energy region $M^2 < 2$ GeV^2. This might be considered an indication of its non-ordinary nature.

2. Regge trajectories

Near a resonance pole, the partial wave reads

$$t_l(s) = \frac{\beta(s)}{l-\alpha(s)} + f(l,s)\,, \tag{1}$$

where $f(l,s)$ is an analytical function near $l=\alpha(s)$. If we consider now that the pole dominates the partial wave behavior, the unitarity condition $\operatorname{Im} t_l(s) = \rho(s)|t_l(s)|^2$ implies that

$$\operatorname{Im}\alpha(s) = \rho(s)\beta(s)\,. \tag{2}$$

Furthermore, the Regge trajectory $\alpha(s)$ and residue $\beta(s)$ satisfy the Schwarz reflection principle, *i.e.*, $\alpha(s^*) = \alpha^*(s)$ and $\beta(s^*) = \beta^*(s)$. The analytic properties of $\alpha(s)$ and $\beta(s)$ and the elastic unitarity condition imply the following system of coupled dispersion relations [5,6]

$$\operatorname{Re}\alpha(s) = \alpha_0 + \alpha' s + \frac{s}{\pi}\,\mathrm{PV}\int_{4m^2}^{\infty} \mathrm{d}s' \frac{\operatorname{Im}\alpha(s')}{s'(s'-s)}\,, \tag{3}$$

$$\begin{aligned}\operatorname{Im}\alpha(s) = {}& \frac{\rho(s) b_0 \hat{s}^{\alpha_0+\alpha' s}}{\left|\Gamma\left(\alpha(s)+\frac{3}{2}\right)\right|} \exp\Bigg(-\alpha' s\left[1-\log\left(\alpha' s_0\right)\right] \\ & + \frac{s}{\pi}\,\mathrm{PV}\int_{4m^2}^{\infty} \mathrm{d}s' \frac{\operatorname{Im}\alpha(s')\log\frac{\hat{s}}{\hat{s}'} + \arg\Gamma\left(\alpha(s')+\frac{3}{2}\right)}{s'(s'-s)}\Bigg)\,,\end{aligned} \tag{4}$$

$$\begin{aligned}\beta(s) = {}& \frac{b_0 \hat{s}^{\alpha_0+\alpha' s}}{\Gamma\left(\alpha(s)+\frac{3}{2}\right)} \exp\Bigg(-\alpha' s\left[1-\log\left(\alpha' s_0\right)\right] \\ & + \frac{s}{\pi}\int_{4m^2}^{\infty} \mathrm{d}s' \frac{\operatorname{Im}\alpha(s')\log\frac{\hat{s}}{\hat{s}'} + \arg\Gamma\left(\alpha(s')+\frac{3}{2}\right)}{s'(s'-s)}\Bigg)\,,\end{aligned} \tag{5}$$

where PV denotes the principal value. For real s, the last two equations reduce to Eq. (2). The three equations are solved numerically with the free parameters fixed by demanding that the pole on the second sheet of the amplitude in Eq. (1) is at the pole positions associated to the resonance under study.

For the $K_0^*(800)$, one should also make explicit in $\beta(s)$ the Adler zero required by the chiral symmetry. In that case, b_0 will not be dimensionless.

We solve the system of Eqs. (4) and (5) iteratively. The values of the parameters are fixed by fitting only three inputs, namely, the real and imaginary parts of the resonance pole position $s_M = (M_{\rm R} - i\Gamma_{\rm R}/2)^2$, where $M_{\rm R}$ and $\Gamma_{\rm R}$ are the pole mass and width of the resonance, together with the absolute value of the pole residue $|g_M|$. Namely, we fit the resonance pole on the second Riemann sheet to: $\beta_M(s)/(l - \alpha_M(s)) \to |g_M^2|/(s - s_M)$.

The pole parameters of the $K^*(892)$ and the $K_0^*(800)$ are taken from a dispersive analysis of πK scattering [7]. For the $K_1(1400)$ and the $K_0^*(1430)$, we use a usual Breit–Wigner description, taking their values from the Review of Particle Physics [8].

In Fig. 1, we show the resulting trajectories. The resulting parameters are given in Table I. For the $K^*(892)$, $K_1(1400)$ and $K_0^*(1430)$ trajectories, the imaginary part is much smaller than the real part near the resonances, growing linearly in s with a typical slope for Regge trajectories of $\alpha' \simeq 0.9$ GeV^{-2}.

TABLE I

Parameters of the $K^*(892)$, $K_1(1400)$, $K_0^*(1430)$ and $K_0^*(800)$ Regge trajectories calculated from their poles in meson–meson scattering. For the $K_0^*(1430)$ and $K_0^*(800)$, b_0 is not dimensionless because we have factorized explicitly in $\beta(s)$ the Adler zero required by chiral symmetry. Note the similar α' for all resonances except $K_0^*(800)$.

	α_0	α' $\left[\text{GeV}^{-2}\right]$	b_0
$K^*(892)$	0.32±0.01	0.83±0.01	0.48±0.03
$K_1(1400)$	$-0.72^{+0.13}_{-0.03}$	$0.90^{+0.01}_{-0.07}$	$6.02^{+0.39}_{-1.13}$
$K_0^*(1430)$	$-1.15^{+0.23}_{-0.15}$	$0.81^{+0.08}_{-0.1}$	$4.04^{+1.26}_{-2.43}$
$K_0^*(800)$	0.28±0.02	0.15±0.01	0.44±0.04

In contrast, as one can observe in Table I, the $K_0^*(800)$ slope of the resulting curve is almost one order of magnitude smaller than that of ordinary mesons. This provides support for the non-ordinary nature of the resonance. The tiny trajectory excludes that any of the known isoscalar resonances may lie on the $K_0^*(800)$ trajectory. We have also tried to force a typical linear

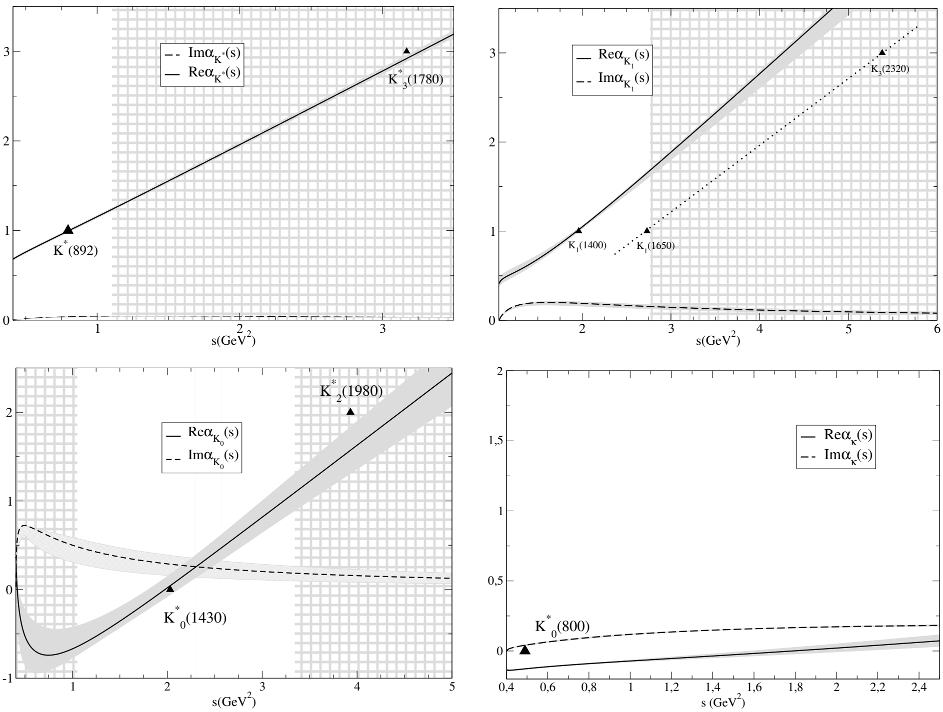

Fig. 1. Regge trajectories calculated from their poles. The continuous lines correspond to the real part of the trajectory (to be identified with spin at integer values), whereas the dashed lines stand for the imaginary parts. The gray bands cover the uncertainties in our calculation, mostly due to using the elastic approximation. The checked area is the mass region where our approach should be considered cautiously as a mere extrapolation.

trajectory for the $K_0^*(800)$, but that completely spoils the data description and the $K_0^*(800)$ pole fit. Hence, the approximation does not hold near the resonance and the method is not applicable.

In Fig. 2, we show the similarities between the $K_0^*(800)$ and the $f_0(500)$ trajectories, both also very similar to those of Yukawa potentials in non-relativistic scattering [4]. The trajectory of the $f_0(500)$ is almost equal to the $G = 2$ Yukawa curve up to $s = 2$ GeV^2, while the curve with $G = 1.5$ is near the curve of the κ. Hence, we can estimate $a_{\pi\pi} = 0.5$ GeV^{-1} and $a_{\pi K} = 0.3$ GeV^{-1} being $a_{\pi\pi}/a_{\pi K} \approx \mu_{\pi K}/\mu_{\pi\pi}$. Being the resonance a kind of a molecular state between two mesons with a short-range potential.

In summary, our formalism is able to predict the Regge trajectory of a resonance using just the parameters associated to the pole while describing the amplitude near the resonance. We have calculated the Regge trajectories

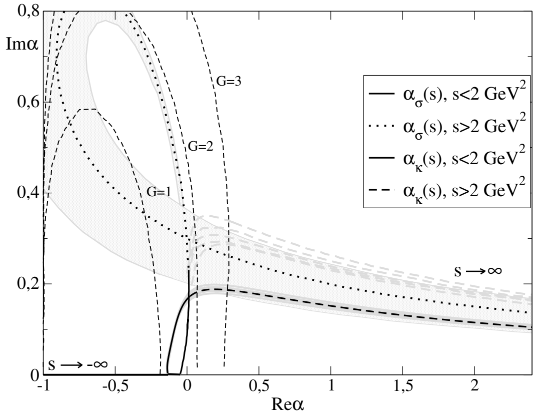

Fig. 2. At low and intermediate energies (thick continuous lines), the trajectories of the $f_0(500)$ and the $K_0^*(800)$ are similar to those of Yukawa potentials $V(r) = Ga\exp(-r/a)/r$ [4] (thin dashed lines). Beyond 2 GeV^2, we plot our results as thick discontinuous lines because they should be considered just as extrapolations.

of the $K^*(892)$, $K_1(1400)$ and $K_0^*(1430)$ obtaining similar results as in [1, 2] for the $\rho(770)$, $f_2(1270)$ and $f_2'(1525)$ typical $q\bar{q}$ behavior. However, this is not the case for the $K_0^*(800)$, which comes out to be similar to the $f_0(500)$ meson.

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