

# A COVARIANT NONLOCAL LAGRANGIAN FOR THE DESCRIPTION OF THE SCALAR KAONIC SECTOR\*

MILENA SOŁTYSIAK<sup>a</sup>, FRANCESCO GIACOSA<sup>a,b</sup>

<sup>a</sup>Institute of Physics, Jan Kochanowski University  
25-406 Kielce, Poland

<sup>b</sup>Institute for Theoretical Physics, Goethe University  
60438 Frankfurt am Main, Germany

(Received June 30, 2016)

Mesons are extended objects, hence their interaction can be described by utilizing form factors. At the Lagrangian level, one can use nonlocal interaction terms. Here, we describe two possible nonlocal Lagrangians leading to a 3D form factor: the first one is simple but does not fulfill covariance (if one insists on a 3D cutoff), the second extension is more involved but guarantees covariance. Such form factors are useful when calculating mesonic loops. As an important example, we discuss the scalar kaonic sector,  $I(J^P) = \frac{1}{2}(0^+)$ . The Lagrangian contains a single scalar kaon (the well-established state  $K_0^*(1430)$ ), but through loops  $K_0^*(800)$  emerges as a dynamically generated companion pole (which disappears in the large- $N_c$  limit).

DOI:10.5506/APhysPolBSupp.9.467

## 1. Introduction

Mesonic Lagrangians are often used to describe the decays and the spectral functions of resonances listed in the PDG [1]. Such Lagrangians make use of some symmetries of the underlying QCD (such as flavor or chiral symmetry), *e.g.* chiral perturbation theory [2] and (extended) sigma models [3]. In particular, in the case of some enigmatic resonances, such as the light  $K_0^*(800)$  meson addressed in this work, the role of mesonic loops turns out to be extremely important, see *e.g.* Refs. [4–9] and references therein.

Yet, mesons are not elementary particles but are extended objects with a radius of about 0.5 fm. Some type of form factor is needed. Already in the  $^3P_0$  model, *e.g.* Ref. [10], a form factor reducing the decay for increasing

---

\* Presented at “Excited QCD 2016”, Costa da Caparica, Lisbon, Portugal, March 6–12, 2016.

phase space is present. This form factor is also useful when calculating quantum fluctuations (loops), since all contributions become finite (see Sec. 2). It is possible to include a form factor directly in the Lagrangian by using non-local interaction terms, *e.g.* Refs. [11–13]. In many studies of mesons, a 3D form factor is employed for simplicity. This is usually regarded as a breaking of covariance. Here, we discuss how to introduce nonlocal terms which deliver the desired 3D cutoff (Sec. 3). Interestingly, an extension which preserves covariance is possible [14]. Implications for the scalar kaonic sector and conclusions are discussed in Sec. 4.

## 2. The ‘ad hoc’ introduction of a form factor

In general, effective Lagrangians contain both derivative and nonderivative interaction terms [2, 3]. A prototype of such a Lagrangian in which a (scalar) state  $S$  interacts with two (pseudoscalar) particles  $\varphi_1$  and  $\varphi_2$  reads

$$\mathcal{L}_{S\varphi_1\varphi_2} = aS\varphi_1\varphi_2 + bS\partial_\mu\varphi_1\partial^\mu\varphi_2. \quad (1)$$

The scalar kaonic sector is obtained upon identifying  $S = K_0^{*+}$  (in first approximation corresponding to  $K_0^*(1430)$ ) and  $\varphi_1 \equiv \pi^0$  and  $\varphi_2 \equiv K^-$  (other isospin combinations are here neglected). The tree-level decay width  $S \rightarrow \varphi_1\varphi_2$  as a function of the ‘running’ mass  $m$  of  $S$  reads

$$\Gamma_{S \rightarrow \varphi_1\varphi_2}(m) = \frac{|\vec{k}_1|}{8\pi m^2} \left[ a - b \frac{m^2 - m_1^2 - m_2^2}{2} \right]^2, \quad (2)$$

where  $m_1$  is the mass of  $\varphi_1$  and  $m_2$  the mass of  $\varphi_2$ . The quantity  $|\vec{k}_1|$  is the modulus of the three-momentum of (one of the) outgoing particle(s)

$$|\vec{k}_1| = \frac{\sqrt{m^4 + (m_1^2 - m_2^2)^2 - 2m^2(m_1^2 + m_2^2)}}{2m}. \quad (3)$$

The actual value of the tree-level decay width is obtained by setting  $m$  to the tree-level (nominal) mass  $m_S$  of the field  $S$ ,  $\Gamma_{S \rightarrow \varphi_1\varphi_2}(m_S)$ . Lagrangian (1), as it stands, is not suitable for loop calculations. One can regularize the theory by considering an *ad hoc* modification of both vertices via a form factor

$$a \rightarrow af_\Lambda(\vec{k}_1^2), \quad b \rightarrow bf_\Lambda(\vec{k}_1^2) \quad \text{with} \quad f_\Lambda(\vec{k}_1^2) = e^{-\vec{k}_1^2/\Lambda^2}. \quad (4)$$

The quantity  $\Lambda$  is often called ‘cutoff’ (a smooth cutoff here). The choice of a Gaussian is part of the modelling and is obviously not unique. Anyway,

there is a clear physical motivation behind  $\Lambda$ : it is the energy scale which takes into account that mesons are extended objects. The dimension of the system is roughly given by  $1/\Lambda$ . Numerically  $\Lambda$  lies between 0.5 and 1 GeV [7]. The ‘local’ limit is recovered for  $\Lambda \rightarrow \infty$ . As a consequence of Eq. (4), the decay width changes as

$$\Gamma_{S \rightarrow \varphi_1 \varphi_2}(m) \rightarrow \Gamma_{S \rightarrow \varphi_1 \varphi_2}^\Lambda(m) = \Gamma_{S \rightarrow \varphi_1 \varphi_2}(m) f_\Lambda^2(\vec{k}_1^2), \tag{5}$$

and is similar to the form factor implemented in the  $^3P_0$  model (see *e.g.* [10] and the ‘quark model’ review in Ref. [1]).

The contribution of the loop  $\Pi(m^2)$  in which the particles  $\varphi_1$  and  $\varphi_2$  circulate as calculated from the original local Lagrangian (1) reads

$$\Pi(m^2) = \int \frac{d^4 k_1}{(2\pi)^4} \frac{[a - b(k_1 \cdot k_2)]^2}{[k_1^2 - m_1^2 + i\varepsilon][k_2^2 - m_2^2 + i\varepsilon]}, \tag{6}$$

where the constraint  $k_2 = p - k_1$  is understood and  $p$  is the momentum of the unstable particle  $S$ . In its reference frame,  $p = (m, \vec{0})$ . As mentioned above, this loop contribution is divergent (with  $\Lambda^4$ ). Substitution (4) makes it convergent thanks to the form factor

$$\Pi(m^2) \rightarrow \Pi_\Lambda(m^2) = \int \frac{d^4 k_1}{(2\pi)^4} \frac{[a - b(k_1 \cdot k_2)]^2 f_\Lambda^2(\vec{k}_1^2)}{[k_1^2 - m_1^2 + i\varepsilon][k_2^2 - m_2^2 + i\varepsilon]}. \tag{7}$$

At this point, one may object that the form factor breaks covariance, since it depends on the three-momentum only. We will show in the next section that this is not necessarily the case. Once the form factor is introduced, the full propagator of the particle  $S$  is calculated as  $\Delta_S(p^2 = m^2) = [m^2 - M_0^2 + \Pi_\Lambda(m^2) + i\varepsilon]^{-1}$ , where  $M_0$  is the bare mass of  $S$  and  $\text{Im}\Pi_\Lambda(m^2) = m\Gamma_{S \rightarrow \varphi_1 \varphi_2}^\Lambda(m)$ . At the one-loop level, the Breit–Wigner mass of  $S$  is defined as  $m_S^2 - M_0^2 + \text{Re}\Pi(m_S^2) = 0$ , while the pole mass is obtained by solving the equation  $s - M_0^2 + \Pi(s) = 0$  in the complex  $s$ -plane, see *e.g.* Ref. [8, 15] for the scalar  $I = 1/2$  and  $I = 1$  sectors.

The real part of the loop can be also obtained as  $\text{Re}\Pi_\Lambda(s = m^2) = c + \frac{1}{\pi} P P \int_{m_1+m_2}^\infty \frac{\text{Im}\Pi_\Lambda(s')}{s' - s} ds'$ , where  $c$  is an additional constant due to a subtlety of QFT containing derivatives, see Ref. [15] for details. For  $\Lambda \rightarrow \infty$ ,  $\text{Im}\Pi_\Lambda(s')$  scale as  $s'^2$ , then a three-time subtracted dispersion relation would be needed. This is quite cumbersome and reinforces the viewpoint that — for the particular case of hadronic physics — a physical cutoff is a meaningful procedure.

### 3. Nonlocal Lagrangians

The modification of Eq. (4) and, consequently, the form factor in Eqs. (5) and (7) have been introduced as an *ad hoc* change of the Feynman rules. Yet, it is possible to modify the Lagrangian in order that the modified equations automatically follows from it.

**Nonlocal extension 1:** The easiest way (see Refs. [11,12]) is to consider the following nonlocal Lagrangian (for simplicity, we discuss here only the *a*-term in Eq. (1), but the *b*-term is very similar)

$$aS\varphi_1\varphi_2 \rightarrow aS \int d^4y\varphi_1(x+y/2)\varphi_2(x-y/2)\Phi(y). \quad (8)$$

The Feynman rule at the *a*-vertex is modified (upon defining  $k_1 = p/2 + q$ ,  $k_2 = p/2 - q$ , hence:  $q = (k_1 - k_2)/2$ )

$$a \rightarrow a \int d^4ye^{-iqy}\Phi(y) = a\varphi(q). \quad (9)$$

For  $\Phi(y) = \delta^{(4)}(y)$ , one reobtains the local limit. A covariant form factor requires a dependence  $\Phi(y^2)$ , hence  $\varphi(q^2)$  follows. Quite interestingly, if the masses of  $\varphi_1$  and  $\varphi_2$  are equal and on-shell,  $q^2 = -\vec{k}_1^2$ , in agreement with Eq. (5) upon setting  $\varphi(q^2) = e^{q^2/\Lambda^2}$ . However, the loop is different from Eq. (7) since it involves  $\varphi^2(q^2) = \varphi^2(4(k_1 - p)^2)$  into the integral [and not simply  $f_A^2(\vec{k}_1^2)$ ]. A generalization in the case of unequal masses is discussed in Ref. [13].

The choice  $\Phi(y) = \delta(y^0)\phi(\vec{y})$  leads to the desired result  $\varphi(q) = f_A(\vec{k}_1^2)$  of Eq. (4) and also of Eqs. (5) and (7), but it explicitly breaks covariance. This result is anyway valuable since it shows that it is possible to get the desired 3D cutoff form, but should be used only in the reference frame of the decaying particle.

**Nonlocal extension 2:** We aim to determine the Lagrangian which generates the vertex function (4) in a covariant manner [14]. We start from the general nonlocal expression

$$aS\varphi_1\varphi_2 \rightarrow g \int d^4zd^4y_1d^4y_2S(x+z)\varphi_1(x+y_1)\varphi_2(x+y_2)\Phi(z, y_1, y_2), \quad (10)$$

where the vertex function  $\Phi(z, y_1, y_2)$  in position space has been introduced. The case  $\Phi(z, y_1, y_2) = \delta(z)\delta(y_1)\delta(y_2)$  delivers the local limit (1). The vertex function in momentum space is given by

$$\varphi(p, k_1, k_2) = \int d^4zd^4y_1d^4y_2e^{ipz}e^{-ik_1y_1}e^{-ik_2y_2}\Phi(z, y_1, y_2).$$

Here, we assume that  $\Phi(z, y_1, y_2)$  is such that

$$\varphi(p, k_1, k_2) = \varphi\left(p, q = \frac{k_1 - k_2}{2}\right) = f_\Lambda\left(\frac{q^2 p^2 - (q \cdot p)^2}{p^2}\right). \quad (11)$$

It respects covariance because the final form factor is a function of Lorentz products. Nevertheless, in the rest frame of  $S$ , one recovers the desired dependence

$$\frac{q^2 p^2 - (q \cdot p)^2}{p^2} = \vec{k}_1^2 \quad (\text{for } p = (m, 0)).$$

Note, in order to get the desired expression,  $\Phi(z, y_1, y_2) = \xi(z, y_1 - y_2)\delta(y_1 + y_2)$ , out of which  $\varphi(p, k_1, k_2) = \int d^4 z d^4 y e^{ipz} e^{-i2qy} \xi(z, y)$  with  $y = y_1 - y_2$  and  $Y = y_1 + y_2$ . This line of reasoning shows that it is — at least in principle — possible to reconcile covariance with a 3D form factor in the rest frame of the decaying particle.

#### 4. Discussions and conclusions

In this work, we have discussed form factors entering in mesonic interaction terms. We have started from a local Lagrangian and we have *ad hoc* modified the interaction vertex by introducing a function of the three-momentum of one outgoing particle. Then, we have presented two nonlocal extensions of the Lagrangian that deliver the desired expressions without the need of modifying by hand the Feynman rules. The first nonlocal extension is relatively simple but delivers the desired form factor at the price of breaking covariance. The second extension is more involved but delivers a 3D cutoff in the rest frame of the decaying particle by respecting covariance.

Recently, the expressions of the propagator described in Sec. 2 (which are then justified by our study of Sec. 3) were used to study the positions of the poles in the complex plane in both the isodoublet- and isovector-scalar sectors. In both cases, the Lagrangians contains only one seed state, corresponding to the quark–antiquark resonances  $a_0(1450)$  and  $K_0^*(1430)$ , respectively. Yet, the loops (with the necessary presence of a form factor) are strong enough to generate two additional resonances:  $a_0(980)$  and  $K_0^*(800)$ . These are companion poles, and hence dynamically generated states. Moreover, these states disappear in the large- $N_c$  limit, confirming their nonconventional nature. In particular,  $K_0^*(800)$  is not yet confirmed in the PDG [1]. The study based on the formalism described in these proceedings unequivocally finds a pole:  $(0.745 \pm 0.029) - i(0.263 \pm 0.027)$  GeV [8]. This is in agreement with other works, *e.g.* Ref. [9] and references therein, and reinforces the need of accepting this state in the PDG.

In both Refs. [8, 15], one obtains a similar values of the cutoff:  $\Lambda \simeq 0.6$  GeV. Moreover, in Ref. [8], different form factors have been tested. It is found that they cannot fit the pion–kaon scattering data as well as the

Gaussian form factor. Thus, even if there is, in principle, no fundamental reason behind a Gaussian function, it nevertheless seems to be the best choice in order to describe hadronic phenomenology.

We thank T. Wolkanowski and G. Pagliara for valuable discussions. F.G. thanks also for past conversations on the topic with Th. Gutsche and V. Lyubovitskij.

## REFERENCES

- [1] K.A. Olive *et al.* [Particle Data Group], *Chin. Phys. C* **38**, 090001 (2014).
- [2] S. Scherer, *Adv. Nucl. Phys.* **27**, 277 (2003) [arXiv:hep-ph/0210398].
- [3] D. Parganlija *et al.*, *Phys. Rev. D* **87**, 014011 (2012) [arXiv:1208.0585 [hep-ph]]; S. Janowski, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **90**, 114005 (2014) [arXiv:1408.4921 [hep-ph]].
- [4] E. van Beveren *et al.*, *Z. Phys. C* **30**, 615 (1986) [arXiv:0710.4067 [hep-ph]].
- [5] J.A. Oller, E. Oset, J.R. Peláez, *Phys. Rev. D* **59**, 074001 (1999) [arXiv:hep-ph/9804209]; J.R. Peláez, *Phys. Rev. Lett.* **92**, 102001 (2004) [arXiv:hep-ph/0309292].
- [6] D. Black, A.H. Fariborz, F. Sannino, J. Schechter, *Phys. Rev. D* **58**, 054012 (1998) [arXiv:hep-ph/9804273].
- [7] F. Giacosa, G. Pagliara, *Phys. Rev. C* **76**, 065204 (2007) [arXiv:0707.3594 [hep-ph]]; F. Giacosa, T. Wolkanowski, *Mod. Phys. Lett. A* **27**, 1250229 (2012) [arXiv:1209.2332 [hep-ph]].
- [8] T. Wolkanowski, M. Sołtysiak, F. Giacosa, *Nucl. Phys. B* **909**, 418 (2016) [arXiv:1512.01071 [hep-ph]].
- [9] A.H. Fariborz, E. Pourjafarabadi, S. Zarepour, S.M. Zebarjad, *Phys. Rev. D* **92**, 113002 (2015) [arXiv:1511.01623 [hep-ph]].
- [10] C. Amsler, F.E. Close, *Phys. Rev. D* **53**, 295 (1996) [arXiv:hep-ph/9507326].
- [11] Y.V. Burdanov, G.V. Efimov, S.N. Nedelko, S.A. Solunin, *Phys. Rev. D* **54**, 4483 (1996) [arXiv:hep-ph/9601344]; F. Giacosa, T. Gutsche, A. Faessler, *Phys. Rev. C* **71**, 025202 (2005) [arXiv:hep-ph/0408085]; R.D. Bowler, M.C. Birse, *Nucl. Phys. A* **582**, 655 (1995) [arXiv:hep-ph/9407336]; F. Giacosa, “Glueball Phenomenology within a Nonlocal Approach”, Ph.D. Thesis, Tuebingen, 2005.
- [12] A. Faessler *et al.*, *Phys. Rev. D* **68**, 014011 (2003) [arXiv:hep-ph/0304031].
- [13] A. Faessler, T. Gutsche, V.E. Lyubovitskij, Y.L. Ma, *Phys. Rev. D* **76**, 014005 (2007) [arXiv:0705.0254 [hep-ph]]; **76**, 114008 (2007) [arXiv:0709.3946 [hep-ph]].
- [14] F. Giacosa, G. Pagliara, *Phys. Rev. D* **88**, 025010 (2013) [arXiv:1210.4192 [hep-ph]].
- [15] T. Wolkanowski, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **93**, 014002 (2016) [arXiv:1508.00372 [hep-ph]].